Inference Related to Common Breaks in a Multivariate System with Joined Segmented Trends with Applications to Global and Hemispheric Temperatures*

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Abstract

What transpires from recent research is that temperatures and forcings seem to be characterized by a linear trend with two changes in the rate of growth. The first occurs in the early 60s and indicates a very large increase in the rate of growth of both temperatures and radiative forcings. This was termed as the "onset of sustained global warming". The second is related to the more recent so-called hiatus period, which suggests that temperatures and total radiative forcings have increased less rapidly since the mid-90s compared to the larger rate of increase from 1960 to 1990. There are two issues that remain unresolved. The first is whether the breaks in the slope of the trend functions of temperatures and radiative forcings are common. This is important because common breaks coupled with the basic science of climate change would strongly suggest a causal effect from anthropogenic factors to temperatures. The second issue relates to establishing formally via a proper testing procedure that takes into account the noise in the series, whether there was indeed a 'hiatus period' for temperatures since the mid 90s. This is important because such a test would counter the widely held view that the hiatus is the product of natural internal variability. Our paper provides tests related to both issues. The results show that the breaks in temperatures and forcings are common and that the hiatus is characterized by a significant decrease in the rate of growth of temperatures and forcings. The statistical results are of independent interest and applicable more generally.

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1 Introduction

Significant advances have been made in documenting how global and hemispheric temperatures have evolved and in learning about the causes of these changes. On the one hand, large efforts have been devoted to investigate the time series properties of temperature and radiative forcing variables (Gay-Garcia et al., 2009; Kaufmann et al., 2006; Mills, 2013; Tol and de Vos, 1993). In addition, a variety of methods have been applied to detect and model the trends in climate variables, as well as some of their features such as breaks and non-linearities (Estrada, Perron and Martínez-López, 2013; Gallagher et al., 2013; Harvey and Mills, 2002; Karl et al., 2000; Pretis et al., 2015; Reeves et al., 2007; Seidel and Lanzante, 2004; Stocker et al., 2013; Tomé and Miranda, 2004). Also, the multivariate modeling of temperature and radiative forcing series has allowed to provide strong empirical evidence regarding the existence of a common secular trend between these variables, and to evaluate the relative importance of its natural and anthropogenic drivers (Estrada, Perron and Martínez-López, 2013; Estrada, Perron, Gay-García and Martínez-López, 2013; Kaufmann et al., 2006; Tol and Vos, 1998). The methodological contributions of the econometrics literature to this field have been notable; e.g., Dickey and Fuller (1979), Engle and Granger (1987), Johansen (1991), Perron (1989, 1997), Bierens (2000), Ng and Perron (2001), Kim and Perron (2009), Perron and Yabu (2009), among many others, see Estrada and Perron (2014) for a review. Regardless of the differences in assumptions and methods (statistical- or physical-based), there is a general consensus about the existence of a common secular trend between temperatures and radiative forcing variables.

Currently, some of the most relevant questions about the attribution of climate change are concerned with understanding particular periods in which the rate of warming changed, producing rapid warming, slowdowns or pauses. An example of this is the apparent slowdown in the warming experienced since the 1990s, for which a variety of methods have been applied to temperature series in order to try to document or reject the existence of this feature (e.g., Fyfe et al., 2016; Lewandowsky et al., 2015, 2016). However, such issues are delicate to handle when dealing with observed temperature series as the object of interest is the common secular trend behind the observed warming. Furthermore, this underlying trend is affected by low-frequency natural variability oscillations (Swanson et al., 2009; Wu et al., 2011) and changes in its rate of warming are difficult to detect and attribute. New tests and approaches designed to investigate common features in temperature and radiative forcing variables can make attribution studies more relevant for climate science and policy by providing a better understanding of the drivers behind them and of the effectiveness of climate policies.

One way to tackle this problem is to devise procedures to extract the common secular trend between temperature and radiative forcing series and then framing this problem in a univariate context where the available structural change tests can be applied. Estrada and Perron (2016) used this approach to investigate the existence and causes of the current slowdown in the warming. Their multivariate analysis strongly suggests that these variables share a common secular trend and common breaks, largely determined by the anthropogenic radiative forcing. Their analysis is based on the results of co-trending analyses and the application of a principal component analysis to separate the common long-term trend imparted by radiative forcing from the natural variability component in global temperature series. As discussed in that paper, filtering the effects of physical modes of natural variability from temperature series is necessary to obtain a proper assessment of the features and drivers of the warming trend. This problem has seldom been addressed within the time-series based attribution literature (e.g., Estrada, Perron and Martínez-López, 2013; Estrada and Perron, 2016) and it constitutes a relevant research topic that requires the development of new procedures and techniques. What transpires from this research is that temperatures and radiative forcing are most likely characterized by a linear trend with two changes in the rate of growth. The first is occurs in the early 60s and indicates a very large increase in the rate of growth of both temperatures and radiative forcing. This was termed as the "onset of sustained global warming". The second is related to the more recent so-called hiatus period, which suggests that temperatures and total radiative forcing have increased less rapidly since the mid-90s compared to the larger rate of increase from 1960 to 1990.

The alternative approach taken in this paper consists in designing statistical tests within a multivariate setting involving joint-segmented trends with which the existence of common breaks and the 'hiatus' can directly be investigated. There are two issues that until now remain unresolved and that this approach can formally address. The first is whether the breaks in the slope of the trend functions of temperatures and radiative forcing are common. This is important because common breaks coupled with the basic science of climate change would strongly suggest a causal effect from anthropogenic factors to temperatures. As is well known, a common linear trend can occur when the series are spuriously correlated. A common break can eliminate such concerns of spurious correlation and foster the claim that the relationship from forcing to temperatures is causal. Accordingly, one aim of this paper is to develop a test for common breaks across a set of series modeled as joint segmented trends with correlated noise. The theoretical framework follows Qu and Perron (2007). It extends the results of Oka and Perron (2016) who considered testing for common breaks in a system of equations. Their framework, however, precludes joint segmented trends since these involve regressors that are functions of the break dates, which makes the problem very different from those analyzed in, e.g., Bai and Perron (1998, 2003). To obtain the relevant limit distribution, the results of Perron and Zhu (2005) are useful. They show that the limit distribution of the estimate of the break date in a single time series with a joint segmented trend follows a normal limit distribution. We build on that result to show that our common break test follows a standard chi-square distribution. From a theoretical perspective, our results are more general and cover a wide range of cases to test various hypotheses on the break dates in a multivariate system, either within or across equations. Our empirical results show that, once we filter the temperature data for the effect of the Atlantic Mutidecadal Oscillation (AMO) and the North Atlantic Oscillation (NAO) for reasons explained in the text, the breaks in the slope of radiative forcing and temperatures are common, both for the large increase in the 60s and the recent 'hiatus' period.

The second issue relates to establishing formally via a proper testing procedure that takes into account the noise in the series, whether there was indeed a 'hiatus period' for temperatures since the mid-90s. This is important because such a test would counter the widely held view that the hiatus is the product of natural internal variability (Kosaka and Xie, 2013; Trenberth and Fasullo, 2013; Meehl et al., 2011; Balmaseda, Trenberth and Källén, 2013). Using standard univariate tests (e.g., Perron and Yabu, 2009), the results are mixed across various series and sometimes borderline. Our aim is to provide tests with enhanced power by casting the testing problem in a bivariate framework involving temperatures and radiative forcing.Our results indicate that indeed the 'hiatus" represents a significant slowdown in the rate of increase in temperatures, especially when considering global or southern hemisphere series, for which our test point to a rejection of the null of no change for all data sources.

We consider a multivariate system with n equations where the dependent variables are modeled as joint-segmented trends with multiple changes in the slope. The errors are allowed to be serially correlated and correlated across equations. Of interest is testing for general linear restrictions on the break dates, which includes testing for common breaks across equations. The test used is a (quasi-) likelihood ratio test assuming serially uncorrelated errors. Under the stated conditions, the LR test has a pivotal chi-square distribution. However, it is non-pivotal in the general case of interest. Accordingly, we also consider a corrected Wald test which has a pivotal limit distribution. This test can be constructed using break dates estimated one equation at a time, labelled OLS-Wald, or with the break dates estimated via the complete system, labelled GLS-Wald. The latter can offer more efficient estimates of the break dates when there is correlation across the errors from different equations. It is, however, very computationally demanding as it requires least squares operations of order $O(T^m)$ where T is the sample size and m is the total number of breaks. The OLS-Wald test requires least-squares operations of order $O(T^{m_1} + ... + T^{m_n})$ where m_i $(i = 1, ..., n)$ is the number of breaks in the i^{th} equation. Hence, in a bivariate system with one break in each equation, the OLS-Wald is much easier to compute. Simulations show that the two Wald tests have finite sample sizes close to the nominal size when the extent of the serial correlation in the noise is small. They, however, suffer from potentially severe liberal size distortions for moderate to strong serial correlation. Hence, for all three tests (since the LR is non-pivotal), we suggest a bootstrap procedure to obtain the relevant critical values. These bootstrap tests are shown to have exact sizes close to the nominal 5% level in all cases considered.

We consider a test for the presence of an additional break in some series. For instance, in our applications, the null and alternative hypotheses specify a break in the slope of the radiative forcing series (for which it is easy to obtain a rejection using a single equation test), while under the null hypothesis there is no break in the temperature series but there is one under the alternative. Hence, the aim is to see whether a break in temperatures is present using a bivariate system. The theoretical framework is more general and allows multiple breaks in a general multivariate system. The test considered is a quasi-likelihood ratio test. The limit distribution is shown to be non-pivotal and depends in a complex way on a number of nuisance parameters. Hence, we resort to a bootstrap procedure to obtain to relevant critical values.

The rest of the paper is structured as follows. Section 2 presents the statistical model used and the assumptions imposed on the structure. Section 3 considers testing for linear restrictions on the break dates, which includes as a special case testing for common breaks across equations. The various tests considered and their limit distributions are discussed. Section 4 presents the test for the presence of an additional break in some series withinh a multivariate system. Section 5 presents simulation results about the size and power of the tests and introduces the bootstrap versions recommended. Section 6 presents the applications. The data are discussed in Section 6.1. Section 6.2 presents the results for the common breaks tests, while Section 6.3 addresses the issue of testing for the hiatus using a bivariate system consisting of temperatures and forcing. Brief conclusions are offered in Section 7. All proofs are contained in an appendix.

