Global Currency Hedging

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Abstract

This paper considers the risk management problem of an investor who holds a diversified portfolio of global equities and chooses long or short positions in currencies to manage the risk of the equity portfolio. Over the period 1975-2005, we find that the Australian dollar, Canadian dollar, Japanese yen, and British pound are positively correlated with their domestic stock markets and with the global equity market, but the euro, the Swiss franc, and especially the US dollar are negatively correlated with global equities. These correlations imply that risk-minimizing global investors should short the Australian and Canadian dollars, yen, and pound but should hold long positions in the US dollar, the euro, and the Swiss franc. Accordingly, US investors should overhedge foreign equity positions except those in euro countries, which should only be partially hedged. These conclusions are robust to variations in the investment horizon. In the past 15 years the negative equity correlation of the dollar appears to have weakened slightly, while that of the euro has strengthened.

JEL classification: G12.

1 Introduction

What role should foreign currency play in a diversified investment portfolio? In practice, many investors appear reluctant to hold foreign currency directly, perhaps because they see currency as an investment with high volatility and low average return. At the same time, many investors hold indirect positions in foreign currency when they buy foreign equities and fail to hedge the currency exposure implied by the equity holding. Such investors receive the excess return on foreign equities over foreign bills—the foreign-currency excess return on foreign equity—plus the return on foreign bills, that is, the return on foreign currency.

The academic finance literature has explored a number of reasons why investors might want to hold foreign currency. These can be divided into speculative demands, resulting from positive expected excess returns on foreign currency over the minimum-variance portfolio, and risk management demands, resulting from covariances of foreign currency with other assets that investors may wish to hold.[2]

Obviously it is possible that a particular currency may have a high expected return at a particular time, generating a speculative demand for that currency. For example, the literature on the forward premium puzzle shows that currencies with high short-term interest rates deliver high returns on average. This type of speculative demand is inherently asymmetric. For every currency with a high expected return, there must be another with a low expected return, and investors will tend to short currencies with low expected returns just as they go long those currencies with high expected returns. Investors whose domestic currency has a low expected return will tend to go long all foreign currencies and short their own, but investors whose domestic currency has a high expected return will tend to short foreign currencies.

A unique feature of currencies, however, is that investors in each country can simultaneously perceive positive expected excess returns on foreign currencies over their own domestic currencies. That is, a US investor can perceive a positive expected excess return on euros over dollars, while a European investor can at the same time perceive a positive expected excess return on dollars over euros. This possibility arises from Jensen's inequality and is known as the Siegel paradox (Siegel 1972). It can

[2]Risk management demands are more commonly called hedging demands, but this can create confusion in the context of foreign currency because hedging a foreign currency corresponds to taking a short position to cancel out an implicit long position in that currency. In this paper we use foreign currency terminology and avoid the use of the term hedging demand for assets.

explain symmetric speculative demand for foreign currency by investors based in all countries. In practice, however, the currency demand generated by this effect is quite modest. If currency movements are lognormally distributed and the expected excess *log* return on foreign currency over domestic currency is zero (a condition that can be satisfied for all currency pairs simultaneously), then the expected excess *simple* return on foreign currency is one-half the variance of the foreign currency return. With a foreign currency standard deviation of about 10% per year, the expected excess foreign currency return is 50 basis points and the corresponding Sharpe ratio is only 5%. If no other risky investments were available, an investor with log utility would put half her portfolio in foreign currency, but a conservative investor with relative risk aversion of 5 would have only a 10% portfolio weight on foreign currency.

Since conservative investors have small speculative currency demands, their foreign currency holdings are primarily explained by their desire to manage portfolio risks. One type of risk management demand arises if there is no domestic asset that is riskless in real terms, for example because only nominal bills are available and there is uncertainty about the rate of inflation. In this case, the minimum-variance portfolio may contain foreign currency (Adler and Dumas 1983). This effect can be substantial in countries with extremely volatile inflation, such as some emerging markets, but is quite small in developed countries over short time intervals. Campbell, Viceira, and White (2003) show that it can be more important for investors with long time horizons, because nominal bills subject investors to fluctuations in real interest rates, while nominal bonds subject them to inflation uncertainty which is relatively more important at longer horizons. If domestic inflation-indexed bonds are available, however, they are riskless in real terms if held to maturity and thus drive out foreign currency from the minimum-variance portfolio.

Another type of risk management demand for foreign currency arises if an investor holds other assets for speculative reasons, and foreign currency is correlated with those assets. For example, an investor may wish to hold a globally diversified equity portfolio. If the foreign-currency excess return on foreign equities is negatively correlated with the return on the foreign currency (as would be the case, for example, if stocks are real assets and the shocks to foreign currency are primarily related to foreign inflation), then an investor holding foreign equities can reduce portfolio risk by holding a long position in foreign currency.

In this paper we explore the particular demand for foreign currency that results from the desire to manage equity risks. We assume that a domestic asset exists that is riskless in real terms, so that an infinitely conservative investor would hold only this asset and would hold neither equity nor foreign currency. We consider an investor with a given portfolio of equities, and we ask what foreign currency positions this investor should hold in order to minimize the risk of the total portfolio. We consider seven major currencies, the dollar, euro, Japanese yen, Swiss franc, pound sterling, Canadian dollar and Australian dollar, over the period 1975–2002. (Before 1999, we use the German deutschmark in place of the euro.) We consider investment horizons ranging from one month to four years.

We find that our seven currencies can be divided into two groups. The yen, pound, Canadian dollar and Australian dollar are positively correlated with the world equity market, and particularly with the Japanese, British, Canadian and Australian equity markets measured in local-currency terms. These correlations could result from shocks to fundamentals that affect both the profitability of corporations and the fiscal positions of the governments in these countries; or from capital flows, driven by investor sentiment, that move these equity markets jointly with their currency markets; or from the effects of exchange rate movements on the costs and output prices of corporations (Pavlova and Rigobon 2003). The implied portfolio demands for these currencies are negative. Investors with diversified international equity positions should short these currencies in order to minimize overall portfolio risk.

The dollar, and to a lesser extent the Swiss franc and the euro, behave differently. These currencies are negatively correlated with the world equity market and almost uncorrelated with their own domestic equity markets. It is striking that the dollar, the Swiss franc, and the euro are widely used as reserve currencies by central banks, and more generally as stores of value by corporations and individuals around the world. The correlations we observe in the data are consistent with the idea that shocks to risk aversion drive down equity prices and drive up the values of the major reserve currencies. The implied portfolio demands for these currencies are positive. Risk-minimizing investors with diversified international equity positions should hold long positions in these currencies.

Many international equity investors think not about the foreign currency positions they would like to hold, but about the currency hedging strategy they should follow. An unhedged position in international equity corresponds to a long position in foreign currency equal to the equity holding. A fully hedged position corresponds to a net zero position in foreign currency. When currencies and equities are uncorrelated, full hedging is optimal (Solnik 1974). Our empirical results imply that equity investors should more than fully hedge the yen, pound, and Australian and Canadian dollars to achieve net short positions, but should less than fully hedge the dollar, euro, and Swiss franc to maintain net long positions in these currencies.

The organization of the paper is as follows. Section 2 lays out the analytical framework we use for our empirical analysis. We begin by defining returns on internationally diversified portfolios of equities and currencies, then show how to work with log (continuously compounded) returns over short time intervals. We state and solve the problem of choosing currency positions to minimize portfolio variance, given a set of equity holdings. Importantly, we show conditions under which variance-minimizing currency positions do not depend on the base currency of the investor. Section 3 presents empirical results for different equity portfolios, sets of available currencies, investment horizons, and sample periods. Section 4 concludes.

2 Portfolio Choice with Multiple Equities and Currencies

We consider the problem of a domestic investor who invests in stocks from n foreign countries as well as in domestic stocks, and must decide how much currency risk she wants to hedge or, equivalently, her currency exposure. The investor adjusts her exposure to foreign currencies by entering into forward exchange rate contracts or, equivalently, by borrowing and lending in her own currency and in foreign currencies. For convenience, throughout this section we set the domestic country to be the US, and hence refer to the domestic investor as a US investor, and to the domestic currency as the dollar.

In our analysis, we assume that the investor has one-period mean-variance preferences over the currency composition of her portfolio, and that she chooses her optimal exposure to foreign currencies taking as given the composition of her equity portfolio. We make these assumptions about preferences and about optimization both because of tractability reasons and also because they reflect common practice at institutional investors. In future research we would like to relax them, and allow for simultaneous choice of equity portfolio weights and currency ratios under more general preferences, along the lines of the models in Campbell, Chan, and Viceira (2003) and Jurek and Viceira (2005).

2.1 Portfolio returns with currency hedging

Let $R_{c,t+1}$ denote the gross return in currency c from holding country c stocks from the beginning to the end of period $t+1$, and let $S_{c,t+1}$ denote the spot exchange rate in dollars per foreign currency c at the end of period $t+1$. By convention, we index the domestic country by $c=1$ and the n foreign countries by $c=2,...,n+1$. Of course, the domestic exchange rate is constant over time and equal to 1: $S_{1,t+1}=1$ for all t.

At time t, the investor exchanges a dollar for $1/S_{c,t}$ units of currency c in the spot market which she then invests in the stock market of country c. After one period, stocks from country c return $R_{c,t+1}$, which the US investor can exchange for $S_{c,t+1}$ dollars, to earn an unhedged gross return of $R_{c,t+1}S_{c,t+1}/S_{c,t}$. For an arbitrarily weighted portfolio, the unhedged gross portfolio return is given by

$$R^{uh}_{p,t+1} = \mathbf{R}'_{t+1}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right),$$

where $\boldsymbol{\omega}_t = \text{diag}\left(\omega_{1,t}, \omega_{2,t}, ..., \omega_{n+1,t}\right)$ is the $(n+1 \times n+1)$ diagonal matrix of weights on domestic and foreign stocks at time t, $\mathbf{R}_{t+1}$ is the $(n+1\times 1)$ vector of gross nominal stock returns in local currencies, $\mathbf{S}_{t+1}$ is the $(n+1\times 1)$ vector of spot exchange rates, and $\div$ denotes the element-by-element ratio operator, so that the c-th element of $(\mathbf{S}_{t+1} \div \mathbf{S}_t)$ is $S_{c,t+1}/S_{c,t}$. The weights add up to 1 in each period t:

$$\sum_{c=1}^{n+1} \omega_{c,t} = 1 \qquad \forall t. \tag{1}$$

We next consider the hedged portfolio. Let $F_{c,t}$ denote the one-period forward exchange rate in dollars per foreign currency c,[3] and $\theta_{c,t}$ the dollar value of the amount of forward exchange rate contracts for currency c the investor enters into at time t per dollar invested in her stock portfolio. At the end of period $t+1$, the investor gets to exchange $\theta_{c,t}/S_{c,t}$ units of the foreign-currency denominated return $R_{c,t+1}\omega_{c,t}/S_{c,t}$ back into dollars at an exchange rate $F_{c,t}$. She then exchanges the rest, which amounts to $(R_{c,t+1}\omega_{c,t}/S_{c,t} - \theta_{c,t}/S_{c,t})$ units of foreign currency c, at the spot exchange rate $S_{c,t+1}$. Collecting returns for all countries leads to a hedged portfolio return $R^h_{p,t+1}$ of

$$R^h_{p,t+1} = \mathbf{R}'_{t+1}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \boldsymbol{\Theta}'_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) + \boldsymbol{\Theta}'_t\left(\mathbf{F}_t \div \mathbf{S}_t\right), \tag{2}$$

[3] That is, at the end of month t, the investor can enter into a forward contract to sell one unit of currency c at the end of month $t+1$ for a forward price of $F_{c,t}$ dollars.

where $\mathbf{F}_t$ is the $(n+1 \times 1)$ vector of forward exchange rates, and $\boldsymbol{\Theta}_t = (\theta_{1,t}, \theta_{2,t}, ..., \theta_{n,t}, \theta_{n+1,t})'$. Of course, since $S_{1t} = F_{1,t} = 1$ for all t, the choice of domestic hedge ratio $\theta_{1,t}$ is arbitrary. For convenience, we set it so that all hedge ratios add up to 1:

$$\theta_{1,t} = 1 - \sum_{c=2}^{n+1} \theta_{c,t}. \tag{3}$$

Under covered interest parity, the forward contract for currency c trades at $F_{c,t} = S_{c,t}(1 + I_{1,t})/(1 + I_{c,t})$, where $I_{1,t}$ denotes the domestic nominal short-term riskless interest rate available at the end of period t, and $I_{c,t}$ is the corresponding country c nominal short-term interest rate. Thus the hedged dollar portfolio return (2) can be written as

$$R^h_{p,t+1} = \mathbf{R}'_{t+1}\boldsymbol{\omega}_t \left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \boldsymbol{\Theta}'_t \left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) + \boldsymbol{\Theta}'_t \left[\left(\mathbf{1} + \mathbf{I}^d_t\right) \div \left(\mathbf{1} + \mathbf{I}_t\right)\right], \tag{4}$$

where $\mathbf{I}_t = (I_{1,t}, I_{2,t}..., I_{n+1,t})$ is the $(n + 1 \times 1)$ vector of nominal short-term interest rates and $\mathbf{I}^d_t = I_{1,t}\mathbf{1}$.