2 Statistical Model and Assumptions

We adopt a framework similar to that of Perron and Zhu (2005), extended to have multiple breaks in a system of n variables. Each variable is represented by a linear trend with multiple changes in slope such that the trend function is joined at each break date. More specifically, there are m_i breaks in the slope of the i^{th} variable, and the n-variate system is for $t = 1, ..., T$ with T being the sample size:

$$\begin{aligned} y_{1t} &= \mu_1 + \beta_1 t + \textstyle\sum_{j=1}^{m_1} \delta_{1j} b_t(k_{1j}^0) + u_{1t} \\ &\vdots \\ y_{nt} &= \mu_n + \beta_n t + \textstyle\sum_{j=1}^{m_n} \delta_{nj} b_t(k_{nj}^0) + u_{nt} \end{aligned} \tag{1}$$

where $b_t(k_{ij}^0) = 1(t \geqslant k_{ij}^0)(t - k_{ij}^0)$ and k_{ij}^0 is the j^{th} break date for the change (with magnitudes δ_{ij}) in the trend of the i^{th} variable. We let $k_i^0 = (k_{i1}^0, \ldots, k_{im_i}^0)'$ be the vector of break dates for the i^{th} variable and k^0 be the vector of break dates for the entire system, $k^0 = (k_1^{0\prime}, \ldots, k_n^{0\prime})'$. We let $m = \sum_{i=1}^n m_i$ denote the total number of breaks in the entire system. Hence, k^0 is an $m \times 1$ vector.

The i^{th} variable is written in matrix form as

$$y_i = X(k_i^0)\theta_i + u_i \tag{2}$$

where $X(k_i^0) = [c, \tau, b(k_{i1}^0), \ldots, b(k_{im_i}^0)]$ and $\theta_i = (\mu_i, \beta_i, \delta_{i1}, \ldots, \delta_{im_i})'$ with $c = (1, \ldots, 1)'$, $\tau = (1, \ldots, T)'$ and $b(k_{ij}^0) = (b_1(k_{ij}^0), \ldots, b_T(k_{ij}^0))'$.

The entire system is expressed in matrix notation as

$$y = X^0\theta + u \tag{3}$$

where

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}, \; X^0 = \begin{pmatrix} X(k_1^0) & & 0 \\ & \ddots & \\ 0 & & X(k_n^0) \end{pmatrix}, \; \theta = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_n \end{pmatrix} \text{ and } u = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix}.$$

We are interested in the case where the number of breaks for each variable is known but the break dates are unknown. For a vector of generic break dates, we will use the notation without a 0 superscript, that is, k and k_i. Also, we will simply write X to denote the regressor matrix corresponding the break dates specified by k, that is, $X = diag(X(k_1), \ldots, X(k_n))$.

To motivate our quasi-likelihood ratio test, we first assume that u is multivariate normal with 0 mean and covariance $\Sigma \otimes I_T$. This assumption will be relaxed when we derive the asymptotic distribution of our test. For a generic break date vector k and the corresponding regressor matrix X, the log-likelihood function of y is then given by

$$l(k, \theta, \Sigma) = -\frac{nT}{2} \log 2\pi - \frac{T}{2} \log |\Sigma| - \frac{1}{2}(y - X\theta)'(\Sigma^{-1} \otimes I_T)(y - X\theta).$$

Let $\hat{\theta} = (\hat{\theta}_1', \ldots, \hat{\theta}_n')'$ and $\hat{\Sigma}(k)$ be the maximum likelihood estimators for θ and Σ, which jointly solve

$$\hat{\theta} = [X'(\hat{\Sigma}^{-1}(k) \otimes I_T)X]^{-1}[X'(\hat{\Sigma}^{-1}(k) \otimes I_T)y]$$

and

$$\hat{\Sigma}(k) = \frac{1}{T}\hat{U}_k'\hat{U}_k,$$

where

$$\hat{U}_k = [y_1 - X(k_1)\hat{\theta}_1, \ldots, y_n - X(k_n)\hat{\theta}_n].$$

Thus, the maximum of the log-likelihood function is given by

$$l(k) = -\frac{nT}{2}(\log 2\pi + 1) - \frac{T}{2} \log \left|\hat{\Sigma}(k)\right|. \tag{4}$$

It is also useful to work with break fractions relative to the sample size T. Define $\lambda_{ij} = k_{ij}/T$, $\lambda_i = T^{-1}k_i$ and $\lambda = T^{-1}k$. The true break fractions λ_{ij}^0, λ_i^0 and λ^0 are equivalently defined. With the given definition of break fractions, the functions in λ such as $l(\lambda)$ and $\hat{\Sigma}(\lambda)$ will be used interchangeably with $l(k)$ and $\hat{\Sigma}(k)$.

We now state the assumptions needed for the asymptotic analysis.

Assumption 1 $0 < \lambda_{i1}^0 < ... < \lambda_{im_i}^0 < 1$, *with* $k_{ij}^0 = [T\lambda_{ij}^0]$. $i = 1, ..., n$.

Assumption 2 $\delta_{ij} \neq 0$ *for* $j = 1, ..., m_i$ *and* $i = 1, \ldots, n$.

Assumption 3 *Let* $u_t = (u_{1t}, \ldots, u_{nt})'$. *Then,* u_t *is a stationary vector process with* $E(u_t) =$

0 *and* $Var(u_t) = \Sigma$. *In addition,* $T^{-1/2}\sum_{t=1}^{[Tr]} u_t = T^{-1/2}\Psi^{1/2}\sum_{t=1}^{[Tr]} e_t + o_p(1)$ *and*

$$T^{-1/2}\sum_{t=1}^{[Tr]} e_t \Rightarrow W(r),$$

where "⇒" denotes weak convergence under the Skorohod topology, $W(r)$ *is the* n *dimensional standard Wiener process and* $\Psi = \lim T^{-1}E(\sum_{t=1}^{T} u_t)(\sum_{t=1}^{T} u_t)'$.

Assumptions 1 and 2 are standard and simply state that the break dates are asymptotically distinct (i.e., each regime increases proportionally with the sample size T) and the changes in the parameters are non-zero at the break dates. Assumption 3 states that u_t is a stationary process and its partial sum follows a functional central limit theorem.

3 Testing for Linear Restrictions on the Break Dates

We now consider the following null and alternative hypotheses,

$$\begin{aligned} H_0 &: R\lambda^0 = r, \\ H_1 &: R\lambda^0 \neq r. \end{aligned} \tag{5}$$

Some examples of this type of hypotheses are as follows.

1. (specific break factions) $R = I_m$ and $r = \bar{\lambda}$;
2. (break fractions with a fixed distance) $R = [0, \ldots, 1, 0, \ldots, -1, 0, \ldots]$ and $r = c > 0$;
3. (common breaks) $R = [0, \ldots, 1, 0, \ldots, -1, 0, \ldots]$ and $r = 0$.

For the first case, historic events can be used to determine the value of $\bar{\lambda}$. The second case can be used across two equations as well as within an equation. The third case can be viewed as a special case of the second one and should be used across two equations only.

The LR test statistic is

$$LR = -2\left[\max_{\lambda:R\lambda=r} l(\lambda) - \max_{\lambda} l(\lambda)\right] = T\left[\min_{\lambda:R\lambda=r} \log\left|\hat{\Sigma}(\lambda)\right| - \min_{\lambda} \log\left|\hat{\Sigma}(\lambda)\right|\right]$$

Theorem 1 *Suppose that Assumptions 1∼3 hold with* $\Sigma = \Psi$. *Then, as* $T \to \infty$, *we have under the null hypothesis in (5) that* $LR \xrightarrow{d} \chi_q^2$, *where* $q = rank(R)$.

Our model of the broken segmented trend is the only case in structural change problems where we have the usual asymptotic normal distribution for the estimated break dates; see Perron and Zhu (2005). Hence, the associated likelihood ratio test also has a usual χ^2 distribution.

When the short-run and long-run variances differ $\Sigma \neq \Psi$, the asymptotic distribution depends on nuisance parameters. It is difficult to directly modify the LR test above. Instead, we develop a Wald test for the hypothesis in (5) that is robust to serial correlation in the error processes. To express the limiting distribution properly, we define the following notations. Let $f_i(r) = (1, r, (r-\lambda_{i1}^0)^+, \ldots, (r-\lambda_{im_i}^0)^+)'$ and $g_i(r) = (1(r > \lambda_{i1}^0), \ldots, 1(r > \lambda_{im_i}^0))'$, with $(\cdot)^+$ being the integer part of the argument. Then, define $FF_{ij} = \int f_i(r) f_j(r)' dr$, $FG_{ij} = \int f_i(r) g_j(r)' dr$, $GF_{ij} = \int g_i(r) f_j(r)' dr$ and $GG_{ij} = \int g_i(r) g_j(r)' dr$, where all integrals are taken from 0 to 1. Denote by s_{ij} the $(i,j)^{th}$ element of Σ^{-1} and by κ_{ij} that of $\Sigma^{-1}\Psi\Sigma^{-1}$. Now, we define

$$Q_{FF} = \begin{bmatrix} s_{11}FF_{11} & \ldots & s_{1n}FF_{1n} \\ \ldots & \ddots & \ldots \\ s_{n1}FF_{n1} & \ldots & s_{nn}FF_{nn} \end{bmatrix}$$

and

$$\Gamma_{FF} = \begin{bmatrix} \kappa_{11}FF_{11} & \ldots & \kappa_{1n}FF_{1n} \\ \ldots & \ddots & \ldots \\ \kappa_{n1}FF_{n1} & \ldots & \kappa_{nn}FF_{nn} \end{bmatrix}.$$

We also define Q_{FG}, Q_{GF}, Q_{GG}, Γ_{FG}, Γ_{GF} and Γ_{GG} equivalently. Finally, we define

$$\begin{aligned} \Xi_0 &= D_\delta \left[\Gamma_{GG} - \Gamma_{GF} Q_{FF}^{-1} Q_{FG} - Q_{GF} Q_{FF}^{-1} \Gamma_{FG} + Q_{GF} Q_{FF}^{-1} \Gamma_{FF} Q_{FF}^{-1} Q_{FG}\right] D_\delta \\ \Xi_1 &= D_\delta \left[Q_{GG} - Q_{GF} Q_{FF}^{-1} Q_{FG}\right] D_\delta. \end{aligned}$$

where $D_\delta = diag(D_{\delta_1}, \ldots, D_{\delta_n})$ with $D_{\delta_i} = diag(\delta_i)$ and $\delta_i = (\delta_{i1}, \ldots, \delta_{im_i})'$.

We provide the limiting distribution of the break fraction estimator obtained as the maximizer of the log-likelihood function in (4), which allows us to obtain the limit distribution of the Wald test.

Theorem 2 *Suppose that Assumptions 1∼3 hold. Let* $\hat{\lambda} = \hat{k}/T$ *and* $\hat{k} = \arg\min_k l(k)$.

Then, as $T \to \infty$, we have the following results: (i)

$$T^{3/2}(\hat{\lambda} - \lambda^0) \xrightarrow{d} N(0, \Xi_1^{-1}\Xi_0\Xi_1^{-1\prime}).$$

(ii) Under the null hypothesis in (5),

$$Wald \equiv T^3(R\hat{\lambda} - r)' \left(R\hat{\Xi}R'\right)^{-1} (R\hat{\lambda} - r) \xrightarrow{d} \chi_q^2$$

where $q = rank(R)$ and $\hat{\Xi}$ is a consistent estimate of $\Xi = \Xi_1^{-1}\Xi_0\Xi_1^{-1\prime}$.

The total number of regressions required to obtain the above break date estimator $\hat{\lambda}$ is $O(T^m)$. This can pose a considerable amount of computational burden especially if the procedure is to be bootstrapped. Hence, it is worthwhile to devise a test assuming diagonality in Σ because the break dates are then estimated separately from each variable and the total number of regressions required is only $O(T^{m_1} + \cdots + T^{m_n})$. While this modification lessens computational cost significantly, the resulting break date estimators can be less efficient due to the neglected correlation between variables in the system.

Suppose the break dates are estimated separately for each variable from (2). Thus, $\widetilde{\lambda} = (\widetilde{\lambda}_1', \ldots, \widetilde{\lambda}_n')'$ and

$$\widetilde{\lambda}_i = \arg\min_{\lambda_i} SSR_i(\lambda_i) \tag{6}$$

where $SSR_i(\lambda_i) = y_i' M_{\lambda_i} y_i$ with $M_{\lambda_i} = I_T - X(k_i)(X(k_i)'X(k_i))^{-1}X(k_i)'$. The limiting distribution of $\widetilde{\lambda}$ is obtained as a special case of Theorem 2 by letting $\Sigma = I$. The limiting covariance Ξ can be simplified somewhat. Define

$$\Xi_s = \begin{bmatrix} \psi_{11} D_{\delta_1}^{-1} P_{11} D_{\delta_1}^{-1} & \ldots & \psi_{1n} D_{\delta_1}^{-1} P_{1n} D_{\delta_n}^{-1} \\ \vdots & \ddots & \vdots \\ \psi_{n1} D_{\delta_n}^{-1} P_{n1} D_{\delta_1}^{-1} & \ldots & \psi_{nn} D_{\delta_n}^{-1} P_{nn} D_{\delta_n}^{-1} \end{bmatrix},$$

where

$$P_{ij} = (\int p_i(r) p_i'(r) dr)^{-1} \int p_i(r) p_j'(r) dr (\int p_j(r) p_j'(r) dr)^{-1}$$

with

$$p_i(r) = g_i(r) - \int g_i(r) f_i'(r) dr \left(\int f_i(r) f_i'(r) dr\right)^{-1} f_i(r)$$

and ψ_{ij} is the $(i,j)^{th}$ element of Ψ defined in Assumption 3.

Corollary 1 *Suppose that Assumptions 1∼3 hold. Let* $\widetilde{\lambda} = (\widetilde{\lambda}_1', \ldots, \widetilde{\lambda}_n')'$ *where each* $\widetilde{\lambda}_i$ *is obtained from (6). Then, as* $T \to \infty$*, we have the following results: (i)*

$$T^{3/2}(\widetilde{\lambda} - \lambda^0) \xrightarrow{d} N(0, \Xi_s).$$

(ii) Under the null hypothesis in (5),

$$Wald = T^3(R\widetilde{\lambda} - r)' \left(R\hat{\Xi}_s R'\right)^{-1} (R\widetilde{\lambda} - r) \xrightarrow{d} \chi_q^2$$

where $q = rank(R)$ *and* $\hat{\Xi}_s$ *is a consistent estimate of* Ξ_s*.*

The result in (i) coincides with one result reported in Perron and Zhu (2005) when there is only one equation with one break.

4 Break Detection

We consider a test for the null of $m = \sum_{i=1}^n m_i$ breaks in the system against the alternative of $m+1$ breaks with the location of the additional break being unspecified. Our testing procedure is again devised under a quasi-likelihood framework. Suppose that the model is now given by

$$y = X^0\theta + a_i(h)\gamma + u,$$

where $a_i(h) := 1_i \otimes b(h)$ for some $h \neq k_{i1}, \ldots, k_{im_i}$ and 1_i is an $n \times 1$ vector which has a one in the i^{th} element and zero elsewhere. The null and alternative hypotheses are now stated as

$$H_0 : \gamma = 0 \quad \text{and} \quad H_1 : \gamma \neq 0. \tag{7}$$

The maximum of the likelihood function under the null hypothesis is again given by (4) for a generic break date vector k. Similarly, we write the maximum of the likelihood function under the alternative hypothesis as

$$l^{(i)}(k,h) = -\frac{nT}{2}(\log 2\pi + 1) - \frac{T}{2}\log\left|\hat{\Sigma}_{(i)}(k,h)\right|,$$

for a generic break date vector k and an additional break date h. The notation $l^{(i)}(k,h)$ and $\hat{\Sigma}_{(i)}(k,h)$ is used to indicate the fact that an additional break is inserted in the i^{th} equation.

Just like we have defined $\lambda = k/T$ and used $l(k)$ and $l(\lambda)$ interchangeably, we define $\nu = h/T$ and will use $l^{(i)}(k,h)$ and $l^{(i)}(\lambda,\nu)$ interchangeably. The test statistic we consider is given by

$$LR = -2\left[l(\hat{\lambda}) - \sup_{\nu \in C_T^{(i)}} l^{(i)}(\hat{\lambda},\nu)\right]$$

where $\hat{\lambda} = \arg\sup_{\lambda} l(\lambda)$ and

$$C_T^{(i)} = \left\{ \begin{array}{c} \nu \text{ is such that } \varepsilon_{ps} \le \nu \le 1-\varepsilon_{ps} \text{ and } |\nu - \hat{\lambda}_{ij}| \ge \varepsilon_{ps},\ j = 1,\ldots,m_i \\ \text{for some } \varepsilon_{ps},\ 0 < \varepsilon_{ps} < \min\{\hat{\lambda}_{i1}, \hat{\lambda}_{im_i}\}. \end{array} \right\}$$

As is well known in the structural break literature, γ is not identified under the null hypothesis and ν must be restricted to ensure a non-divergent limiting distribution of the test statistic. The set $C_T^{(i)}$ imposes the relevant restrictions.[1]

When the equation with an additional break is not specified a priori, the LR statistic can be extended to be

$$\sup LR = -2\left[l(\hat{\lambda}) - \max_{1\le i\le n} \sup_{\nu \in C_T^{(i)}} l^{(i)}(\hat{\lambda},\nu)\right].$$

In order to express the limiting distribution, we need to define additional terms. Let $f_i(r)$ and $g_i(r)$ be as defined before. Let $b(r,\nu) = (r-\nu)^+$, $FB_i(\nu) = \int f_i(r)b(r,\nu)dr$, $GB_i(\nu) = \int g_i(r)b(r,\nu)dr$ and $BB(\nu) = \int b^2(r,\nu)dr$ where all integrals are taken from 0 to 1. Recall that s_{ij} is the $(i,j)^{th}$ element of Σ^{-1} and κ_{ij} is that of $\Sigma^{-1}\Psi\Sigma^{-1}$, and let $Q_{BB}^{(i)}(\nu) = s_{ii}BB(\nu)$, $\Gamma_{BB}^{(i)}(\nu) = \kappa_{ii}BB(\nu)$,

$$Q_{FB}^{(i)}(\nu) = \begin{bmatrix} s_{1i}FB_1(\nu) \\ \vdots \\ s_{ni}FB_n(\nu) \end{bmatrix} \quad \text{and} \quad \Gamma_{FB}^{(i)}(\nu) = \begin{bmatrix} \kappa_{1i}FB_1(\nu) \\ \vdots \\ \kappa_{ni}FB_n(\nu) \end{bmatrix}$$

[1] Instead of using $\hat{\lambda}$ under both the null and alternative hypothesis, it is possible to jointly minimize (λ,ν) under the alternative hypothesis. However, this requires a more complex asymptotic analysis and we will focus on the simpler case.

as well as

$$Q_{GB}^{(i)}(\nu) = \begin{bmatrix} s_{1i}GB_1(\nu) \\ \vdots \\ s_{ni}GB_n(\nu) \end{bmatrix} \quad \text{and} \quad \Gamma_{GB}^{(i)}(\nu) = \begin{bmatrix} \kappa_{1i}GB_1(\nu) \\ \vdots \\ \kappa_{ni}GB_n(\nu) \end{bmatrix}.$$

Now, using these functions, we define

$$\begin{aligned}
\xi_0^{(i)}(\nu) &= \Gamma_{BB}^{(i)}(\nu) - Q_{BF}^{(i)}(\nu)Q_{FF}^{-1}\Gamma_{FB}^{(i)}(\nu) - \Gamma_{BF}^{(i)}(\nu)Q_{FF}^{-1}Q_{FB}^{(i)}(\nu) + Q_{BF}^{(i)}(\nu)Q_{FF}^{-1}\Gamma_{FF}Q_{FF}^{-1}Q_{FB}^{(i)}(\nu) \\
\xi_1^{(i)}(\nu) &= Q_{BB}^{(i)}(\nu) - Q_{BF}^{(i)}(\nu)Q_{FF}^{-1}Q_{FB}^{(i)}(\nu) \\
\varsigma_0^{(i)}(\nu) &= D_\delta[\Gamma_{GB}^{(i)}(\nu) - Q_{GF}Q_{FF}^{-1}\Gamma_{FB}^{(i)}(\nu) - \Gamma_{GF}Q_{FF}^{-1}Q_{FB}^{(i)}(\nu) + Q_{GF}Q_{FF}^{-1}\Gamma_{FF}Q_{FF}^{-1}Q_{FB}^{(i)}(\nu)] \\
\varsigma_1^{(i)}(\nu) &= D_\delta[Q_{GB}^{(i)}(\nu) - Q_{GF}Q_{FF}^{-1}Q_{FB}^{(i)}(\nu)],
\end{aligned}$$

where D_δ, Q_{GF}, Q_{FF}, Γ_{GF} and Γ_{FF} are as previously defined. Finally, let $\eta_{(i)}(\nu)$ be a mean zero Gaussian process defined over the unit interval with

$$\begin{aligned}
Var(\eta_{(i)}(\nu)) &= \xi_0^{(i)}(\nu) - \varsigma_1^{(i)}(\nu)'\Xi_1^{-1}\varsigma_0^{(i)}(\nu) - \varsigma_0^{(i)}(\nu)'\Xi_1^{-1}\varsigma_1^{(i)}(\nu) \\
&\quad + \varsigma_1^{(i)}(\nu)'\Xi_1^{-1}\Xi_0\Xi_1^{-1}\varsigma_1^{(i)}(\nu).
\end{aligned}$$

Also, the limit counterpart of the set $C_T^{(i)}$ is denoted by $C^{(i)}$, which is the same as $C_T^{(i)}$ but with $\hat{\lambda}_{ij}$ replaced by λ_{ij}^0. The limit distribution of the tests are stated in the following theorem.