Equation (4) shows that selling currency forward—i.e., setting $\theta_{c,t} > 0$—is analogous to a strategy of shorting foreign bonds and holding domestic bonds, i.e. borrowing in foreign currency and lending in domestic currency.[4] That the hedged portfolio includes long and short positions in domestic and foreign bonds is intuitive. A long foreign stock position implies a long position in the currency of that country; thus an investor can hedge this currency exposure by simultaneously shorting bonds denominated in that currency and investing the proceeds in bonds denominated in her domestic currency.

By convention, an investor is said to fully hedge the currency risk exposure in her foreign stock portfolio when she sets $\theta_{c,t} = \omega_{c,t}$. Note that when $\omega_{c,t} > 0$, full currency hedging of the stock position implies that the investor shorts currency c one for one with the currency position implicit in her long stock market investment in

[4]Note, however, that the two strategies are not completely equivalent except in the continuous time limit. Let us write the hedged return for an investor borrowing $\boldsymbol{\Theta}_{c,t}$ dollars (i.e. shorting bonds) in foreign currency c and lending $\boldsymbol{\Theta}_{c,t}$ dollars in domestic currency (i.e. holding domestic bonds) for each dollar invested in her stock portfolio. The return on this strategy is

$$R^{BL}_{p,t+1} = \mathbf{R}'_{t+1}\boldsymbol{\omega}_t \left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \boldsymbol{\Theta}'_t \left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right)\left(1 + \mathbf{I}_t\right) + \boldsymbol{\Theta}'_t \left(1 + \mathbf{I}^d_t\right),$$

which is slightly different from that of an investor hedging through forward contracts. We show in the appendix that, in continuous time, the two strategies are exactly equivalent.

country c at time t. Of course, the investor has not literally fully hedged all currency risk in her foreign stock investment, because this position will fluctuate with the realized return at time $t+1$. For example, if the stock return is positive, the units of currency c held by the investor at time $t+1$ will exceed $\omega_{c,t}/S_{c,t}$. The investor then benefits if the exchange rate has increased, and loses otherwise. It is also important to note that currency hedging instruments, whether bonds or forward contracts, are imperfect because they imply an exposure to the foreign risk-free interest rate that cannot be separated from the pure exchange rate risk. Similarly, the investor is said to under-hedge currency risk when $\theta_{c,t} < \omega_{c,t}$, and to over-hedge when $\theta_{c,t} > \omega_{c,t}$.

To capture the fact that the investor can alter the currency exposure implicit in her foreign stock position using forward contracts or lending and borrowing, we now define a new variable $\psi_{c,t}$ as $\psi_{c,t} \equiv \omega_{c,t} - \theta_{c,t}$. A fully hedged portfolio, in which the investor does not hold any exposure to currency c, corresponds to $\psi_{c,t} = 0$. A positive value of $\psi_{c,t}$ means that the investor wants to hold exposure to currency c, or equivalently that the investor does not want to fully hedge the currency exposure implicit in her stock position in country c. Of course, a completely unhedged portfolio corresponds to $\psi_{c,t} = \omega_{c,t}$. Thus $\psi_{c,t}$ is a measure of currency demand or currency exposure. Accordingly we refer to $\psi_{c,t}$ as currency demand or currency exposure indistinctly.

For convenience, we now rewrite equation (4) in terms of currency demands:

$$\begin{aligned} R^h_{p,t+1} &= \mathbf{R}'_{t+1}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \mathbf{1}'\boldsymbol{\omega}_t\left[\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \left(\mathbf{1} + \mathbf{I}^d_t\right) \div \left(\mathbf{1} + \mathbf{I}_t\right)\right] \\ &\quad + \boldsymbol{\Psi}'_t\left[\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \left(\mathbf{1} + \mathbf{I}^d_t\right) \div \left(\mathbf{1} + \mathbf{I}_t\right)\right], \end{aligned}$$

where $\boldsymbol{\Psi}_t = \left(\psi_{1,t}, \psi_{2,t}, ..., \psi_{n+1,t}\right)'$.

Note that $\boldsymbol{\Psi}_t = \boldsymbol{\omega}_t\mathbf{1} - \boldsymbol{\Theta}_t$. Given the definition of $\psi_{c,t}$, equations (1) and (3) imply that

$$\psi_{1,t} = -\sum_{c=2}^{n+1}\psi_{c,t}. \tag{5}$$

or $\Psi'_t\mathbf{1} = \mathbf{0}$, so that $\psi_{1,t}$ indeed represents the domestic currency exposure. That currency demands must add to zero is intuitive. Since the investor is fully invested in stocks, she can achieve a long position in a particular currency c only by borrowing—or equivalently, by shorting bonds—in her own domestic currency, and investing the proceeds in bonds denominated in that currency. Thus the currency portfolio is a zero investment portfolio. Section 2.2 next develops this point in more detail.

2.2 Log portfolio returns over short time intervals

For convenience, we work with log (or continuously compounded) returns, interest rates, and exchange rates, which we denote with lower case letters. To this end, we compute a log version of equation (4) which holds exactly in the continuous time limit where investors adjust their hedge ratios continuously, and it is approximate otherwise. We show in the appendix that the continuously compounded (or log) hedged portfolio excess return over the domestic interest rate is approximately equal to

$$r^h_{p,t+1} - i_{1,t} = \mathbf{1}'\boldsymbol{\omega}_t \left(\mathbf{r}_{t+1} - \mathbf{i}_t\right) + \boldsymbol{\Psi}'_t \left(\Delta\mathbf{s}_{t+1} + \mathbf{i}_t - \mathbf{i}^d_t\right) + \frac{1}{2}\Sigma^h_t, \tag{6}$$

where bold case letters denote the column vector of $(n+1)$ country observations, and small case letters denote logs. Thus $\mathbf{r}_{t+1} = \log\left(\mathbf{R}_{,t+1}\right)$, $\Delta\mathbf{s}_{t+1} = \log\left(\mathbf{S}_{t+1}\right) - \log\left(\mathbf{S}_t\right)$, and $\mathbf{i}_t = \log\left(1 + \mathbf{I}_t\right)$ and $\mathbf{i}^d_t = \log\left(1 + I_{1,t}\right)\mathbf{1}$.

Equation (6) provides an intuitive decomposition of the hedged portfolio excess return. The first term represents the excess return on a fully hedged stock portfolio. The second term involves only the vector of excess returns on currencies, $\Delta\mathbf{s}_{t+1}+\mathbf{i}_t-\mathbf{i}^d_t$, and thus represents pure currency exposure. Recall that $\psi_{c,t}$ is the position taken in currency c in excess of perfect hedging, for $c = 1, 2..., n+1$. Of course, this term vanishes when the investor chooses to avoid currency exposure and sets $\boldsymbol{\Psi}_t$ to a vector of zeroes. Finally, the third term in equation (6) is a Jensen's variance correction equal to

$$\begin{aligned} \Sigma^h_t \;=\; & \mathbf{1}'\boldsymbol{\omega}_t \operatorname{diag}\left(\operatorname{Var}_t\left(\Delta\mathbf{s}_{t+1} - \mathbf{i}^d_t + \mathbf{i}_t\right)\right) - \left(\boldsymbol{\omega}_t\mathbf{1} - \Psi_t\right)' \operatorname{diag}\left(\operatorname{Var}_t\left(\Delta\mathbf{s}_{t+1} - \mathbf{i}^d_t + \mathbf{i}_t\right)\right) \\ & - \operatorname{Var}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right) + \boldsymbol{\Psi}'_t\left(\Delta\mathbf{s}_{t+1} - \mathbf{i}^d_t + \mathbf{i}_t\right)\right). \end{aligned} \tag{7}$$

2.3 Mean-variance optimization

We consider the optimal currency exposure for a given stock portfolio. In terms of the expression for log hedged portfolio return (6), we assume that the vector $\boldsymbol{\omega}_t$ of portfolio weights is given, and that the choice variable is Ψ_t, the vector of currency demands. More specifically, we assume that the investor optimally chooses each period t a vector of currency demands

$$\widetilde{\boldsymbol{\Psi}}_t = \left(\psi_{2,t}, ..., \psi_{n+1,t}\right)'$$

to minimize the conditional variance of the log excess return on the hedged portfolio over that period, subject to a constraint on the expected return. Note that the demand for domestic currency $\psi_{1,t}$ is not included because it is given once the other currency demands are determined.

Formally, the investor solves the following mean-variance problem:

$$\begin{aligned} & \min_{\widetilde{\Psi}_t} \frac{1}{2} \operatorname{Var}_t \left(r^h_{p,t+1} - i_{1,t} \right) \\ \text{s.t.} \quad & \mathrm{E}_t \left(r^h_{p,t+1} - i_{1,t} \right) + \frac{1}{2} \operatorname{Var}_t \left(r^h_{p,t+1} - i_{1,t} \right) = \mu^h_p. \end{aligned}$$

The Lagrangian associated with this problem is

$$\begin{aligned} \pounds \left(\widetilde{\Psi}_t \right) &= \frac{1}{2} \operatorname{Var}_t \left(r^h_{p,t+1} \right) + \lambda \left[\mu^h_p - \mathrm{E}_t \left(r^h_{p,t+1} - i_{1,t} \right) - \frac{1}{2} \operatorname{Var}_t \left(r^h_{p,t+1} \right) \right] \\ &= \frac{1}{2} (1 - \lambda) \operatorname{Var}_t \left(r^h_{p,t+1} \right) + \lambda \left[\mu^h_p - \mathrm{E}_t \left(r^h_{p,t+1} - i_{1,t} \right) \right], \end{aligned}$$

where the multiplier λ is typically interpreted as a measure of the investor's risk tolerance.

Simple algebraic manipulation of the problem shown in the appendix leads to the following vector of optimal mean-variance currency demands:

$$\begin{aligned} \widetilde{\mathbf{\Psi}}^*_t (\lambda) &= \lambda \operatorname{Var}_t \left(\widetilde{\Delta \mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t \right)^{-1} \left[\mathrm{E}_t \left(\widetilde{\Delta \mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t \right) + \frac{1}{2} \operatorname{diag} \left(\operatorname{Var}_t \widetilde{\Delta \mathbf{s}}_{t+1} \right) \right] \\ & \quad - \operatorname{Var}_t \left(\widetilde{\Delta \mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t \right)^{-1} \left[\operatorname{Cov}_t \left(\mathbf{1}' \boldsymbol{\omega}_t \left(\mathbf{r}_{t+1} - \mathbf{i}_t \right), \left(\widetilde{\Delta \mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t \right) \right) \right] \end{aligned} \tag{8}$$

where we denote by $\widetilde{M}$ the $(n \times m)$ submatrix that selects rows 2 to $n+1$ of the corresponding $(n+1 \times m)$ matrix M, i.e., $\widetilde{M}$ includes the values of M corresponding to foreign countries only.

To build intuition, we also consider a constrained case in which the investor chooses identical demand ratios across all currencies. In that case $\psi_{c,t} = \psi_t \; \forall c$ and $\widetilde{\Psi}_t = \psi_t \widetilde{\mathbf{1}}$.