Theorem 3 *Suppose that Assumptions 1∼3 hold. Then, as $T \to \infty$, we have the following results under the null hypothesis in (7): (i)*

$$LR \xrightarrow{d} \sup_{\nu \in C^{(i)}} \frac{\eta_{(i)}^2(\nu)}{\xi_1^{(i)}(\nu)}.$$

(ii)

$$\sup LR \xrightarrow{d} \sup_{1 \le i \le n} \sup_{\nu \in C^{(i)}} \frac{\eta_{(i)}^2(\nu)}{\xi_1^{(i)}(\nu)}.$$

The limiting distributions depend on various nuisance parameters. First, all of the true break fractions λ_{ij}^0 for $j = 1, \ldots, m_i$ and $i = 1, \ldots, n$ matter via terms such as $Q_{FB}^{(i)}(\nu)$, $Q_{GB}^{(i)}(\nu)$, $\Gamma_{FB}^{(i)}(\nu)$, $\Gamma_{GB}^{(i)}(\nu)$, Q_{FF}, Q_{GF}, Γ_{FF} and Γ_{GF}. Second, both the short-run and long-run variances matter via the various Q and Γ matrices. Third, the trimming parameter ε_{ps}

in the set $C^{(i)}$ matters. Given the complexity of the limiting distributions, we neither seek a way to eliminate nuisance parameters nor attempt to tabulate critical values. Instead, we resort to using a bootstrap method to generature p-values. The various terms are evaluated in a manner similar to that of the tests for common breaks described in the next section. Note that if the short-run and long-run variances coincide and the true break fractions λ_{ij}^0 are used instead of their estimates, $Var(\eta_{(i)}(\nu))$ reduces to be $\xi_1^{(i)}(\nu)$.

5 Monte Carlo Simulations

In this section, we provide Monte Carlo simulation results to assess the adequacy of the asymptotic distributions derived in the previous section. All results are based on 1,000 replications. We focus on the test for common breaks as the tests for an additional change in slope are too computationally demanding so that a reasonable simulation experiment is prohibitive. The data generating process (DGP) is specified by:

$$\begin{aligned} y_{1t} &= \mu_1 + \beta_1 t + \delta_{11} b_t(k_{11}^0) + u_{1t} \\ y_{2t} &= \mu_2 + \beta_2 t + \delta_{21} b_t(k_{21}^0) + u_{2t}. \end{aligned}$$

The simulation results are exactly invariant to the values of μ_i and β_i. For the slope change parameters δ_{11} and δ_{21}, the cases of 0.5, 1.0 and 1.5 are considered. The error terms are such that

$$\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} = L \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \quad \text{with} \quad LL' = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix},$$

and

$$\begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} = \alpha \begin{pmatrix} e_{1t-1} \\ e_{2t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

with

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim IIN\left(0, (1-\alpha)^2 I_2\right).$$

The parameter ρ stands for the correlation across equations and α stands for the autoregressive parameter in each of the error terms. We consider (-0.5, 0, 0.5) for the values of ρ and

(0, 0.3, 0.7) for the values of α. What influences the precision of the break date estimate is the ratio between the break magnitude and the long-run variance. In our simulations, the long-run variance is always set to be unity.

The sample size is set to $T = 100$ and the true break dates are in the middle of the sample, $k_{11}^0 = k_{21}^0 = 50$. For each generated data, we test four null hypotheses, which are (1) $R = I_2$ and $r = (0.5, 0.5)'$, (2) $R = I_2$ and $r = (0.525, 0.475)'$, (3) $R = [1, -1]$ and $r = 0$ and (4) $R = [1, -1]$ and $r = 0.05$. Hence the null rejection probabilities for (1) and (3) pertian to the finite sample sizes with the nominal size being 5% while those for (2) and (4) pertain to powers. In testing these hypotheses, we consider the LR and Wald tests based on the SUR system (Theorems 1 and 2) as well as the Wald test based on estimating the break dates equation by equation using OLS regressions (Corollary 1). We refer to the two Wald tests as the GLS-Wald and OLS-Wald, respectively. For both versions, the long-run variance is estimated using the Quadratic Spectral window where the bandwidth parameter is selected using Andrews' (1991) data dependent method with an AR(1) approximation.

We also simulate the bootstrap version of the aforementioned tests. To obtain bootstrap p-values, we first estimate break dates imposing the null hypothesis and remove the estimated trend. Then, we fit a VAR(1) on the resulting residuals. Following Kilian (1998), we compute the bias-corrected estimates of the VAR coefficients and obtain the corresponding residuals of the VAR. From them, we construct a pseudo VAR process and add to it the estimated trend functions to obtain a bootstrap sample.

Simulating bootstrap tests requires a lot of computing time especially for those based on the full system. We make use of the Warp-Speed method suggested by Giacomini, Politis and White (2013). Also, we use the feasible GLS estimate instead of the ML estimator in the construction of the LR and GLS-Wald tests.

In Table 1, the results for the LR test are reported. Regarding the finite sample size obtained using the asymptotic critical values, two features emerge. First, the size is near or below the nominal level when $\alpha = 0$, but it climbs up quickly as α increases. In the worst case, it can go up to 50%. This result is well expected since the asymptotic validity of the LR test holds only when $\alpha = 0$. Second, the size inflation is more evident when δ_i is small. In fact, when δ_i is very large, the estimated break dates coincide with the true ones making the test statistic literally zero with a large probability. However, this type of conservativeness is of little concern because the power function does not appear to be decreasing as δ_i gets large. On the other hand, the finite sample size obtained by the bootstrap procedure stays near the nominal 5% level across all simulation designs, offering significant improvements over the asymptotic test. The bootstrapped test gives slightly smaller power than the asymptotic test, but the difference is marginal especially when the large improvement in the size is considered.

Table 2 reports the results for the GLS-Wald test. Overall, the GLS-Wald test using the asymptotic critical values are more liberal than the corresponding LR test, which is a well known characteristic of Wald tests. Despite its liberal nature, the GLS-Wald is less sensitive to the value of α, since we are using a robust covariance matrix. The GLS-Wald test also gets conservative as δ_i gets large for the same reason as the LR test. The bootstrapped GLS-Wald test controls the size as well as the bootstrapped LR test.

Table 3 shows the results for the OLS-Wald test. The OLS-Wald test performs very similarly to the GLS-Wald test for both the asymptotic and bootstrap versions.

Lastly, the power is close to one in almost all cases. The hypotheses that are rejected misspecifies the break dates only by 5% of the sample size, yet our tests are powerful enough to detect them.

To sum up, all three tests, the LR, GLS-Wald, and OLS-Wald suffer from size distortions if the magnitude of the breaks is not large enough. However, the bootstrap procedures significantly help control the size without losing much power.

6 Applications

We investigate the commonality of the break dates across the temperature and anthropogenic forcing series as well as test whether the recent "hiatus" is significant. We use two sets of global, northern and southern hemispheric temperature series, each of which will be denoted as G, N and S, respectively. The first set comes from the Climate Research Unit's HadCRUT4 (Morice et al., 2012) and the second from the NASA database (GISTEMP Team, 2015; Hansen et al., 2010). For global temperatures, we also use the data from Berkeley Earth (Rohde et al., 2013) and the dataset in Karl et al. (2015). We first discuss the data used, then the results for the common breaks tests and finally the results for the test on a change in slope in temperatures related to the so-called 'hiatus period'.

6.1 The data

The annual temperature data used are from the HadCRUT4 (1850-2014) (http://www.metoffice.gov.uk /hadobs/hadcrut4/data/current/download.html) and the GISS-NASA (1880-2014) datasets (http://data.giss.nasa.gov/gistemp/). The Atlantic Multidecadal Oscillation (AMO) and the North Atlantic Oscillation (NAO) series (1856-2014) are from NOAA; http://www.esrl .noaa.gov/psd/data/timeseries/AMO/ and http://www.esrl.noaa.gov/psd/ gcos_wgsp/ Timeseries/NAO/). As stated above, for global temperatures, we also use the data from Berkeley Earth (Rohde et al., 2013) and the dataset in Karl et al. (2015).

We also use series from databases related to climate model simulations by the Goddard Institute for Space Studies (GISS-NASA). The radiative forcing data obtained from GISS-NASA (https://data.giss.nasa.gov/modelforce/; Hansen et al., 2011) for the period 1880-2010 include the following (in W/m2): well-mixed greenhouse gases, WMGHG, (carbon dioxide, methane, nitrous oxide and chlorofluorocarbons); ozone; stratospheric water vapor; solar irradiance; land use change; snow albedo; stratospheric aerosols; black carbon; reflective tropospheric aerosols; and the indirect effect of aerosols. The aggregated radiative forcing series are constructed as follows: WMGHG is the radiative forcing of the well-mixed greenhouse gases; Total Radiative Forcing (TRF) is WMGHG plus the radiative forcing of ozone, stratospheric water vapor, land use change; snow albedo, black carbon, reflective tropospheric aerosols, the indirect effect of aerosols and solar irradiance.

6.2 Common breaks tests

The temperature series are affected by various modes of natural variability such as the Atlantic Multidecadal Oscillation (AMO) and the North Atlantic Oscillation (NAO), which are characterized by low frequency movements (Kerr, 2000; Hurrell, 1995). Since trends and breaks are low frequency features, it is important to purge the temperature series from natural low frequency components [2]. This allows more precise estimates of the break dates. Other high frequency fluctuations in temperature series do not affect the precision of the estimates of the break dates and the magnitudes of the changes in slope. Accordingly, we filter out the effect of these modes of variability by regressing each temperature series on these modes and a constant. Since the effect of natural variability might have occurred with a time lag, we choose an appropriate lag using the Bayesian Information Criterion (BIC); Schwarz (1978). The candidate regressors for the filtering are the current value and lags (up to order$kmax-1$) of AMO and NAO. We first work with G from HadCRUT4. We start with $kmax = 2$ so that the candidates are the current value and the first lag only. BIC chooses the

[2] We also considered filtering the effect of the Pacific Decadal Oscilliation (PDO) index (https://www.ncdc.noaa.gov/teleconnections/pdo/) and the Southern Oscilliation Index (SOI) (http://www.cru.uea.ac.uk/cru/data/soi/); see Wolter and Timlin (1998) and Mantua and Hare (2002). The results were qualitatively similar showing robustness. Given the similarity of the results, these are not reported.

current value of AMO and the first lag of NAO. Since the maximum lag allowed is selected, we increase $kmax$ to 4. Then, BIC chooses the current value of AMO and the second lag of NAO. When applying the BIC, the number of observations used is limited by $kmax$ (e.g., Perron and Ng, 2005). Having decided on the current value of AMO and the second lag of NAO, we applied the filtering to all available observations, not limited by $kmax$. We could repeat the same procedure to each series. However, it does not make much sense to have the same mode affects each temperature series with a different lag. Hence, we filter all series with the current value of AMO and the second lag of NAO.[3] The filtered temperature series are denoted as $\tilde{\mathrm{G}}$, $\tilde{\mathrm{N}}$ and $\tilde{\mathrm{S}}$. Figures 1 $\sim$ 3 display the time plots of the temperature series, both original and filtered.

For the anthropogenic forcing variables, we use WMGHG and TRF. Figure 4 displays the time plots of WMGHG and TRF as well as the AMO and NAO. The data set used covers the period 1880$\sim$2014 and the filtering is done with the full sample. We use the data for 1900$\sim$2014 for the purpose of the common break date tests.[4]

We use two sample periods: 1900$\sim$1992 and 1963$\sim$2014. The reason we split the sample is to reduce the number of breaks in the system, thereby lessening the computational burden. Our choice of sample periods is made with the intention to minimize any unwanted effect from a second break that might exist in any of the variables in the system. The break dates in WMGHG are estimated most precisely since it has a very small noise component (see Figure 3) and they are 1963 and 1992.

First we consider bivariate systems with one temperature series and one anthropogenic forcing. The null hypothesis of a common break date is tested using the LR test from Theorem 1. Since this test is not asymptotically pivotal, we supply bootstrap p-values as well. The bootstrap is carried out in the same way as done in the Monte Carlo simulations reported earlier. We also report the GLS-Wald test from Theorem 2. Although the GLS-Wald test is asymptotically pivotal, we still supply the bootstrap p-values because of the size distortions observed in our Monte Carlo study.

In addition to the 4 global, 2 northern hemispheric and 2 southern hemispheric temperature series, we also consider the respective average of the global, northern and southern

[3] If we choose the filtering regressors for each series, G and SH from NASA and G from Berkeley require the fourth lag of NAO instead of the second lag. But the break date estimates in the filtered series changes by two years at most.

[4] For the forcing variables, the data is available only up to 2011. We used forecast values for 2012$\sim$2014 based on an autoregressive model with a broken linear trend. In the case of TRF the forecast was produced using an AR(2) and a broken linear trend with 1960 and 1991 as the break dates. For WMGHG, we use an AR(8) and and a broken linear trend with break dates at 1960 and 1994.

hemispheric temperature series across different datasets. Hence, there are 11 temperature series and 11 filtered temperature series, each of which is paired with either WMGHG or TRF, yielding a total of 44 bivariate systems.

Table 4 (a) reports the results for the original temperature series for the sample period of 1900∼1992. The asymptotic p-values of the Wald and LR test are very small in most cases and the null of a common break date is rejected with only a few exceptions. However, the bootstrap p-values are well over the conventional thresholds 0.05 or 0.01 in all cases. Given the simulation results in the previous section, we are inclined to view the bootstrap p-values as more reliable than the asymptotic ones. Note that the break date for the forcing variable is almost unchanging at either 1963 or 1966, regardless of the temperature series in the bivariate system. Also, the estimated common break date, the restricted break date estimates, always coincide with the break date for the forcing variable. On the other hand, the break date estimates for the temperature series are quite spread out. Then, it is fair to say that the break dates in the forcing variables are being estimated more precisely than those in the temperature series. The large p-values produced by the bootstrapping is a consequence of the high level of noise in the temperature series relative to the magnitude of the break. Hence, we fail to reject the null of a common break date despite the large discrepancy in the estimated break dates.

Table 4 (b) shows the results obtained with the filtered temperature series. The picture changes quite dramatically. The break dates for the forcing variables are still the same at 1963 or 1966, but those for the temperature series are now also concentrated around 1960. Especially, when TRF enters the bivariate system, the estimate of the break date is 1960, 1965 or 1966, the exception being the southern hemispheric temperature from HadCRUT4 and the average of the two southern hemispheric temperature series, for which the estimate is still 1956. As a result, both the asymptotic and bootstrap p-values are much larger than those seen in Table 4 (a).

Tables 4 (c) and (d) present the results for the second sample 1963∼2014. In Table 4 (c), the original temperature series are used and the estimates of the break dates for the temperature series are again quite spread out, while those for the forcing variables are consistently around 1990∼1992. Just like in Table 4 (a), the asymptotic p-values suggest strong rejection of the null hypothesis, but the bootstrap p-values turns out much larger reflecting again on the high level of correlated noise in the original series. From Table 4 (d), the estimates of the break dates for the filtered temperature series are 1990∼1992 with only a few exceptions. Also, the p-values are larger than the corresponding ones in Table 4 (c).

We repeat the analysis for the bivariate systems with the OLS-Wald test as a robustness check and report the results in Table 5. The results are very similar when the filtered series are considered. The break date estimates obtained equation by equation coincide with those obtained from the system or differ only by a few years at most. When the original series are used, the break date estimates for some series largely deviate from those computed by the system method. However, the fact that the bootstrap p-values are much larger than usual thresholds is unchanging. Thus, our conclusions remain the same.

An advantage of the OLS-Wald test is that it can handle a larger system without extra computational burden. The break date estimates will not change if we consider a bigger system, because they are computed equation by equation. However, the long-run variance estimate and the bootstrap p-values can change as we consider a bigger system. We consider three five variables systems. Each system has a global, northern and southern hemispheric temperature and the two forcing variables. The first one uses the temperature series from HadCRUT4, the second one from NASA and the third one has average global, northern and southern hemispheric temperature series. The results are reported in Table 6. They show results broadly similar to those obtained bivariate systems.

6.3 Testing for the hiatus

The literature on the existence and drivers of the current slowdown in the rate of warming has expanded quickly (Tollefson, 2014, 2016). In this section, we focus on testing if the hiatus can be explained by natural variability or if it is a feature of the underlying warming trend (Estrada, Perron and Martinez, 2013). A relatively small part of the literature investigating the slowdown in the warming in global temperatures are based on formal structural change/change-point tests (Cahill et al., 2015; Pretis et al., 2015), while some others are based simply on testing for trends during arbitrarily selected periods of time (Cahill et al., 2015; Foster and Rahmstorf, 2011; Lewandowsky et al., 2015). These articles, as well as most of physically-based studies, have typically failed to find convincing evidence for the existence of a change in the slope of global temperatures during the last decades. According to one of the commonly accepted hypothesis, the apparent hiatus is produced by the effects of low-frequency natural variability that result from coupled ocean-atmosphere processes and heat exchange between the ocean and the atmosphere. It has been proposed that effects of natural variability modes such as AMO, NAO and PDO were able to mask the warming trend since the 1990s, creating the illusion of a slowdown in the underlying warming trend (e.g., Guan et al ., 2015; Steinman et al., 2015; Li, Sun and Jin, 2013; Trenberth and Fasullo, 2013). However, Estrada and Perron (2016) argue that low-frequency oscillations do indeed distort the underlying warming trend but that they cannot account for the current slowdown. Instead, the main effect of these oscillations has been to make more difficult to detect the drop in the rate of warming, which is a real feature of the warming trend that is imparted by the slowdown in the radiative forcing from well-mixed greenhouse gases.

Establishing whether the 'hiatus period' is statistically significant is important because such a result would counter the widely held view that the hiatus is the product of natural internal variability (Kosaka and Xie, 2013; Trenberth and Fasullo, 2013; Meehl et al., 2011; Balmaseda, Trenberth and Källén, 2013). Using standard tests (e.g., Perron and Yabu, 2009), the results are mixed across various series and sometimes borderline. Our aim is to provide tests with enhanced power by casting the testing problem in a bivariate framework involving temperatures and radiative forcing.

We consider bivariate systems with one temperature series and one forcing variable. We use the LR to test the presence of a break in temperature series given the presence of a structural break in forcing series. We first estimate a break date in the forcing series under the likelihood framework for a bivariate system of a pair of forcing and temperature series. Then, given the estimated break date in the forcing series, we apply the LR test for the null hypothesis of no break in a temperature series against the alternative of one break. To obtain p-values for our test, we apply the bootstrap procedure described in Section 5 with the number of resampling being 1,000.

The results are presented in Table 7. First, as expected from prior results, the LR test is more likely to reject the null hypothesis of no break when using filtered temperature series. Hence, we report results only for that case. For the filtered series from 1900 to 1992, we reject the null at less than 5% significance level in all pairs of forcing and temperature series. Hence, there is clear evidence of a break in temperatures that is near 1960 (varying between 1954 and 1966 depending on the series and the forcing). This concur with the common break results obtained in the previous section. When using the sample from 1963 to 2014, we reject the null in seven and eight cases out of eleven filtered series at the 10% significance level with well-mixed green-house gases and total radiating forcing, respectively. Since the sample size of time series from 1963 to 2014 is small ($T = 52$), we might expect that the LR test may have little power, but our result suggests strong evidence for the presence of a break in temperature series. Note that the evidence for a break is stronger when using bivariate system involving TRF. This is due to the fact that TRF exhibit a larger decrease in slope compared to WMGHG. The errors are also more strongly correlated. For instance, solar irradiance is a part of both TRF and temperatures and an important source of variations. When considering systems with TRF, the only pairs that do not allow a rejection at the 10% level are those associated with Northern hemisphere temperatures, For the latter, the p-values range from .14 to .19. For all other pair, the p-values are below 10%. Given, the small sample available, the overall evidence strongly indicate a break in temperature that is located in the early 90s for most series supporting the hiatus.