The appendix shows that the solution to this constrained case is

$$\begin{aligned}\psi_t^*(\lambda) &= \lambda\left[\frac{\mathbf{1}'\,\mathrm{E}_t\left(\widetilde{\Delta\mathbf{s}}_{t+1}+\widetilde{\mathbf{i}}_t-\widetilde{\mathbf{i}}_t^d\right)+\frac{1}{2}\mathbf{1}'\,\mathrm{diag}\left(\mathrm{Var}_t\,\widetilde{\Delta\mathbf{s}}_{t+1}\right)}{\mathbf{1}'\,\mathrm{Var}_t\left(\widetilde{\Delta\mathbf{s}}_{t+1}+\widetilde{\mathbf{i}}_t-\widetilde{\mathbf{i}}_t^d\right)\mathbf{1}}\right] \\ &\quad -\frac{\mathbf{1}'\,\mathrm{Cov}_t\left(\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\widetilde{\Delta\mathbf{s}}_{t+1}+\widetilde{\mathbf{i}}_t-\widetilde{\mathbf{i}}_t^d\right)\mathbf{1}}{\mathbf{1}'\,\mathrm{Var}_t\left(\widetilde{\Delta\mathbf{s}}_{t+1}+\widetilde{\mathbf{i}}_t-\widetilde{\mathbf{i}}_t^d\right)\mathbf{1}}. \end{aligned} \tag{9}$$

Equations (8) and (9) show that the optimal mean-variance demand for currency has two components that correspond to two possible motives to take on currency risk. The first component is a speculative demand that is proportional to the expected excess currency return. The investor wants to hold currency risk in proportion to the Sharpe ratio of the excess return on foreign currency over the domestic interest rate, and in proportion to her risk tolerance λ.

The speculative component of currency demand is zero when the expected excess return on foreign currency over domestic bonds is zero or, equivalently, when uncovered interest parity (UIP) holds. To see this, note that UIP implies that the forward rate $F_{c,t}$ is an unbiased predictor of the spot rate $S_{c,t+1}$,

$$\mathrm{E}_t\left(S_{c,t+1}\right)=F_{c,t}=S_{c,t}\left(1+I_{1,t}\right)/\left(1+I_{c,t}\right),\qquad c=1,...,n+1, \tag{10}$$

which we can rewrite in logs and in vector form as

$$\mathrm{E}_t\left(\mathbf{s}_{t+1}\right)=\mathbf{f}_t=\mathbf{s}_t+\mathbf{i}_t^d-\mathbf{i}_t-\frac{1}{2}\,\mathrm{diag}\left(\mathrm{Var}_t\left(\mathbf{s}_{t+1}\right)\right). \tag{11}$$

When equation (11) holds, the term in brackets in (8) and (9) is zero.

It is important to note that UIP as we have defined it in (10) cannot hold simultaneously for all base currencies. This is known as Siegel's paradox (Siegel 1972); it results from the facts that an exchange rate is a ratio of two prices, and that the expectation of the inverse of a ratio differs from the inverse of the expectation of that ratio when there is uncertainty. Thus speculative demand cannot be zero for all base currencies.

The second component of currency demand corresponds to a risk management (RM) demand for currency aimed at minimizing total portfolio return volatility regardless of expected return. For convenience, we rewrite this component of currency demand separately as

$$\widetilde{\boldsymbol{\Psi}}^{*}_{RM,t} = -\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t\right)^{-1}\left[\operatorname{Cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right), \left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t\right)\right)\right]. \tag{12}$$

In the constrained case $\widetilde{\Psi}_t = \psi_t\widetilde{\mathbf{1}}$, RM currency demand takes the form

$$\psi^{*}_{RM,t} = -\frac{\mathbf{1}'\operatorname{Cov}_t\left(\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right), \widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t\right)\mathbf{1}}{\mathbf{1}'\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t\right)\mathbf{1}}. \tag{13}$$

Equations (12) and (13) show that, for given portfolio weights, $\widetilde{\Psi}^{*}_{RM,t}$ is proportional to the negative of the covariance between stock returns and exchange rates. If stock returns and exchange rates are uncorrelated, the RM component of currency demand is zero. In this case holding currency exposure adds volatility to the investor's portfolio and, unless this volatility is compensated, the investor is better off by holding no currency exposure at all or, equivalently, by fully hedging her portfolio.

If stock returns and exchange rates are positively correlated, the domestic currency tends to appreciate when the foreign stock market falls. Thus the investor can reduce portfolio return volatility by over-hedging, that is, by shorting foreign currency in excess of what would be required to fully hedge the currency exposure implicit in her stock portfolio. Conversely, a negative correlation between stock returns and exchange rates implies that the foreign currency appreciates when the foreign stock market falls. Thus the investor can reduce portfolio return volatility by under-hedging, that is, by holding foreign currency.

In our subsequent empirical analysis, we ignore the speculative component of currency demand, and instead focus exclusively on the risk management component of currency demand (12) and (13). We ignore the speculative component of currency demand for two reasons. First, this demand depends on expected excess returns on currencies, which are notoriously difficult to estimate. Second, many institutional investors do not have a strong opinion about the expected excess return on currencies, and instead are primarily interested in determining the degree of currency exposure that minimizes portfolio return volatility. That is, they are exclusively interested in the RM component of currency demand. In the rest of the paper we will refer to the RM component of currency demand simply as optimal currency demand or currency exposure.

2.4 From conditional to unconditional moments

Our empirical analysis is based on the estimation of optimal currency demands for a set of stock portfolios and currencies. To facilitate the estimation of optimal currency demands, we make some additional assumptions about the conditional moments of stock returns and exchange rates that allow us to move from conditional moments to unconditional moments. Specifically we make three assumptions. First, we assume that the risk premia on stock returns over the local risk-free rate are constant over time; second, we assume that expected excess currency returns are also constant; third, we assume that second moments are constant.

Under these assumptions, we can rewrite optimal currency demands (12) and (13) in terms of unconditional moments of returns and exchange rates as follows:

$$\widetilde{\mathbf{\Psi}}^{*}_{RM,t} = -\operatorname{Var}\left(\widetilde{\Delta\mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t\right)^{-1} \mathbf{1}'\boldsymbol{\omega}_t \operatorname{Cov}\left(\mathbf{r}_{t+1} - \mathbf{i}_t, \widetilde{\Delta\mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t\right), \qquad (14)$$

and

$$\psi^{*}_{RM,t} = -\frac{\mathbf{1}'\boldsymbol{\omega}_t \operatorname{Cov}\left(\mathbf{r}_{t+1} - \mathbf{i}_t, \widetilde{\Delta\mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t\right)\mathbf{1}}{\mathbf{1}'\operatorname{Var}\left(\widetilde{\Delta\mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t\right)\mathbf{1}}. \qquad (15)$$

Equations (14) and (15) show that, for fixed portfolio weights $\boldsymbol{\omega}_t \equiv \boldsymbol{\omega}$, we can compute optimal currency exposures by estimating simple regression coefficients of portfolio excess returns $\mathbf{1}'\boldsymbol{\omega}(\mathbf{r}_{t+1} - \mathbf{i}_t)$, where returns are measured as local excess stock returns $r_{c,t+1} - i_{c,t}$, onto a constant and the vector of currency excess returns $\widetilde{\Delta\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}^d_t + \widetilde{\mathbf{i}}_t$, and switching the sign of the slopes.

A very useful property of these optimal currency demands is that for a given stock portfolio, they are invariant to changes in the base currency, provided that the set of available currencies (which always includes an investor's own domestic currency) does not change. If we restrict the set of available currencies to a pair, for example the US dollar and the euro, this means that residents of both the US and Germany will have the same optimal demands for dollars and euros corresponding to a given equity portfolio. Residents of a third country, however, have another domestic currency available to them and so they will not necessarily have the same demands for dollars and euros even if they hold the same equity portfolio. If we allow a larger set of available currencies, then residents of all the countries in the set will have the same vector of optimal currency demands for a given equity portfolio.

In our empirical analysis we consider several particular cases of (14) and (15) of practical relevance. First, we consider the case of an investor who is fully invested in a single-country stock portfolio and optimally decides how much exposure to a single currency c to hold in order to minimize total portfolio return volatility. In that case both (14) and (15) reduce to

$$\psi^*_{RM,t} = -\frac{\text{Cov}\left(r_{1,t+1} - i_{1,t}, \Delta s_{c,t+1} + i_{c,t} - i_{1,t}\right)}{\text{Var}\left(\Delta s_{c,t+1} - i_{1,t} + i_{c,t}\right)}, \quad (16)$$

where for simplicity we assume that the stock market is the investor's own domestic stock market.

Thus the optimal currency demand is given by the negative of the slope coefficient estimated by a regression of the local excess stock return on the domestic market onto a constant and the excess return on currency c. A positive value of $\psi^*_{RM,t}$ means that the investor can reduce the volatility of her single-country stock portfolio by simultaneously borrowing $\psi^*_{RM,t}$ units of her own domestic currency per dollar invested in the domestic stock market, and investing them in bills denominated in currency c. We label this case as "single-country stock portfolio, single foreign currency."

Second, we consider the case of an investor who is fully invested in a single-country stock portfolio and uses the whole range of available currencies to minimize total portfolio return volatility. In that case the vector of optimal currency demands is given by the negative of the slopes of a multiple regression of the excess stock return on the domestic market onto a constant and the vector of currency excess returns. We label this case as "single-country stock portfolio, multiple currencies".

Third, we consider a case where the investor holds a global portfolio of stocks with equal or value weights, whether she uses a single currency or the whole vector of available currencies to minimize total portfolio return volatility. We label these cases as "world portfolio, single foreign currency" or "world portfolio, multiple currencies".

Finally, we consider an investor who holds a large fraction of her wealth in her domestic stock market, and the rest in a value-weighted portfolio of international stocks. We label this case as "home-biased portfolio."

3 Estimating Currency Demands

3.1 Data

Our empirical analysis uses data on exchange rates and interest rates from the International Financial Statistics database published by the International Monetary Fund, and stock return data from Morgan Stanley Capital International.[5] These data series are available on a monthly frequency. Our basic analysis is based on monthly regressions of overlapping quarterly excess returns. We report results for seven countries: Australia, Canada, Germany, Japan, Switzerland, the UK and the US. The sample period is 1975:7-2005:12, the longest sample period for which we have data available for all variables and for all seven countries. With regard to currencies, we will refer to the German currency as the euro, even though prior to 1999 the exchange rate we use is based on the deutschmark.

Table 1 reports the full sample average and standard deviation of nominal log stock returns, log stock returns in excess of their local short-term interest rates, changes in log exchange rates with respect to the US dollar, currency excess returns with respect to the dollar, and short-term nominal interest rates. Annualized average stock excess returns vary widely; they are lowest for Australia and Canada, which have a high market weight in commodity producers, and highest for Switzerland. Annual stock return volatilities are in the range 18%-24%, except for the US, whose volatility is considerably smaller at 15% over this period. Stock return volatility and excess stock return volatility are almost identical, reflecting the fact that short-term interest rates exhibit very low volatility. Annualized short-term interest rate volatility is 1% or less for all countries.

Average changes in exchange rates with respect to the US dollar over this period are negative for the Australian dollar, the Canadian dollar and the British pound, reflecting an appreciation of the US dollar with respect to these currencies over this period, and positive for the euro, the Swiss franc and the yen. Exchange rate volatility relative to the dollar is around 10% for all currencies except the Canadian dollar, which moves closely with the US dollar giving a volatility of only 5.4%. Excess returns to currencies are small on average and exhibit annual volatility similar to that of exchange rates, a result once again of the stability of short-term interest rates.

[5] In the case of the Swiss short-term interest rate, our data source is the OECD. We use euromoney rates up to 1989, and LIBOR rates afterwards, as published by the OECD.

Using the usual formula for the mean of a serially uncorrelated random variable, it is easy to verify that average excess returns to currencies are insignificantly different from zero.