7 Conclusion

We consider a multivariate system with n equations where the dependent variables are modeled as joint-segmented trends with multiple changes in the slope. The errors are allowed to be serially correlated and correlated across equations. We consider testing for general linear restrictions on the break dates, which includes testing for common breaks across equations. The test used is a (quasi-) likelihood ratio test assuming serially uncorrelated errors that have a diagonal covariance matrix. Under the stated conditions, the LR test has a pivotal chi-square distribution. However, it is non-pivotal in the general case of interest. Accordingly, we also consider a corrected Wald test which has a pivotal limit distribution. This test can be constructed using break dates estimated one equation at a time or with the break dates estimated via the complete system. The limit distribution of the test is standard chi-square. However, simulations showed that the two Wald tests suffer from potentially severe liberal size distortions for moderate to strong serial correlation. Hence, for all three tests, we suggest a bootstrap procedure to obtain the relevant critical values. These bootstrap tests are shown to have the correct size in all cases considered. Our empirical results show that, once we filter the temperature data for the effect of the Atlantic Mutidecadal Oscillation (AMO) and the North Atlantic Oscillation (NAO), the breaks in the slope of radiative forcing and temperatures are common, both for the large increase in the 60s and the recent 'hiatus' period.

We also consider a test for the presence of an additional break in some series. The theoretical framework is general and allows multiple breaks in a general multivariate system. The test considered is a quasi-likelihood ratio test. The limit distribution is shown to be non-pivotal and depends in a complex way on a number of nuisance parameters. Hence, we use a bootstrap procedure to obtain relevant critical values. In our applications, the null and alternative hypotheses specify a break in the slope of the radiative forcing series, while under the null hypothesis there is no break in the temperature series but there is one under the alternative hypothesis. Our results indicate that indeed the 'hiatus" represents a significant slowdown in the rate of increase in temperatures, especially when considering global or southern hemisphere series, for which our test point to a rejection of the null of no change for all data sources considered.

The statistical results are of independent interest and applicable more generally beyond the climate change applications considered.

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Appendix

Proof of Theorems 1 and 2: Let $\hat{\lambda}_r = \arg\min_{\lambda \text{ s.t. } R\lambda=r} |\hat{\Sigma}(\lambda)|$ and $\hat{\lambda} = \min_\lambda |\hat{\Sigma}(\lambda)|$. Given the consistency of $\hat{\Sigma}(\hat{\lambda}_r)$, $\hat{\Sigma}(\hat{\lambda})$ and $\hat{\Sigma}(\lambda^0)$ under the null hypothesis, we can Taylor expand LR such that

$$\begin{aligned} LR &= T\ tr\left[\hat{\Sigma}^{-1}(\hat{\lambda})\left(\hat{\Sigma}(\hat{\lambda}_r) - \hat{\Sigma}(\hat{\lambda})\right)\right] + o_p(1) \\ &= T\ tr\left[\hat{\Sigma}^{-1}(\hat{\lambda})\left(\hat{\Sigma}(\hat{\lambda}_r) - \hat{\Sigma}(\lambda^0)\right)\right] \\ &\quad -T\ tr\left[\hat{\Sigma}^{-1}(\hat{\lambda})\left(\hat{\Sigma}(\hat{\lambda}) - \hat{\Sigma}(\lambda^0)\right)\right] + o_p(1) \end{aligned} \tag{A.1}$$

Consider the first term in the above decomposition. We may write

$$\begin{aligned} T\ tr\left[\hat{\Sigma}^{-1}(\hat{\lambda})\left(\hat{\Sigma}(\hat{\lambda}_r) - \hat{\Sigma}(\lambda^0)\right)\right] &= tr\left[\hat{\Sigma}^{-1}(\hat{\lambda})\left(\hat{U}'_{\hat{\lambda}_r}\hat{U}_{\hat{\lambda}_r} - \hat{U}'_{\lambda^0}\hat{U}_{\lambda^0}\right)\right] \\ &= tr\left[\Sigma^{-1}\left(\widetilde{U}'_{\hat{\lambda}_r}\widetilde{U}_{\hat{\lambda}_r} - \widetilde{U}'_{\lambda^0}\widetilde{U}_{\lambda^0}\right)\right] + o_p(1) \\ &= \min_{\lambda \text{ s.t. } R\lambda=r} tr\left[\Sigma^{-1}\left(\widetilde{U}'_{\lambda}\widetilde{U}_{\lambda} - \widetilde{U}'_{\lambda^0}\widetilde{U}_{\lambda^0}\right)\right] + o_p(1), \end{aligned}$$

where $\widetilde{U}_\lambda$ is the matrix of estimated residuals from the infeasible GLS estimator, instead of the ML estimator, for λ.

For notational simplicity, write $\Omega = \Sigma \otimes I_T$ and $M(\lambda) = I - \Omega^{-1/2}X(X'\Omega^{-1}X)^{-1}X'\Omega^{-1/2}$. Then the test statistic admits the following decomposition.

$$\begin{aligned} tr\left[\Sigma^{-1}\left(\widetilde{U}'_{\lambda}\widetilde{U}_{\lambda} - \widetilde{U}'_{\lambda^0}\widetilde{U}_{\lambda^0}\right)\right] &= vec(\widetilde{U}_\lambda)'\Omega^{-1}vec(\widetilde{U}_\lambda) - vec(\widetilde{U}_{\lambda^0})'\Omega^{-1}vec(\widetilde{U}_{\lambda^0}) \\ &= \theta'(X^0 - X)'\Omega^{-1/2\prime}M(\lambda)\Omega^{-1/2}(X^0 - X)\theta \\ &\quad +2\theta'(X^0 - X)'\Omega^{-1/2\prime}M(\lambda)\Omega^{-1/2}u \\ &\quad +u'\Omega^{-1/2\prime}\left[M(\lambda) - M(\lambda^0)\right]\Omega^{-1/2}u \\ &\equiv (XX) + 2(XU) + (UU). \end{aligned} \tag{A.2}$$

Note that

$$\begin{aligned} \left[X(k_i^0) - X(k_i)\right]\theta_i &= [0, 0, b(k_{i1}^0) - b(k_{i1}), \ldots, b(k_{im_i}^0) - b(k_{im_i})]\theta_i \\ &= [0, 0, (k_{i1} - k_{i1}^0)\widetilde{\iota}(k_{i1}), \ldots, (k_{im_i} - k_{im_i}^0)\widetilde{\iota}(k_{im_i})]\theta_i \\ &= [(k_{i1} - k_{i1}^0)\widetilde{\iota}(k_{i1}), \ldots, (k_{im_i} - k_{im_i}^0)\widetilde{\iota}(k_{im_i})]\delta_i \\ &\equiv \widetilde{\iota}(k_i)diag(\delta_i)(k_i - k_i^0) \end{aligned}$$

where $\delta_i = (\delta_{i1}, \ldots, \delta_{im_i})'$, $\widetilde{\iota}(k_{ij}) = (k_{ij} - k_{ij}^0)^{-1}(b(k_{ij}^0) - b(k_{ij}))$, $k_i = (k_{i1}, \ldots, k_{im_i})'$ and

$\widetilde{\iota}(k_i) = [\widetilde{\iota}(k_{i1}), \ldots, \widetilde{\iota}(k_{im_i})]$. Now,

$$\begin{aligned}
\left[X^0 - X\right]\theta &= \begin{pmatrix} [X(k_1^0) - X(k_1)]\,\theta_1 & & 0 \\ & \ddots & \\ 0 & & [X(k_n^0) - X(k_n)]\,\theta_n \end{pmatrix} \\
&= \begin{pmatrix} \widetilde{\iota}(k_1) & & 0 \\ & \ddots & \\ 0 & & \widetilde{\iota}(k_n) \end{pmatrix} \begin{pmatrix} D_{\delta_1} & & 0 \\ & \ddots & \\ 0 & & D_{\delta_n} \end{pmatrix} \begin{pmatrix} k_1 - k_1^0 \\ \vdots \\ k_n - k_n^0 \end{pmatrix} \\
&\equiv \Psi(\delta)(k - k^0).
\end{aligned}$$

with $D_{\delta_i} = diag(\delta_i)$. Thus

$$\begin{aligned}
|(XX)| &\leq \left\|\Omega^{-1/2}(X^0 - X)\theta\right\|^2 \\
&\leq \left\|\Omega^{-1/2}\right\|^2 \left\|(X^0 - X)\theta\right\|^2 \\
&= \left\|\Omega^{-1/2}\right\|^2 \left\|\Psi(\delta)(k - k^0)\right\|^2 \\
&\leq \left\|\Omega^{-1/2}\right\|^2 \left\|\Psi(\delta)\right\|^2 \left\|k - k^0\right\|^2 \\
&= O(T) \left\|k - k^0\right\|^2,
\end{aligned}$$

where the last equality follows from Lemma 1 of Perron and Zhu (2005). In addition, the $O(T)$ term in the above equation is not $o(T)$ and $(XX) > 0$.

$$\begin{aligned}
|(XU)| &= \left|\theta'(X^0 - X)'\Omega^{-1/2\prime}M(\lambda)\Omega^{-1/2}u\right| \\
&= \left|(k - k^0)'\Psi(\delta)'\Omega^{-1/2\prime}M(\lambda)\Omega^{-1/2}u\right| \\
&\leq \left\|k - k^0\right\| \left\|\Psi(\delta)'\Omega^{-1/2\prime}M(\lambda)\Omega^{-1/2}u\right\| \\
&= \left\|k - k^0\right\| O_p(T^{1/2}).
\end{aligned}$$

For (UU), note that

$$\begin{aligned}
(UU) &= u'\Omega^{-1/2\prime}\left[M(\lambda)-M(\lambda^0)\right]\Omega^{-1/2}u \\
&= u'\Omega^{-1}X^0(X^{0\prime}\Omega^{-1}X^0)^{-1}X^{0\prime}\Omega^{-1}u - u'\Omega^{-1}X(X'\Omega^{-1}X)^{-1}X'\Omega^{-1}u \\
&= u'\Omega^{-1}(X^0-X)(X^{0\prime}\Omega^{-1}X^0)^{-1}X^{0\prime}\Omega^{-1}u \\
&\quad +u'\Omega^{-1}X(X'\Omega^{-1}X)^{-1}\left(X'\Omega^{-1}X-X^{0\prime}\Omega^{-1}X^0\right)(X^{0\prime}\Omega^{-1}X^0)^{-1}X^{0\prime}\Omega^{-1}u \\
&\quad +u'\Omega^{-1}X(X'\Omega^{-1}X)^{-1}(X_0-X)'\Omega^{-1}u \\
&= \left\|k-k^0\right\| O_p(T^{-1}).
\end{aligned}$$

These orders of magnitude show that (XX), which is strictly positive, is the dominant term in (A.2). When these terms are evaluated at $k=\hat{k}_r$, (XX) is still the dominant term and it contradicts to the fact that $tr[\Sigma^{-1}(\widetilde{U}'_{\hat{\lambda}_r}\widetilde{U}_{\hat{\lambda}_r}-\widetilde{U}'_{\lambda^0}\widetilde{U}_{\lambda^0})]<0$. The only way to avoid the contradiction is to have $T^{1/2}||\hat{k}_r-k^0||=T^{3/2}(\hat{\lambda}_r-\lambda^0)=O_p(1)$.

Since the rate of convergence is obtained as $T^{3/2}$, the minimization can be carried out over the set

$$\Lambda_T=\{\lambda|\lambda \text{ is such that } |\lambda_{ij}-\lambda^0_{ij}|\le MT^{-3/2} \text{ for all } i,\, j \text{ and some large constant } M\}.$$

On Λ_T, (UU) is asymptotically negligible and can be ignored. Thus,

$$\begin{aligned}
&\min_{\lambda \text{ s.t. } R\lambda=r \text{ on } \Lambda_T} tr\left[\Sigma^{-1}\left(\widetilde{U}'_\lambda\widetilde{U}_\lambda-\widetilde{U}'_{\lambda^0}\widetilde{U}_{\lambda^0}\right)\right] \\
=&\min_{\lambda \text{ s.t. } R\lambda=r \text{ on } \Lambda_T}(XX)+2(XU)+o_p(1).
\end{aligned}$$

Defining $\varphi_T=T^{3/2}(\lambda-\lambda^0)$ yields that

$$(XX)=\varphi'_T\left[\frac{1}{T}\Psi(\delta)'\Omega^{-1/2}M(\lambda)\Omega^{-1/2}\Psi(\delta)\right]\varphi_T$$

and

$$(XU)=\varphi'_T\left[\frac{1}{\sqrt{T}}\Psi(\delta)'\Omega^{-1/2\prime}M(\lambda)\Omega^{-1/2}u\right].$$

Define a $Tn\times m$ matrix $Z=T^{-1/2}M(\lambda^0)\Omega^{-1/2}\Psi(\delta)$, a $Tn\times 1$ vector $z=M(\lambda^0)\Omega^{-1/2}u$ and $Q=Z(Z'Z)^{-1}R'_\perp$ where the $(m-q)\times m$ matrix $R_\perp$ is such that $R(Z'Z)^{-1}R'_\perp=0$ for a

$q \times m$ matrix of restrictions R. Then,

$$\begin{aligned}
& \min_{\lambda \text{ s.t. } R\lambda=r \text{ on } \Lambda_T} (XX) + 2(XU) \\
= & \min_{\varphi_T \text{ s.t. } R\varphi_T=0 \text{ on } \Lambda_T} \left[Z\varphi_T + z\right]' \left[Z\varphi_T + z\right] - z'z + o_p(1) \\
= & z\left[I - Q(Q'Q)^{-1}Q'\right] z - z'z + o_p(1) \\
= & -z'Q(Q'Q)^{-1}Q'z + o_p(1).
\end{aligned}$$

The second term in (A.1) is basically the same except that there is no restriction on the break dates, so that $Q = Z$. It follows that

$$T\ tr\left[\hat{\Sigma}^{-1}(\hat{\lambda})\left(\hat{\Sigma}(\hat{\lambda}) - \hat{\Sigma}(\lambda^0)\right)\right] = -z'Z(Z'Z)^{-1}Z'z + o_p(1),$$

and

$$\begin{aligned}
LR &= -z'Q(Q'Q)^{-1}Q'z + z'Z(Z'Z)^{-1}Z'z + o_p(1) \\
&= z'Z(Z'Z)^{-1}R\left(R'(Z'Z)^{-1}R\right)^{-1}R'(Z'Z)^{-1}Z'z + o_p(1).
\end{aligned}$$

Note that

$$\begin{aligned}
Z'Z &= \frac{1}{T}\Psi(\delta)'\Omega^{-1/2\prime}M(\lambda^0)\Omega^{-1/2}\Psi(\delta) \\
&= \frac{1}{T}\Psi(\delta)'\Omega^{-1}\Psi(\delta) - \frac{1}{T}\Psi(\delta)'\Omega^{-1}X^0S_T\left(\frac{1}{T}S_TX^{0\prime}\Omega^{-1}X^0S_T\right)^{-1}\frac{1}{T}S_TX^{0\prime}\Omega^{-1}\Psi(\delta) \\
&\to D_\delta\left[Q_{GG} - Q_{GF}Q_{FF}^{-1}Q_{FG}\right]D_\delta = \Xi_1,
\end{aligned}$$

where

$$S_T = diag\left\{\begin{bmatrix} 1 & & \\ & T^{-1} & \\ & & T^{-1}I_{m_1}\end{bmatrix}, \ldots, \begin{bmatrix} 1 & & \\ & T^{-1} & \\ & & T^{-1}I_{m_n}\end{bmatrix}\right\}.$$

For $Z'z$, let $\epsilon = \Omega^{-1}u$, which means that

$$\epsilon_t = \begin{bmatrix}\epsilon_{1t} \\ \vdots \\ \epsilon_{nt}\end{bmatrix} = \Sigma^{-1}\begin{bmatrix}u_{1t} \\ \vdots \\ u_{nt}\end{bmatrix}$$

and the partial sum process of ϵ_t obeys the functional central limit theorem from Assumption 3, that is,

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{[Tr]}\epsilon_t \Rightarrow \bar{B}(r) = \Sigma^{-1}\Psi^{1/2}W(r).$$

Denote the i^{th} element of $\bar{B}(r)$ by $\bar{B}_i(r)$. We can then express the limit of $Z'z$ as the follows:

$$\begin{aligned}
Z'z &= \frac{1}{\sqrt{T}}\Psi(\delta)'\Omega^{-1/2\prime}M(\lambda^0)\Omega^{-1/2}u \\
&= \frac{1}{\sqrt{T}}\Psi(\delta)'\Omega^{-1}u - \frac{1}{T}\Psi(\delta)'\Omega^{-1}X^0S_T\left(\frac{1}{T}S_TX^{0\prime}\Omega^{-1}X^0S_T\right)^{-1}\frac{1}{\sqrt{T}}S_TX^{0\prime}\Omega^{-1}u \\
&= \begin{bmatrix} -\frac{1}{T}\Psi(\delta)'\Omega^{-1}X^0S_T\left(\frac{1}{T}S_TX^{0\prime}\Omega^{-1}X^0S_T\right)^{-1} & I \end{bmatrix}\begin{bmatrix} \frac{1}{T^{1/2}}S_TX^{0\prime}\epsilon \\ \frac{1}{T^{1/2}}\Psi(\delta)'\epsilon \end{bmatrix} \\
&\xrightarrow{d} \begin{bmatrix} -D_\delta Q_{GF}Q_{FF}^{-1} & I_m \end{bmatrix}\begin{bmatrix} \int f_1(r)d\bar{B}_1(r) \\ \vdots \\ \int f_n(r)d\bar{B}_n(r) \\ \int g_1(r)d\bar{B}_1(r) \\ \vdots \\ \int g_n(r)d\bar{B}_n(r) \end{bmatrix}
\end{aligned}$$

where the covariance of the vector of stochastic integrals in the above expression is given by

$$\begin{bmatrix} \Gamma_{FF} & \Gamma_{FG}D_\delta \\ D_\delta\Gamma_{GF} & D_\delta\Gamma_{GG}D_\delta \end{bmatrix}.$$

Thus, we have

$$Z'z \xrightarrow{d} N\left(0, \Xi_0\right).$$

Theorem 1 assumes that $\Sigma = \Psi$, which gives $\Xi_0 = \Xi_1$. Then, it follows that $LR \xrightarrow{d} \chi^2_q$. For Theorem 2, $T^{3/2}(\hat{\lambda} - \lambda^0) = \hat{\varphi}_T + o_p(1) = -(Z'Z)^{-1}Z'z \xrightarrow{d} N(0, \Xi_1^{-1}\Xi_0\Xi_1^{-1\prime})$. $\square$

Proof of Corollary 1: Similarly to (A.2), we have the following decomposition

$$\begin{aligned} SSR_i(\lambda_i) - SSR_i(\lambda_i^0) &= \theta_i'(X(k_i^0) - X(k_i))' M_{\lambda_i}(X(k_i^0) - X(k_i))\theta_i \\ &\quad + 2\theta_i'(X(k_i^0) - X(k_i))' M_{\lambda_i} u_i \\ &\quad + u_i'(M_{\lambda_i} - M_{\lambda_i^0}) u_i \\ &\equiv (XX)_i + 2(XU)_i + (UU)_i \end{aligned}$$

The results in Perron and Zhu directly applies and $\widetilde{\lambda}_i$ is consistent at rate $T^{3/2}$. Thus, we can again focus on $T^{-3/2}$ neighborhood of λ_i^0, say Λ_{iT}, and

$$\begin{aligned} \widetilde{\lambda}_i &= \arg \min_{\lambda_i \text{ on } \Lambda_{iT}} SSR_i(\lambda_i) - SSR_i(\lambda_i^0) \\ &= \arg \min_{\lambda_i \text{ on } \Lambda_{iT}} (XX)_i + 2(XU)_i + o_p(1). \end{aligned}$$

Define $\varphi_i = T^{3/2}(\lambda_i - \lambda_i^0)$ and $\Psi_i(\delta_i) = \widetilde{\iota}(k_i) D_{\delta_i}$. It follows that

$$\begin{aligned} & T^{3/2}(\widetilde{\lambda}_i - \lambda_i^0) \\ = \; & \arg \min_{\varphi_i \text{ on } \Lambda_{iT}} \varphi_i' \left[\frac{1}{T}\Psi_i(\delta_i)' M_{\lambda_i} \Psi_i(\delta_i)\right] \varphi_i + 2\varphi_i' \left[\frac{1}{T^{1/2}}\Psi_i(\delta_i)' M_{\lambda_i} u_i\right] + o_p(1) \\ = \; & -\left[\frac{1}{T}\Psi_i(\delta_i)' M_{\lambda_i^0} \Psi_i(\delta_i)\right]^{-1} \left[\frac{1}{T^{1/2}}\Psi_i(\delta_i)' M_{\lambda_i^0} u_i\right] + o_p(1). \end{aligned}$$