Table 2 reports the full-sample monthly correlations of foreign currency excess returns, $\Delta \mathbf{s}_{t+1} + \mathbf{i}_t - \mathbf{i}_t^d$ in our notation. We report currency return correlations for each base currency. Table 2 shows that all currency returns are positively cross-correlated. These correlations are large—almost all correlation coefficients are above 30%—but they are far from perfect, implying that we have significant cross-sectional variation in the dynamics of exchange rates. Three correlations stand out as unusually large. The Canadian dollar exhibits a very high correlation with the US dollar (85-91%) regardless of the base currency used to measure exchange rates. It also exhibits a high degree of correlation with the Australian dollar (71-77%), except when the base country used to measure exchange rates is the US. The high correlation of the Canadian dollar with both the US dollar and the Australian dollar reflects the dual role of the Canadian economy as a resource dependent economy that is simultaneously highly integrated with the US. The third high correlation is between the Swiss franc and the euro (86-93%), reflecting the integration of the Swiss economy with the rest of Western Europe, and particularly with Germany.

Table 3 reports full-sample quarterly correlations of stock market returns denominated in local currency. These correlation coefficients are all between 30% and 55%, again with three important exceptions. The Canadian stock market is highly correlated with both the US stock market (72%) and the Australian stock market (60%), and the Swiss stock market is highly correlated with the German stock market (70%). These correlations demonstrate again the dual role of the Canadian economy and the integration of the Swiss economy with the German economy.

While significant, the stock market correlations are still small enough to suggest the presence of substantial benefits of international diversification in this sample period. Not surprisingly, the Japanese stock market exhibits the lowest cross-sectional correlation with all other markets. This is a reflection of the prolonged period of low or negative stock market returns in Japan during the 1990's, at a time when most other markets delivered large positive returns.

3.2 Single-country equity portfolios

We start our empirical analysis of optimal currency demand by examining the case of an investor who is fully invested in a single-country equity portfolio and is considering whether exposure to other currencies would help reduce the volatility of her portfolio return. We assume that the investor has a horizon of one quarter.

Table 4 reports optimal currency exposures for the case in which the investor is considering one currency at time (Panel A), and that in which she is considering multiple currencies at once (Panel B). That is, Panel A reports the regression coefficient (16). In both panels, the reference stock market is reported at the left of each row, while the currency under consideration is reported at the top of each column. In all tables we report Newey-West heteroskedasticity and autocorrelation consistent (h.a.c.) standard errors in parenthesis below each optimal currency exposure. Starred coefficients are those for which we reject the null of zero at a 5% significance level.

To facilitate the interpretation of this table and the remaining tables in the empirical sections, it is useful to recapitulate the exact interpretation of the coefficients shown in this table using a specific example. The cell in the northeast corner of the table, which corresponds to the German stock market and the US dollar, has a value of 0.49. This means that, in order to minimize the overall volatility of her portfolio return, an investor who is fully invested in the German stock market should short (or borrow) forty-nine euro cents worth of German T-bills per euro of stock market exposure, and invest those forty-nine euro cents in US Treasury bills. That is, the portfolio return minimizing strategy for this investor implies that she should optimally hold a 49% exposure to the US dollar.

Panel A of Table 4 shows that optimal demands for foreign currency are statistically significant in most cases. They are particularly large for three stock markets (rows of the table), those of Australia, Japan, and the UK Investors in the Australian, Japanese, and British stock markets are keen to hold foreign currency, regardless of the particular currency under consideration, because the Australian dollar, Japanese yen, and British pound tend to depreciate against all currencies when their stock markets fall; thus any foreign currency serves as a hedge against fluctuations in these stock markets.

Optimal currency demands are also particularly large for two currencies (columns of the table), the US dollar and the euro, and to a lesser extent for two other currencies, the Canadian dollar and the Swiss franc. The demand for US dollars to manage the risks of single-country stock portfolios is significantly positive for every stock market. It is largest for the single-country stock portfolios invested in the stock markets of Australia, Canada, Japan and the UK, which generate dollar demands of 126%, 193%, 101%, and 74% respectively. Demands for the euro and the Swiss franc are also large for all single-country stock portfolios except the US stock market. The demand for the Canadian dollar is large for all single-country stock portfolios except the US stock market, for which it is highly negative.

The last row of this panel describes individual optimal currency demands for a portfolio fully invested in US stocks. In contrast to the other single-country stock portfolios considered in the table, most of these demands are economically small, and statistically not different from zero. These small demands reflect a low correlation between US stock returns and the US dollar exchange rates of the currencies in the table. There are two important exceptions to this pattern. The first exception is the Swiss franc, which generates a positive demand from a US stock portfolio reflecting a small negative correlation between US stock returns and the dollar-Swiss franc exchange rate or, equivalently, a tendency for the Swiss franc to appreciate when the US stock market falls. Interestingly, the euro generates a similar demand, but it is not statistically significant. The second exception, which we have already noted, is a large negative demand for the Canadian dollar.

Panel B of Table 4 reports optimal currency demands for single-country stock portfolios considering all currencies simultaneously. That is, each row of Panel B reports (14) when $\mathbf{r}_{t+1}$ is unidimensional and equal to the stock market shown on the leftmost column. Panel B shows that, when single-country stock market investors consider investing in all currencies simultaneously, they generally choose positive exposures to the US dollar, the euro and the Swiss franc, negative exposures to the Canadian dollar and the Australian dollar, and avoid exposures to the pound and the yen.

Thus the patterns shown in Panel A do not result from the arbitrary assumption that only one foreign currency can be used to hedge equity risk. The Canadian dollar is an important exception. The positive demand for the Canadian dollar from single-country stock portfolios other than the US shown in Panel A reverses its sign and becomes large and negative for all single-country stock portfolios in Panel B. This contrasting result suggests that single-country stock investors demand the Canadian dollar as a substitute for the US dollar: When the US dollar is not available, they choose to hold the Canadian dollar; when the US dollar is available to them, they choose instead to hold the US dollar and to short the Canadian dollar along with the Australian dollar. This result makes sense given the dual nature of Canada as an economy that is both resource dependent and highly integrated with the US.

Panel B also shows that the demand for the euro becomes somewhat smaller when investors are allowed to invest simultaneously in the Swiss franc. This result is consistent with the notion that investors see the euro and the Swiss franc as substitutes for one another.

3.3 Global equity portfolios

Thus far we have considered only investors who are fully invested in a single country stock market, and use currencies to hedge the risk of that stock market. In this section we consider investors who hold internationally diversified stock portfolios, and optimally choose their currency exposure in order to minimize their portfolio return variance.

We start our analysis considering an investor who is equally invested in the seven stock markets included in our analysis: Germany, Australia, Canada, Japan, Switzerland, the UK, and the US Table 5 shows optimal currency demands at a quarterly horizon for such an investor optimizing over a single currency, while Table 6 considers the case of multiple-currency optimization at varying time horizons.

The first row of Table 5 shows the optimal positions of a German investor holding the global equity portfolio and able to trade in only one foreign currency at a time (along with his domestic bonds). That investor would short 0.19 euros worth of Australian bonds, or 0.21 euros worth of Japanese bonds, or 0.20 euros worth of UK bonds. In each of these cases, the investor would simultaneously buy equivalent amounts of euro bonds. The exposure for a dollar position is positive but statistically insignificant. Overall, this row implies that, against all currencies but the dollar, the euro tends to depreciate when global stock markets perform well, generating a positive risk-management demand for the euro. By the symmetry property of exchange rates, this result can also be read directly in the first column of the same table, which shows positive demand for euro bonds by foreign investors holding that same portfolio.

If we now look at other columns of the table, we see that demand for the dollar is also positive (with the exception of demands from German investors and from Swiss investors, which are insignificant), reflecting a negative correlation of the dollar with global stock markets. Conversely, there are negative or insignificant demands for the Australian dollar, the Canadian dollar, the Japanese yen and the British pound because these currencies are positively correlated with global markets. That these three currencies covary positively with global markets is unsurprising given the strong positive correlation with their own local markets uncovered in Panel A of Table 4 and the highly positive stock market cross-correlations shown in Table 3. For the euro, the Swiss franc and the dollar, however, correlations close to zero with their local stock markets give way to significant negative correlations of these currencies with the global equity market.

Table 6 shows results for an investor holding the same equally-weighted global portfolio, but using multiple currency positions to minimize risk. The table reports optimal currency demands at different investment horizons ranging from 1 month to 48 months, or four years. We have already noted in Section 2.4 that, in the multiple-currency case, optimal currency demands generated by a given global stock portfolio are the same regardless of the currency base. Accordingly, we only need to report one set of currency demands for each investment horizon. Note that the identity (5) implies that the numbers in each row add up to zero.

Panel A of the table considers an equally-weighted portfolio of five of the seven countries included in our analysis, and their currencies. Panel A excludes Canada and Switzerland from the analysis because these countries' stock markets are highly correlated with the US and German markets, and their currencies are highly correlated with the dollar and the euro. Thus Panel A considers a case in which investors do not have close currency substitutes available for investment. This helps understand the role of these currencies in investors' portfolios. Panel B of the table considers the case that includes all seven countries.

Once again, it is useful to recapitulate the exact meaning of the numbers we report to facilitate the discussion of the results. The numbers shown in Table 6 are optimal currency exposures. If it is optimal for all investors to fully hedge the currency exposure implicit in their stock portfolios or, equivalently, to hold no currency exposure, the optimal currency demands shown in Table 6 should be equal to zero everywhere. To obtain optimal currency hedging demands from optimal currency exposures, we need only compute the difference between portfolio weights—which in this case are 20% for each country stock market—and the optimal currency exposure corresponding to that country.

Panel A of Table 6 shows that the optimal currency exposure associated with the equally-weighted world portfolio implies a large, statistically significant exposure to the dollar at all horizons. The dollar exposure is highest at a one-month horizon at 90% of the value of the equity portfolio, but is still very large at a four-year horizon at 70%. The optimal currency exposure to the euro is also large, and statistically significant at most investment horizons. By contrast, the optimal exposures to the Australian dollar, the yen and the British pound are generally negative and statistically significant, particularly at short horizons.

If we focus on a one-month horizon, the results imply that, say, a German investor holding our equally-weighted five-country portfolio would borrow in other currencies an amount worth 103 euro cents per euro invested in the stock portfolio, and use the proceeds to buy US T-bills worth 90 euro cents, and German bills worth 13 euro cents. These purchases would be financed with proceeds from borrowing Australian dollars (49 euro cents per euro invested in the stock portfolio), yen (26 cents) and British pounds (28 cents).

We can easily restate these results in terms of hedging demands. For each dollar invested in the stock portfolio, this German investor would underhedge her exposure to the dollar, and overhedge her exposure to the Australian dollar, the yen and the British pound. More precisely, this German investor would not only not hedge the 20% dollar exposure implied by the portfolio, but she would also enter into forward contracts to buy dollars worth today 70 euro cents. She would simultaneously enter into forward contracts to sell Australian dollars, yen and British pounds worth today, respectively, 69, 46, and 48 euro cents per euro invested in the stock portfolio.

At horizons of one year or longer, the optimal combination of currency exposures is similar, essentially equivalent to holding a portfolio long US dollars and euros financed with British pounds and Japanese yen.

Panel B of Table 6 adds the stock markets of Canada and Switzerland to the equally-weighted global portfolio, and their currencies to the set of available currencies. The patterns that emerge from Panel B are consistent with the general patterns shown in Panel A, but now the high correlation of the the Canadian dollar with the US dollar results in very large optimal exposures to the dollar combined with large short positions in the Canadian dollar at horizons up to one year, and small exposures to the dollar and large exposures to the Canadian dollar at longer horizons. Similarly, the high correlation of the Swiss franc and the euro results in small optimal exposures to the euro and the Swiss franc at short horizons, and large exposures to the Swiss franc combined with short positions to the euro at longer horizons.

Overall, Table 6 suggests that the optimal currency exposure associated with the equally-weighted world portfolio implies a large, statistically significant long exposure to the dollar at all horizons, and smaller long positions in the euro and the Swiss franc, combined with either short positions or fully hedged positions in all other major currencies.

It is also interesting to examine the variance-minimizing currency exposures implied by a value-weighted portfolio of international stocks. We focus on a portfolio with weights determined by the relative market capitalization of each of the seven stock markets under consideration at the end of our sample period—December 2005. This would be the value-weighted portfolio relevant to an investor who is estimating optimal currency exposure at the end of our sample period. At this date, the US market was dominant, representing almost 60% of total capitalization. The Japanese and British stock markets follow with weights of 13.5% and 12.7%, respectively. The Australian, Canadian and German markets are much smaller, respectively representing 2.8%, 4% and 3.6% of our seven countries' market capitalization.

Tables 7 and 8, whose structures are identical to that of Tables 5 and 6, report optimal currency exposures implied by this value-weighted world portfolio, in the case of single and multiple currency respectively. In both tables 7 and 8, optimal currency exposures for the value-weighted portfolio are qualitatively similar to those for the equally-weighted portfolio. Investors want economically and statistically significant long exposures to the dollar, the euro, and the Swiss franc, and negative exposures to the yen, the Australian dollar, the Canadian dollar and the British pound. That is, they want to underhedge their exposure to the dollar, the euro, and the Swiss franc, and overhedge their exposure to the other currencies.

One salient feature of Table 7 when compared to Table 5 is that, whereas all other currency positions are very similar, dollar positions are much lower in Table 7 (29% vs 51% for an Australian investor, 22% vs 31% for a Japanese investor, and an insignificant 13% vs a significant 31% for a UK investor). Table 8 similarly shows optimal dollar demands smaller than those in Table 6. The large weight (60%) of the US stock market in the value-weighted world stock portfolio helps explain the differences in optimal currency exposures between the value-weighted portfolio and the equally-weighted portfolio. This large weight makes the value-weighted world stock portfolio very similar to a portfolio fully invested in the US stock market. We have shown in Section 3.2 that such a stock portfolio generates small currency demands, except for moderate long exposures to the euro and the Swiss franc, and a large short exposure to the Canadian dollar, because movements in the US stock market are largely uncorrelated with movements in exchange rates. By contrast, the equal weighted portfolio gives only a relatively small weight (20%) to the US stock market and thus does not inherit the properties of the US stock portfolio.

Our analysis so far has been focused on portfolios that are either completely invested in a single-country stock market, or fully diversified internationally. In practice, it is common for many institutional investors to hold equity portfolios which are heavily biased toward their own local stock market which nonetheless have a significant component of international diversification. Thus it is relevant to look at a case that captures this practice.

Table 9 examines the optimal currency exposures at a one-quarter horizon of "home biased world portfolios" which are 75% invested in the stock market indicated on the leftmost column of the table, and 25% in a value-weighted world portfolio that excludes this market. Panel A shows results for the single-currency case, Panel B for the multiple-currency one. Of course, since the composition of such portfolios changes across countries, Panel B shows an optimal vector of currency exposures for each one. Panel A is similar to Panel A of Table 4 which shows the single-currency case for 100% home-biased portfolios. Panel B shows once again that optimal currency exposures are large, positive and statistically significant for the US dollar, the euro, and the Swiss franc, negative for the Canadian dollar, and insignificantly different from zero for any of the other individual currencies.

The main conclusion that emerges from our discussion is that stock market investors with significant exposures to stock markets other than the US stock market find it optimal to hold economically significant exposures to the US dollar, the euro, and the Swiss franc. These exposures minimize the volatility of their portfolio returns, because these three currencies tend to appreciate when international stock markets fall.

Table 10 quantifies the variance reduction that investors can achieve by combining their international stock market portfolios with optimally chosen currency exposures. We report the annualized volatility of the 3-month return on the equally-weighted world portfolio, the value-weighted world portfolio, and the home-biased world portfolio. For comparison, we also report the volatility of the currency unhedged portfolio—which of course depends on the base currency—and of a portfolio that is fully currency hedged—so currency demands are set to zero.

Table 10 shows that the portfolio of optimal currency exposures achieves significant reductions in return volatility relative to both the unhedged portfolio and the hedged portfolio. These reductions are particularly significant for the equally-weighted world portfolio and the value-weighted world portfolio. For example, the euro denominated, unhedged, value-weighted world portfolio has an annual volatility of 17.06%. A policy of hedging half the currency exposure of the portfolio, which is typical among institutional investors, reduces the volatility to 14.75%. When fully hedged, the volatility of this portfolio is further reduced to 13.62%. But this volatility is still significantly larger than the volatility implied by the optimally hedged portfolio, which is 12.17%. It is interesting to note that a fully currency hedged portfolio does not necessarily have lower volatility than an unhedged portfolio. For example, the unhedged Japanese home-biased world portfolio has an annual volatility of 17.3%. When it is fully hedged, its volatility increases to 18.66%. By contrast, when it is optimally hedged, its volatility decreases significantly to 14.75%.

As one would expect from our discussion of the optimal currency exposures generated by stock portfolios heavily biased toward US stocks, the volatility reduction achieved by optimal hedging is smallest for such portfolios. For example, the volatility of a US-biased world portfolio is 14.17% when unhedged, and falls to 12.51% when optimally hedged. This reduction is still significant in absolute terms, but is somewhat smaller than the volatility reduction that optimal currency hedging can achieve for other stock portfolios.

3.4 Stability across subperiods

This section examines whether our empirical results are sample specific or whether they capture stable relations between excess returns on stocks and currencies. The sample period for which we have estimated optimal currency exposures includes an early period of global high inflation and interest rates, with exceptional performance of the Japanese stock market relative to other stock markets, followed by another subperiod of global lower inflation and interest rates, with subpar performance of the Japanese stock market. It is reasonable to examine if the results we have shown for the full sample hold across these two markedly different subperiods. Accordingly, we divide our sample period into the periods 1975–1989 and 1990–2005.

Figures 1 to 5 show the time series of an equally weighted average of foreign currency excess returns ("world currency") as well as the home stock market return and value weighted global stock market return for each of our base countries. A vertical line divides each graph between the first and the second subperiod. Foreign currency excess returns are defined as above so that, when the world currency line goes up, the home currency depreciates.

It is striking to observe that, throughout our sample period, the world currency and home stock market lines move against one another for Australia, Japan and the UK. This pattern reflects the strong positive correlations between these countries' currencies and their stock markets that we have already discussed. The figures show clearly that these correlations are stable over time. Canada exhibits a similar pattern, but with a weaker correlation..

In the first subperiod Germany and Switzerland look like Australia, Japan, and the UK with strong negative correlations between the world currency and home stock market, corresponding to positive correlations between the euro and Swiss franc and the German and Swiss stock markets. However this pattern weakens toward the end of the subperiod, and largely reverses in the second subperiod. The US has episodes of both positive and negative comovement in both subperiods, but there is a general tendency for the dollar to move against world markets particularly in the first subperiod.

These patterns determine the optimal currency positions that we find when we split the sample into two subsamples divided at December 1989. Table 11 reports results for an investor holding an equally-weighted global stock portfolio, and using the vector of available currencies to manage risk. We report results at three horizons (1, 3 and 12 months) in a fashion analogous to Table 6. Panel A considers stock markets and currencies from five countries excluding Canada and Switzerland. Panel B includes all seven countries.

For the equally weighted world portfolio, optimal currency exposures in the first subperiod are very similar to the full period ones, with the exception of the euro and the Swiss franc exposure, which are lower and not significant (point estimates are even negative at some horizons). For other currencies, we find the familiar large long positions in US dollars combined with large short positions in the Australian dollar, the Canadian dollar, the yen, and the British pound.

In the second subperiod we again obtain the same large short positions for the

Australian dollar, the Canadian dollar, the yen, and pound, but the results are somewhat different for the euro, the Swiss franc, and the dollar. When the Canadian dollar and the Swiss franc are excluded from the analysis, the optimal euro position becomes statistically significant, and much larger than the full-period one with a point estimate of 52% at a 3-month horizon versus 32% in the full-period estimates. The point estimate for the dollar position drops to 62% (vs. 70% in the full period) at a 3-month horizon, and becomes insignificant at a 12-month horizon. In the second subperiod the optimal demand for euros is larger than that for dollars, and statistically significant, at horizons beyond three months. When the Canadian and Swiss stock markets and currencies are included in the analysis, optimal currency demands for the equally-weighted world portfolio reveal a similar pattern, with a strengthening of the demand for the euro and the Swiss franc in the second subperiod, and a slight weakening of the demand for the US dollar.[6]

Overall, this subperiod analysis suggests one major change occurring between the periods 1975-1989 and 1990-2002. In the 1990's the Swiss franc and the euro became currencies with risk-management properties similar to that of the dollar, that is, currencies that covary negatively with global stock markets.

4 Conclusion

In this paper we have studied the correlations of foreign exchange rates with stock returns over the period 1975–2005 and have drawn out the implications for risk management by international equity investors. We have found that many currencies—in particular the Australian dollar, Canadian dollar, Japanese yen, and British pound—are positively correlated with their own domestic stock markets and to some extent with the world market. The US dollar, the euro, and the Swiss franc, however, are negatively correlated with the world equity market. These patterns imply that international equity investors can minimize their equity risk by taking short positions in the Australian and Canadian dollars, Japanese yen, and British pound, and long positions in the US dollar, euro, and Swiss franc. For US investors, the implication is that the currency exposures of international equity portfolios should be at least fully hedged, and probably overhedged, with the exception of the euro and Swiss

[6] Results not shown here to save space, but available upon request, show that overall, the optimal currency demands generated by the value-weighted world portfolio across subperiods are qualitatively similar to those generated by the equally-weighted world portfolio.

franc which should be partially hedged. These results are robust to variation in the investment horizon between one month and four years.[7] We obtain similar results when we consider the 1970's and 1980's in one subsample and the 1990's and 2000's in another, except that risk-minimizing equity investors should hold more euros and Swiss francs in the later period and slightly fewer dollars.

Campbell, Viceira, and White (2003) show that long-term investors interested in minimizing real interest rate risk using international portfolios of bills—or equivalently, currency exposures—also have large demands for bills denominated in euros and dollars, because these two currencies have had relatively stable interest rates. Their results suggest that these two currencies are attractive stores of value for international money market investors. Our results add to this evidence, by showing that the dollar and the euro tend to appreciate when international stock markets fall. This negative correlation generates demands for dollar and euro denominated bills as a way to reduce the volatility of international stock portfolios. In other words, the dollar and the euro are attractive stores of value for international equity investors.

These findings raise some interesting questions about the structure of the world economy. First, one might ask why the dollar, euro, and Swiss franc have these properties. One possible explanation is that they attract flows of capital at times when bad news arrives about the world economy, or when investors become more risk averse. This "flight to quality" drives up the dollar, euro, and Swiss franc at times when the prices of risky financial assets decline. This explanation takes as given that these currencies are regarded as safe assets and therefore benefit from a flight to quality. It is consistent with the role of the dollar, and increasingly the euro, as reserve currencies in the international financial system. Our finding that the risk-minimizing demand for euros has increased over time suggests that the euro has partially displaced the dollar as a reserve currency.

Second, one might ask what are the equilibrium consequences of risk management demand for dollars, euros, and Swiss francs. It is striking that the US, Germany, and Switzerland have had the lowest currency returns in our sample, and the lowest interest rates with the exception of Japan. International investors may be willing to receive lower compensation for holding dollar and euro denominated bills because of

[7] Froot (1994) studies the dollar and the pound over a longer sample period and finds that risk-minimizing foreign currency positions increase with the investment horizon, implying that long-horizon equity investors should not hedge their currency risk. We do not find this horizon effect in our post-1975 dataset.

the hedging properties of these currencies. If this is the case, it suggests that a country benefits from having a reserve currency not only because international demand for its monetary base generates seigniorage revenue, but also because international demand for its Treasury bills reduces the interest cost of financing the government debt.[8]

The analysis of this paper can be extended in several directions. First, we can consider the risk management demand for currencies by international investors who hold long-term bonds rather than equities. Currency hedging is more prevalent among bond market investors than among stock market investors, and it would be interesting to see whether the covariances across bond returns and currency returns justify this practice. Second, we can analyze the risk management demand of long-term investors not by using long-term mean-variance analysis, as in this paper, but using the long-term portfolio choice framework of Merton (1971), as implemented for example by Campbell, Chan, and Viceira (2003) and Jurek and Viceira (2006). Long-horizon mean-variance analysis ignores the fact that investors can rebalance their portfolios over time, and the alternative framework takes this into account. Finally, we can ask whether average currency returns reflect the differences in risk that we have identified here. For example, we can estimate a global CAPM and derive its predictions for currency returns and optimal currency holdings along the lines of Black (1990). If there are important risk-based differences in average currency returns, then these should influence currency demands for all but the most conservative investors.

[8]In a similar spirit, Lustig and Verdelhan (2005) show that currencies with high interest rates have high covariances with US consumption growth. The connection between liquidity preference (the demand for safe assets with low returns) and risk was first made explicitly by Tobin (1958).

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Table 1
Summary Statistics

	Germany	Australia	Canada	Japan	Switzerland	UK	US
Interest rates							
$E(i_{c,t})$	4.81	8.51	7.41	3.71	3.21	8.11	5.81
$\sigma(i_{c,t})$	0.61	1.01	1.01	0.91	0.71	0.91	0.81
Hedged stock excess returns							
$E(r_{c,t} - i_{c,t}) + ½\sigma^2$	8.11	6.31	5.31	8.81	11.11	7.01	7.01
$\sigma(r_{c,t} - i_{c,t})$	21.61	23.81	19.61	22.11	17.81	19.11	14.91
Δ exchange rate							
$E(\Delta s_{c,t}) + ½\sigma^2$	2.11	-1.41	-0.21	3.71	3.11	-0.11	.
$\sigma(\Delta s_{c,t})$	11.01	10.21	5.41	11.51	12.21	10.71	.
Currency excess returns							
$E(\Delta s_{c,t} + i_{c,t} - i_{US,t}) + ½\sigma^2$	1.11	1.31	1.31	1.61	0.61	2.21	.
$\sigma(\Delta s_{c,t} + i_{c,t} - i_{US,t})$	11.11	10.21	5.41	11.61	12.31	10.81	.

Note. Stock market returns are from the Morgan Stanley Capital International database. All other variables are from the IMF's IFS database. Data are monthly. Coverage extends from 1975:7 to 2005:12. Unless otherwise specified, all following tables use data from the full period.

Variables i, r and s respectively denote log nominal short-term interest rates (returns on 3-month treasury bills), log stock return in local currency, and log exchange rates. All statistics reported are in percentage points.

Hedged stock excess returns are the returns on foreign stocks to a fully hedged investor, i.e. the local currency return, in excess of the local nominal interest rate.

Exchange rates are with respect to the dollar, in dollars per unit of foreign currency (i.e. the dollar depreciates when the exchange rate increases).

The currency excess return is the return to a US investor of borrowing in dollars to hold foreign currency.

Table 2
Currency return correlations

	Germany	Australia	Canada	Japan	Switzerland	UK	US
Base country: Germany							
Germany	.						
Australia	.	1.00					
Canada	.	0.72	1.00				
Japan	.	0.39	0.39	1.00			
Switzerland	.	-0.01	-0.03	0.22	1.00		
UK	.	0.38	0.41	0.29	0.06	1.00	
US	.	0.66	0.88	0.44	0.00	0.44	1.00
Base country: Australia							
Germany	1.00						
Australia	.	.					
Canada	0.52	.	1.00				
Japan	0.67	.	0.46	1.00			
Switzerland	0.93	.	0.47	0.69	1.00		
UK	0.76	.	0.51	0.59	0.72	1.00	
US	0.58	.	0.85	0.55	0.54	0.58	1.00
Base country: Canada							
Germany	1.00						
Australia	0.21	1.00					
Canada	.	.	.				
Japan	0.59	0.25	.	1.00			
Switzerland	0.91	0.20	.	0.62	1.00		
UK	0.68	0.24	.	0.50	0.65	1.00	
US	0.31	0.11	.	0.35	0.31	0.34	1.00
Base Country: Japan							
Germany	1.00						
Australia	0.42	1.00					
Canada	0.51	0.74	1.00				
Japan	.	.	.	.			
Switzerland	0.88	0.33	0.39	.	1.00		
UK	0.68	0.49	0.56	.	0.60	1.00	
US	0.51	0.67	0.90	.	0.41	0.58	1.00
Base Country: Switzerland							
Germany	1.00						
Australia	0.38	1.00					
Canada	0.44	0.77	1.00				
Japan	0.27	0.46	0.47	1.00			
Switzerland	.	.	.	.	.		
UK	0.46	0.49	0.53	0.37	.	1.00	
US	0.43	0.71	0.91	0.51	.	0.55	1.00
Base Country: UK							
Germany	1.00						
Australia	0.32	1.00					
Canada	0.39	0.71	1.00				
Japan	0.50	0.42	0.44	1.00			
Switzerland	0.86	0.25	0.30	0.52	1.00		
UK	.	.	.	.	.	.	
US	0.39	0.64	0.88	0.48	0.32	.	1.00
Base Country: US							
Germany	1.00						
Australia	0.24	1.00					
Canada	0.17	0.43	1.00				
Japan	0.55	0.25	0.10	1.00			
Switzerland	0.90	0.21	0.12	0.58	1.00		
UK	0.66	0.25	0.14	0.44	0.61	1.00	
US	.	.	.	.	.	.	.

Note. This table presents cross-country correlations of foreign currency log excess returns $s_{c,t}+i_{c,t}-i_{d,t}$, where d indexes the base country. Correlations are presented separately for investors from each base country. They are computed using monthly returns.

Table 3
Stock market return correlations

Stock Markets	Germany	Australia	Canada	Japan	Switzerland	UK	US
Germany	1.00						
Australia	0.35	1.00					
Canada	0.42	0.61	1.00				
Japan	0.35	0.32	0.34	1.00			
Switzerland	0.70	0.40	0.46	0.43	1.00		
UK	0.51	0.50	0.54	0.40	0.58	1.00	
US	0.50	0.48	0.72	0.30	0.52	0.55	1.00

Note. This table presents correlations of hedged stock market excess returns ($r_{c,t}$-$i_{c,t}$, as explained in Table 1's note). They are computed using monthly returns.

Table 4

Optimal currency exposure for single-country stock portfolios: single and multiple currency cases

Stock market	Currency						
	Germany	Australia	Canada	Japan	Switzerland	UK	US
Panel A : Single currency							
Germany		0.16	0.32*	0.25*	0.28	0.19	0.49*
		(0.11)	(0.15)	(0.12)	(0.25)	(0.16)	(0.15)
Australia	0.82*		1.10*	0.56*	0.68*	0.60*	1.26*
	(0.12)		(0.14)	(0.12)	(0.11)	(0.15)	(0.11)
Canada	0.53*	-0.01		0.25*	0.46*	0.26*	1.93*
	(0.11)	(0.14)		(0.10)	(0.11)	(0.13)	(0.22)
Japan	0.92*	0.51*	0.79*		0.80*	0.74*	1.01*
	(0.12)	(0.10)	(0.12)		(0.13)	(0.12)	(0.13)
Switzerland	0.69*	0.26*	0.44*	0.27*		0.34*	0.54*
	(0.20)	(0.09)	(0.10)	(0.10)		(0.11)	(0.11)
UK	0.75*	0.29*	0.54*	0.31*	0.58*		0.74*
	(0.13)	(0.10)	(0.13)	(0.09)	(0.12)		(0.13)
US	0.19	-0.14	-0.77*	-0.03	0.19*	0.09	
	(0.10)	(0.09)	(0.17)	(0.10)	(0.09)	(0.11)	
Panel B : Multiple currencies at once							
Germany	-0.72*	-0.11	-0.44	0.03	0.31	-0.06	0.98*
	(0.26)	(0.14)	(0.30)	(0.14)	(0.24)	(0.17)	(0.30)
Australia	0.61*	-1.17*	-0.64*	-0.11	0.05	-0.29	1.55*
	(0.21)	(0.14)	(0.20)	(0.13)	(0.22)	(0.17)	(0.25)
Canada	0.33	-0.05	-1.96*	-0.21	0.31	-0.30*	1.89*
	(0.22)	(0.11)	(0.24)	(0.11)	(0.24)	(0.15)	(0.24)
Japan	0.42*	-0.16	-0.59*	-1.24*	0.11	0.03	1.43*
	(0.20)	(0.16)	(0.26)	(0.15)	(0.22)	(0.13)	(0.25)
Switzerland	0.19	-0.13	-0.22	-0.01	-0.63*	-0.03	0.84*
	(0.22)	(0.10)	(0.21)	(0.12)	(0.19)	(0.14)	(0.23)
UK	0.42	-0.12	-0.48*	-0.17	0.19	-0.99*	1.16*
	(0.23)	(0.10)	(0.22)	(0.10)	(0.22)	(0.15)	(0.21)
US	0.12	0.04	-0.91*	-0.23*	0.28	-0.01	0.70*
	(0.19)	(0.09)	(0.18)	(0.10)	(0.18)	(0.12)	(0.20)

Note. This table considers an investor holding a portfolio composed of equity from his own country, who chooses a foreign currency position to minimize the variance of his portfolio. Panel A allows the investor to use only one foreign currency. Panel B allows her to choose a vector of positions in all available foreign currencies. Rows indicate the equity being held (as well as the base country), columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the hedged excess return to the row country stock market onto the excess return on the column country currency. Rows of Panel B (excluding diagonal terms) are obtained by regressing the excess return to the row country stock market onto the vector of all foreign currency excess returns. All regressions include an intercept. Diagonal terms in Panel B are obtained by computing the opposite of the sum of other terms in the same row and the corresponding standard deviation.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table 5

Optimal currency exposure for an equally-weighted global equity portfolio: single-currency case

Base country	Currency						
	Germany	Australia	Canada	Japan	Switzerland	UK	US
Germany		-0.19*	-0.06	-0.21*	-0.11	-0.20*	0.16
		(0.08)	(0.10)	(0.08)	(0.17)	(0.10)	(0.10)
Australia	0.19*		0.32*	0.04	0.15*	0.09	0.51*
	(0.08)		(0.09)	(0.07)	(0.07)	(0.09)	(0.09)
Canada	0.06	-0.32*		-0.11	0.03	-0.08	1.04*
	(0.10)	(0.09)		(0.09)	(0.09)	(0.11)	(0.17)
Japan	0.21*	-0.04	0.11		0.19*	0.05	0.31*
	(0.08)	(0.07)	(0.09)		(0.08)	(0.08)	(0.09)
Switzerland	0.11	-0.15*	-0.03	-0.19*		-0.13	0.14
	(0.17)	(0.07)	(0.09)	(0.08)		(0.09)	(0.09)
UK	0.20*	-0.09	0.08	-0.05	0.13		0.31*
	(0.10)	(0.09)	(0.11)	(0.08)	(0.09)		(0.11)
US	-0.16	-0.51*	-1.04*	-0.31*	-0.14	-0.31*	
	(0.10)	(0.09)	(0.17)	(0.09)	(0.09)	(0.11)	

Note. This table considers an investor holding a portfolio composed of stocks from all countries, with equal weights, who chooses a position in one foreign currency at a time to minimize the variance of his portfolio. Rows indicate the base country of the investor, columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the excess return to the global equity portfolio onto the excess return of the column country currency to an investor based in the row country. All regressions include an intercept.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table 6

Optimal currency exposure for an equally-weighted global equity portfolio: multiple-currency case

Time horizon	Currency						
	Germany	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 5 country optimization							
1 month	0.13	-0.49*		-0.26*		-0.29*	0.90*
	(0.10)	(0.11)		(0.07)		(0.08)	(0.09)
2 months	0.27*	-0.46*		-0.31*		-0.27*	0.78*
	(0.10)	(0.10)		(0.07)		(0.09)	(0.11)
3 months	0.32*	-0.46*		-0.30*		-0.27*	0.70*
	(0.10)	(0.09)		(0.08)		(0.11)	(0.13)
6 months	0.31*	-0.39*		-0.38*		-0.19	0.65*
	(0.13)	(0.13)		(0.10)		(0.15)	(0.18)
12 months	0.28	-0.12		-0.51*		-0.36	0.71*
	(0.18)	(0.18)		(0.15)		(0.19)	(0.22)
24 months	0.34	0.05		-0.60*		-0.61*	0.81*
	(0.28)	(0.20)		(0.26)		(0.21)	(0.21)
48 months	0.47*	0.25		-0.74*		-0.69*	0.70*
	(0.18)	(0.18)		(0.31)		(0.19)	(0.20)
Panel B : 7 country optimization							
1 month	0.01	-0.30*	-0.76*	-0.23*	0.08	-0.25*	1.45*
	(0.14)	(0.11)	(0.14)	(0.07)	(0.12)	(0.08)	(0.15)
2 months	0.16	-0.27*	-0.77*	-0.30*	0.09	-0.24*	1.33*
	(0.14)	(0.09)	(0.15)	(0.08)	(0.13)	(0.09)	(0.16)
3 months	0.20	-0.24*	-0.75*	-0.28*	0.09	-0.24*	1.22*
	(0.15)	(0.09)	(0.17)	(0.09)	(0.15)	(0.11)	(0.19)
6 months	0.13	-0.19	-0.52*	-0.35*	0.15	-0.20	0.99*
	(0.25)	(0.14)	(0.26)	(0.12)	(0.21)	(0.16)	(0.29)
12 months	-0.12	0.08	-0.39	-0.50*	0.39	-0.40	0.94*
	(0.41)	(0.21)	(0.38)	(0.18)	(0.33)	(0.21)	(0.39)
24 months	-0.96	0.03	0.27	-0.56*	1.21*	-0.56*	0.57
	(0.50)	(0.29)	(0.48)	(0.28)	(0.48)	(0.22)	(0.43)
48 months	-1.44	-0.26	1.08*	-0.69*	1.94*	-0.62*	-0.01
	(0.83)	(0.33)	(0.55)	(0.31)	(0.97)	(0.17)	(0.41)

Note. This table considers an investor holding a portfolio composed of stocks from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon *T* of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global equity portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping *T*-months returns, *T* varying from 1 month to 48 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table 7

Optimal currency exposure for a value-weighted global equity portfolio: single-currency case

Base country	Currency						
	Germany	Australia	Canada	Japan	Switzerland	UK	US
Germany		-0.20*	-0.20*	-0.29*	0.02	-0.21*	-0.02
		(0.07)	(0.09)	(0.07)	(0.15)	(0.09)	(0.10)
Australia	0.20*		0.09	-0.01	0.17*	0.09	0.29*
	(0.07)		(0.08)	(0.07)	(0.07)	(0.08)	(0.09)
Canada	0.20*	-0.09		-0.06	0.16*	0.06	0.85*
	(0.09)	(0.08)		(0.08)	(0.08)	(0.10)	(0.16)
Japan	0.29*	0.01	0.06		0.30*	0.11	0.22*
	(0.07)	(0.07)	(0.08)		(0.08)	(0.07)	(0.09)
Switzerland	-0.02	-0.17*	-0.16*	-0.30*		-0.17*	-0.02
	(0.15)	(0.07)	(0.08)	(0.08)		(0.08)	(0.08)
UK	0.21*	-0.09	-0.06	-0.11	0.17*		0.13
	(0.09)	(0.08)	(0.10)	(0.07)	(0.08)		(0.11)
US	0.02	-0.29*	-0.85*	-0.22*	0.02	-0.13	
	(0.10)	(0.09)	(0.16)	(0.09)	(0.08)	(0.11)	

Note. We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table 8

Optimal currency exposure for a value-weighted global equity portfolio: multiple-currency case

Time horizon	Currency						
	Germany	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 5 country optimization							
1 month	0.20*	-0.29*		-0.19*		-0.19*	0.47*
	(0.10)	(0.10)		(0.07)		(0.08)	(0.09)
2 months	0.33*	-0.26*		-0.29*		-0.17*	0.39*
	(0.10)	(0.09)		(0.07)		(0.08)	(0.11)
3 months	0.37*	-0.25*		-0.29*		-0.15	0.32*
	(0.09)	(0.09)		(0.08)		(0.10)	(0.13)
6 months	0.36*	-0.21		-0.36*		-0.08	0.28
	(0.11)	(0.13)		(0.10)		(0.14)	(0.17)
12 months	0.30	0.00		-0.43*		-0.22	0.34
	(0.17)	(0.18)		(0.15)		(0.18)	(0.22)
24 months	0.40	0.10		-0.46		-0.50*	0.46*
	(0.32)	(0.21)		(0.31)		(0.25)	(0.23)
48 months	0.55	0.27		-0.57		-0.59*	0.35
	(0.29)	(0.19)		(0.38)		(0.30)	(0.22)
Panel B : 7 country optimization							
1 month	0.04	-0.13	-0.73*	-0.23*	0.18	-0.19*	1.06*
	(0.16)	(0.10)	(0.14)	(0.07)	(0.13)	(0.08)	(0.15)
2 months	0.17	-0.08	-0.79*	-0.33*	0.19	-0.17*	1.01*
	(0.15)	(0.09)	(0.14)	(0.07)	(0.14)	(0.09)	(0.16)
3 months	0.19	-0.05	-0.81*	-0.34*	0.21	-0.15	0.94*
	(0.15)	(0.08)	(0.16)	(0.09)	(0.15)	(0.11)	(0.18)
6 months	0.10	-0.02	-0.63*	-0.41*	0.28	-0.11	0.77*
	(0.22)	(0.14)	(0.23)	(0.12)	(0.19)	(0.14)	(0.26)
12 months	-0.24	0.21	-0.54	-0.52*	0.59	-0.27	0.76*
	(0.36)	(0.21)	(0.35)	(0.18)	(0.30)	(0.18)	(0.35)
24 months	-0.92*	0.15	0.05	-0.54	1.29*	-0.45	0.42
	(0.40)	(0.32)	(0.49)	(0.31)	(0.46)	(0.23)	(0.42)
48 months	-1.60	-0.32	1.12	-0.69	2.31*	-0.52*	-0.30
	(0.85)	(0.36)	(0.72)	(0.38)	(1.09)	(0.23)	(0.54)

Note. This table considers an investor holding a portfolio composed of stocks from all countries, with constant value weights (reflecting the end-of-period 2005:12 weights as reported in Table 7), who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon T of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global equity portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping T-months returns, T varying from 1 month to 48 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table 9

Optimal currency exposure for a home-biased global equity portfolio: single and multiple currency cases

Base country	Currency						
	Germany	Australia	Canada	Japan	Switzerland	UK	US
PANEL A : Single currency							
Germany		0.07	0.19	0.11	0.21	0.09	0.35*
		(0.10)	(0.13)	(0.10)	(0.22)	(0.13)	(0.13)
Australia	0.66*		0.84*	0.41*	0.55*	0.47*	1.01*
	(0.10)		(0.12)	(0.10)	(0.10)	(0.13)	(0.10)
Canada	0.44*	-0.03		0.17	0.38*	0.21	1.65*
	(0.10)	(0.12)		(0.09)	(0.10)	(0.12)	(0.19)
Japan	0.74*	0.37*	0.58*		0.65*	0.56*	0.78*
	(0.10)	(0.08)	(0.10)		(0.11)	(0.10)	(0.11)
Switzerland	0.50*	0.15	0.28*	0.12			
	(0.18)	(0.08)	(0.10)	(0.09)			
UK	0.59*	0.18	0.37*	0.19*	0.46*		0.56*
	(0.11)	(0.09)	(0.11)	(0.08)	(0.10)		(0.12)
US	0.09	-0.23*	-0.82*	-0.15	0.08	-0.05	
	(0.10)	(0.08)	(0.16)	(0.09)	(0.09)	(0.11)	
Panel B : Multiple currencies at once							
Germany	-0.48*	-0.09	-0.53*	-0.07	0.28	-0.08	0.97*
	(0.23)	(0.12)	(0.25)	(0.12)	(0.20)	(0.14)	(0.26)
Australia	0.50*	-0.88*	-0.68*	-0.17	0.09	-0.26	1.39*
	(0.18)	(0.12)	(0.18)	(0.11)	(0.19)	(0.14)	(0.22)
Canada	0.29	-0.05	-1.66*	-0.24*	0.28	-0.27*	1.64*
	(0.19)	(0.10)	(0.21)	(0.10)	(0.20)	(0.13)	(0.21)
Japan	0.36*	-0.13	-0.65*	-0.98*	0.14	-0.02	1.29*
	(0.16)	(0.13)	(0.21)	(0.12)	(0.18)	(0.11)	(0.21)
Switzerland	0.19	-0.11	-0.37	-0.10	-0.41*	-0.06	0.87*
	(0.19)	(0.09)	(0.19)	(0.11)	(0.17)	(0.12)	(0.21)
UK	0.36	-0.10	-0.58*	-0.22*	0.19	-0.75*	1.10*
	(0.20)	(0.09)	(0.19)	(0.09)	(0.19)	(0.13)	(0.20)
US	0.17	-0.02	-0.85*	-0.29*	0.24	-0.10	0.85*
	(0.17)	(0.09)	(0.17)	(0.09)	(0.16)	(0.11)	(0.19)

Note. This table considers an investor holding a home-biased portfolio of global equity. The portfolio is constructed by assigning a 75% weight to the home country of the investor, and distributing the remaining 25% over the four other countries according to their value weights as of the end of the period. The investor chooses a foreign currency position to minimize the variance of his portfolio. Panel A allows the investor to use only one foreign currency. Panel B allows her to choose a vector of positions in all available foreign currencies. Rows indicate the base country of the investor, columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the excess return on the row country home biased global equity portfolio onto the excess return on the column country currency. Rows of Panel B (excluding diagonal terms) are obtained by regressing the excess return on the row country portfolio on the vector of all foreign currency excess returns. All regressions include an intercept. Diagonal terms in Panel B are obtained by computing the opposite of the sum of other terms and the corresponding standard deviation.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table 10

Variance Reduction: standard deviations of hedged portfolios

Base country	No hedge	Half hedge	Full hedge	Optimal hedge
Equally-weighted portfolio				
Germany	16.66	15.32	14.61	12.66
Australia	15.55	14.43	14.61	12.66
Canada	16.25	15.07	14.61	12.66
Japan	16.01	14.68	14.61	12.66
Switzerland	17.00	15.36	14.61	12.66
UK	15.92	14.80	14.61	12.66
US	18.01	16.04	14.61	12.66
Value-weighted portfolio				
Germany	17.06	14.75	13.62	12.17
Australia	15.11	13.57	13.62	12.17
Canada	13.55	13.30	13.62	12.17
Japan	15.15	13.54	13.62	12.17
Switzerland	17.84	14.99	13.62	12.17
UK	15.71	14.02	13.62	12.17
US	14.66	14.03	13.62	12.17
Home biased portfolio				
Germany	17.84	17.97	18.17	17.26
Australia	18.14	18.71	19.34	15.18
Canada	16.98	17.25	17.54	14.36
Japan	17.30	17.94	18.66	14.75
Switzerland	15.43	15.66	16.01	14.81
UK	15.46	15.81	16.26	13.84
US	14.17	13.93	13.79	12.51

Note. This table reports the variance of portfolios featuring different uses of currency for risk-management.

We present results for three types of global equity portfolios (equally-weighted, value-weighted and home-biased as respectively described in Tables 5, 8 and 10). Within each panel, rows represent base countries and columns represent the risk-management strategy.

"No hedge" refers to the simple equity portfolio. "Half hedge" refers to a portfolio in which half of the implicit currency risk is neutralized. "Full hedge" refers to a portfolio in which all of the implicit currency risk is neutralized. "Optimal hedge" refers to a portfolio in which the currency position is chosen optimally to minimize variance.

Reported standard deviations are annualized, and measured in percentage points.

All results presented are computed considering returns at a quarterly horizon.

Table 11
Subperiod analysis
Equally-weighted world portfolio: multiple-currency case

Time horizon	Currency						
	Germany	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 5 country optimization							
Subperiod I : 1975-1989							
1 month	-0.02	-0.42*		-0.23*		-0.24*	0.92*
	(0.16)	(0.16)		(0.11)		(0.10)	(0.10)
3 months	0.13	-0.35*		-0.31*		-0.27	0.80*
	(0.15)	(0.11)		(0.11)		(0.15)	(0.20)
12 months	-0.16	0.00		-0.41*		-0.28	0.86*
	(0.19)	(0.21)		(0.15)		(0.19)	(0.22)
Suberiod II : 1990-2005							
1 month	0.33*	-0.60*		-0.26*		-0.40*	0.92*
	(0.11)	(0.13)		(0.09)		(0.11)	(0.14)
3 months	0.52*	-0.62*		-0.23*		-0.28*	0.62*
	(0.13)	(0.13)		(0.11)		(0.13)	(0.18)
12 months	0.82*	-0.56*		-0.42*		-0.41	0.56
	(0.21)	(0.24)		(0.19)		(0.25)	(0.34)
Panel B : 7 country optimization							
Subperiod I : 1975-1989							
1 month	0.01	-0.26	-0.83*	-0.17	-0.09	-0.19	1.54*
	(0.18)	(0.16)	(0.22)	(0.12)	(0.13)	(0.11)	(0.23)
3 months	0.09	-0.20	-0.71*	-0.29*	-0.03	-0.23	1.36*
	(0.18)	(0.12)	(0.25)	(0.14)	(0.18)	(0.15)	(0.34)
12 months	-0.47	0.11	-0.27	-0.41	0.27	-0.29	1.06
	(0.49)	(0.25)	(0.64)	(0.24)	(0.38)	(0.23)	(0.61)
Suberiod II : 1990-2005							
1 month	-0.11	-0.37*	-0.66*	-0.27*	0.43	-0.36*	1.35*
	(0.26)	(0.12)	(0.18)	(0.09)	(0.24)	(0.13)	(0.19)
3 months	0.17	-0.28*	-0.81*	-0.20	0.32	-0.26	1.06*
	(0.27)	(0.14)	(0.22)	(0.11)	(0.24)	(0.14)	(0.19)
12 months	0.48	-0.28	-0.46	-0.35	0.29	-0.40	0.71
	(0.56)	(0.28)	(0.40)	(0.22)	(0.56)	(0.26)	(0.41)

Note. This table replicates tables 6 and 8 for two subperiods, respectively extending from 1975:7 to 1989:12 and from 1990:1 to 2005:12. Time horizons include 1, 3 and 12 months only.

Figure 1: Foreign currency and hedged stock market excess returns

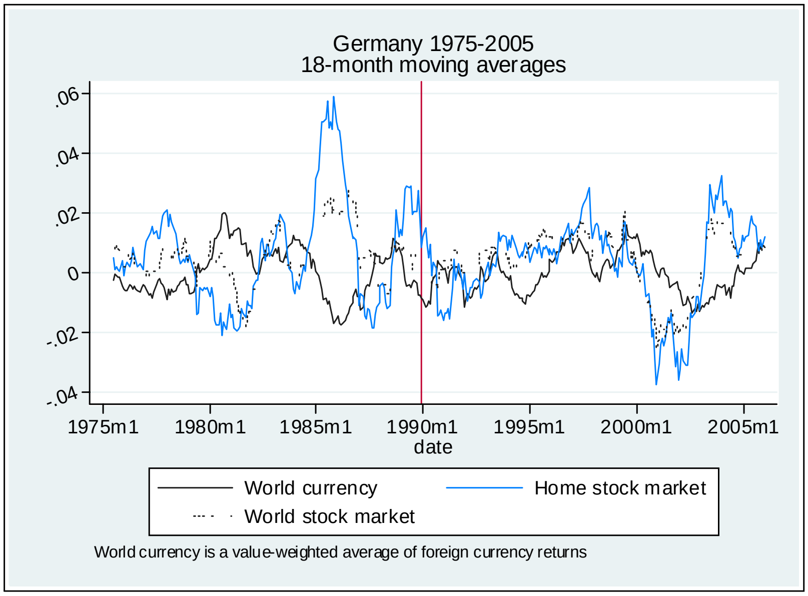

Figure 2: Foreign currency and hedged stock market excess returns

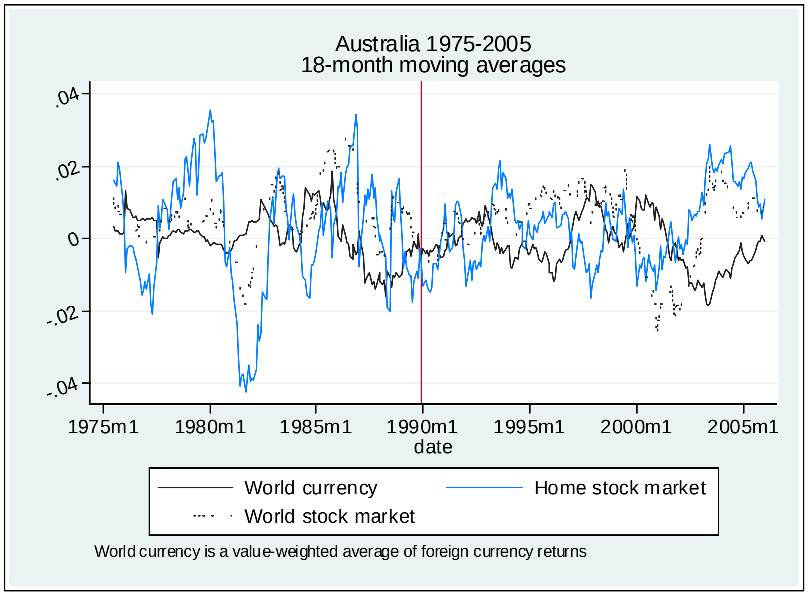

Figure 3: Foreign currency and hedged stock market excess returns

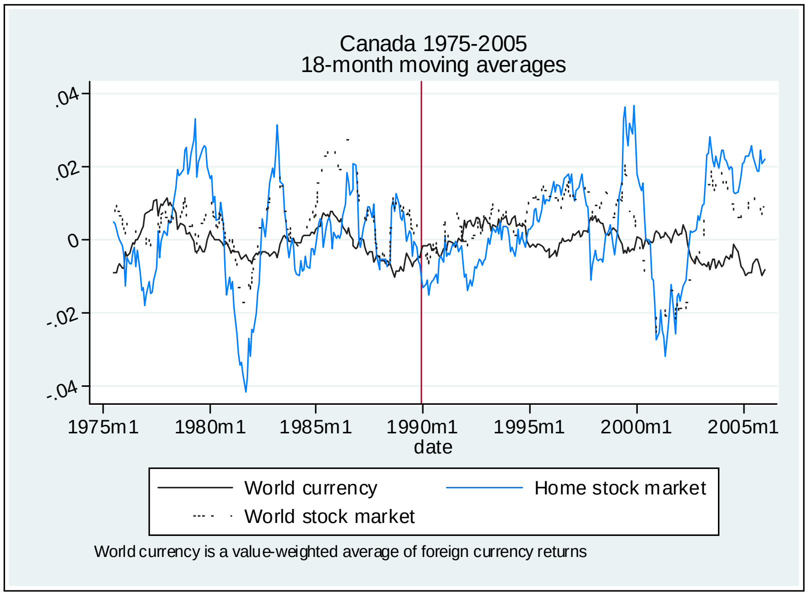

Figure 4: Foreign currency and hedged stock market excess returns

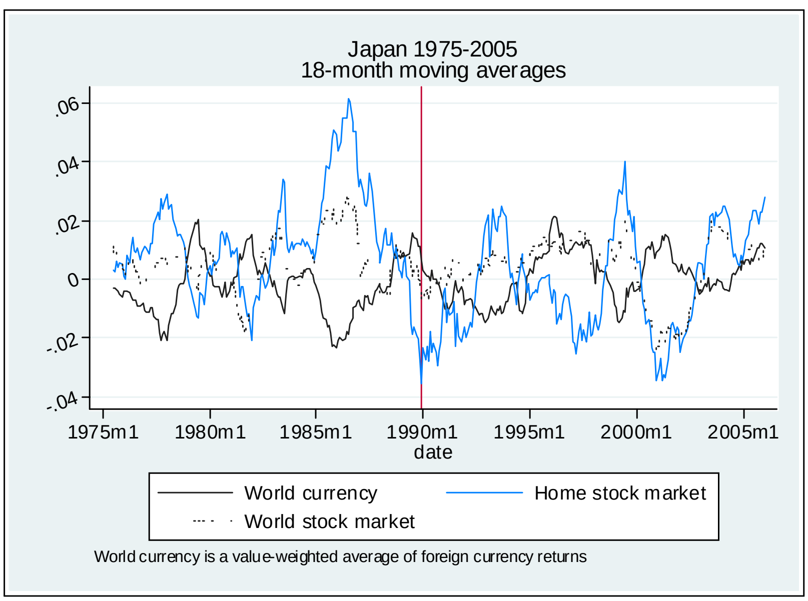

Figure 5: Foreign currency and hedged stock market excess returns

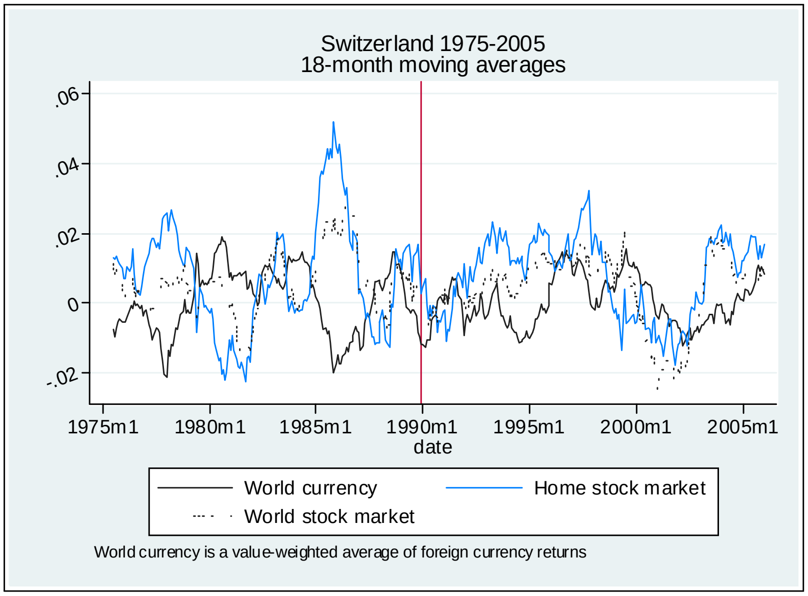

Figure 6: Foreign currency and hedged stock market excess returns

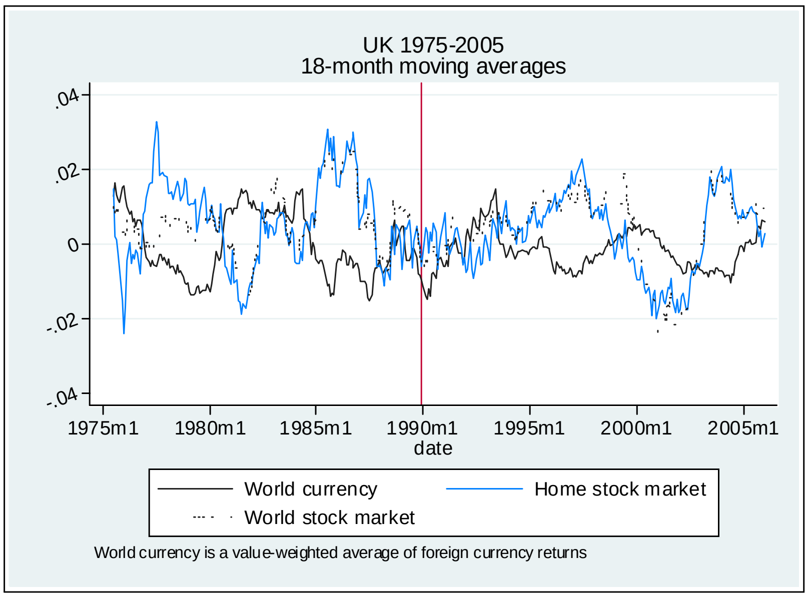

Figure 7: Foreign currency and hedged stock market excess returns