Note that

$$\frac{1}{T}\Psi_i(\delta_i)' M_{\lambda_i^0} \Psi_i(\delta_i) \to D_{\delta_i} \left[\int p_i(r) p_i'(r) dr\right] D_{\delta_i}$$

and

$$\frac{1}{T^{1/2}}\Psi_i(\delta_i)' M_{\lambda_i^0} u_i \Rightarrow D_{\delta_i} \left[\int p_i(r) dB_i(r)\right]$$

where $B_i(r)$ is the i^{th} element of $B(r) = \Psi^{1/2} W(r)$ in Assumption 3. Furthermore, the weak convergence holds jointly in i under Assumption 3. Therefore,

$$T^{3/2} \begin{pmatrix} \widetilde{\lambda}_1 - \lambda_1^0 \\ \vdots \\ \widetilde{\lambda}_n - \lambda_n^0 \end{pmatrix} \xrightarrow{d} N\left(0, \begin{pmatrix} \psi_{11} D_{\delta_1}^{-1} P_{11} D_{\delta_1}^{-1} & \dots & \psi_{1n} D_{\delta_1}^{-1} P_{1n} D_{\delta_n}^{-1} \\ \vdots & \ddots & \vdots \\ \psi_{n1} D_{\delta_n}^{-1} P_{n1} D_{\delta_1}^{-1} & \dots & \psi_{nn} D_{\delta_n}^{-1} P_{nn} D_{\delta_n}^{-1} \end{pmatrix}\right)$$

where $P_{ij} = (\int p_i(r) p_i'(r) dr)^{-1} \int p_i(r) p_j'(r) dr (\int p_j(r) p_j'(r) dr)^{-1}$. $\square$

Proof of Theorem 3: We will use $\hat{\Sigma}_{(i)}(k, h)$ and $\hat{\Sigma}_{(i)}(\lambda, \nu)$ interchangeably. Similarly to (A.1), we can write

$$LR = T\ tr\left[\hat{\Sigma}_{(i)}^{-1}(\hat{\lambda},\hat{\nu})\left(\hat{\Sigma}(\hat{\lambda}) - \hat{\Sigma}_{(i)}(\hat{\lambda},\hat{\nu})\right)\right] + o_p(1),$$

where

$$\hat{\nu} = \arg\min_{\nu\in C_T^{(i)}} \log\left|\hat{\Sigma}_{(i)}(\hat{\lambda},\nu)\right|.$$

Define the projection matrix $P_{(i)}(\lambda,\nu)$ as

$$P_{(i)}(\lambda,\nu) = M(\lambda)\Omega^{-1/2}a_i\left(a_i'\Omega^{-1/2\prime}M(\lambda)\Omega^{-1/2}a_i\right)^{-1}a_i'\Omega^{-1/2\prime}M(\lambda),$$

where $a_i = a_i([T\nu])$ for simplicity and $M(\lambda) = I - \Omega^{-1/2}X(X'\Omega^{-1}X)^{-1}X'\Omega^{-1/2}$. Then, it follows that

$$LR = \sup_{\nu\in C_T^{(i)}} y'\Omega^{-1/2\prime}M(\hat{\lambda})P_{(i)}(\hat{\lambda},\nu)M(\hat{\lambda})\Omega^{-1/2}y + o_p(1).$$

Since

$$\begin{aligned} M(\lambda)\Omega^{-1/2}y &= M(\lambda)\Omega^{-1/2}(X^0 - X)\theta + M(\lambda)\Omega^{-1/2}u \\ &= M(\lambda)\Omega^{-1/2}\Psi(\delta)(k-k^0) + M(\lambda)\Omega^{-1/2}u, \end{aligned}$$

we can write

$$LR = \sup_{\nu\in C_T^{(i)}}\left[\begin{array}{c} (\hat{k}-k^0)'\Psi(\delta)'\Omega^{-1/2\prime}M(\hat{\lambda})P_{(i)}(\hat{\lambda},\nu)M(\hat{\lambda})\Omega^{-1/2}\Psi(\delta)(\hat{k}-k^0) \\ +2(\hat{k}-k^0)'\Psi(\delta)'\Omega^{-1/2\prime}M(\hat{\lambda})P_{(i)}(\hat{\lambda},\nu)M(\hat{\lambda})\Omega^{-1/2}u \\ +u'\Omega^{-1/2\prime}M(\hat{\lambda})P_{(i)}(\hat{\lambda},\nu)M(\hat{\lambda})\Omega^{-1/2}u \end{array}\right] + o_p(1).$$

From the proof of theorem 1, recall that $\sqrt{T}(\hat{k}-k^0) = -(Z'Z)^{-1}Z'z + o_p(1)$ where $Z = T^{-1/2}M(\lambda^0)\Omega^{-1/2}\Psi(\delta)$ and $z = M(\lambda^0)\Omega^{-1/2}u$. Then,

$$LR = \sup_{\nu\in C^{(i)}}\left[\ z'\left(I - Z(Z'Z)^{-1}Z'\right)P_{(i)}(\lambda^0,\nu)\left(I - Z(Z'Z)^{-1}Z'\right)z\ \right] + o_p(1).$$

To further analyze this expression, we write

$$\begin{aligned} & z'\left(I - Z(Z'Z)^{-1}Z'\right)P_{(i)}(\lambda^0,\nu)\left(I - Z(Z'Z)^{-1}Z'\right)z \qquad \text{(A.3)} \\ = & \frac{\left(T^{-3/2}a_i'\Omega^{-1/2\prime}M(\lambda^0)\left(I - Z(Z'Z)^{-1}Z'\right)z\right)^2}{T^{-3}a_i'\Omega^{-1/2\prime}M(\lambda^0)\Omega^{-1/2}a_i} \end{aligned}$$

The numerator of (A.3) weakly converges from Assumption 3 and the continuous mapping theorem. To see this, consider the following decomposition.

$$T^{-3/2}a_i'\Omega^{-1/2\prime}M(\lambda^0)\left(I - Z(Z'Z)^{-1}Z'\right)z = A_1(\nu)A_2(\nu)A_3(\nu), \text{ say,}$$

with

$$\begin{aligned}
A_1(\nu) &= \left[1, -\left(T^{-3/2}a_i'\Omega^{-1/2\prime}M(\lambda^0)Z\right)(Z'Z)^{-1}\right] \\
A_2(\nu) &= \begin{bmatrix} 1 & -T^{-2}a_i'\Omega^{-1}X^0S_T\left(\frac{1}{T}S_TX^{0\prime}\Omega^{-1}X^0S_T\right)^{-1} & 0 \\ 0 & -T^{-1}\Psi(\delta)'\Omega^{-1}X^0S_T\left(\frac{1}{T}S_TX^{0\prime}\Omega^{-1}X^0S_T\right)^{-1} & I_m \end{bmatrix} \\
A_3(\nu) &= \begin{bmatrix} T^{-3/2}a_i' \\ T^{-1/2}S_TX^{0\prime} \\ T^{-1/2}\Psi(\delta)' \end{bmatrix}\Omega^{-1}u.
\end{aligned}$$

For A_1, note that $Z'Z \xrightarrow{p} \Xi_1$ from the proof of Theorem 1 and

$$\begin{aligned}
& T^{-3/2}a_i'\Omega^{-1/2\prime}M(\lambda^0)Z \\
=\; & T^{-2}a_i'\Omega^{-1/2\prime}M(\lambda^0)\Omega^{-1/2}\Psi(\delta) \\
=\; & T^{-2}a_i'\Omega^{-1}\Psi(\delta) \\
& -T^{-2}a_i'\Omega^{-1}X^0S_T\left(\frac{1}{T}S_TX^{0\prime}\Omega^{-1}X^0S_T\right)^{-1}\frac{1}{T}S_TX^{0\prime}\Omega^{-1}\Psi(\delta) \\
\to\; & [Q_{BG}^{(i)}(\nu) - Q_{BF}^{(i)}(\nu)Q_{FF}^{-1}Q_{FG}]D_\delta \quad \text{uniformly in } \nu.
\end{aligned}$$

Thus,

$$\begin{aligned}
A_1(\nu) &\to \left[1, -[Q_{BG}^{(i)}(\nu) - Q_{BF}^{(i)}(\nu)Q_{FF}^{-1}Q_{FG}]D_\delta\Xi_1^{-1}\right] \\
&= \left[1, -\varsigma_1'(\nu)\Xi_1^{-1}\right] \quad \text{uniformly in } \nu.
\end{aligned} \tag{A.4}$$

Similarly, A_2 is such that

$$A_2(\nu) \to \begin{bmatrix} 1 & -Q_{BF}^{(i)}(\nu)Q_{FF}^{-1} & 0 \\ 0 & -D_\delta Q_{GF}Q_{FF}^{-1} & I_m \end{bmatrix} \quad \text{uniformly in } \nu. \tag{A.5}$$

For A_3, let $\bar{B}_i(r)$ be as defined in the proof of Theorem 1. Then, we can write

$$A_3(\nu) = \begin{bmatrix} T^{-3/2}a_i'\epsilon \\ T^{-1/2}S_T X^{0\prime}\epsilon \\ T^{-1/2}\Psi(\delta)'\epsilon \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & & 0 \\ & I_{m+2n} & \\ 0 & & D_\delta \end{bmatrix} \begin{bmatrix} \int b(r,\nu)d\bar{B}_i(r) \\ \int f_1(r)d\bar{B}_1(r) \\ \vdots \\ \int f_n(r)d\bar{B}_n(r) \\ \int g_1(r)d\bar{B}_1(r) \\ \vdots \\ \int g_n(r)d\bar{B}_n(r) \end{bmatrix} \tag{A.6}$$

over $\nu \in (0,1)$. Thus, A_3 is a Gaussian process with covariance matrix given by

$$\begin{bmatrix} \Gamma_{BB}^{(i)}(\nu) & \Gamma_{BF}^{(i)}(\nu) & \Gamma_{BG}^{(i)}(\nu)D_\delta \\ \Gamma_{FB}^{(i)}(\nu) & \Gamma_{FF} & \Gamma_{FG}D_\delta \\ D_\delta\Gamma_{GB}^{(i)}(\nu) & D_\delta\Gamma_{GF} & D_\delta\Gamma_{GG}D_\delta \end{bmatrix}.$$

Therefore, combining (A.4), (A.5) and (A.6) yields the weak convergence result

$$T^{-3/2}a_i'\Omega^{-1/2\prime}M(\lambda^0)[I - Z(Z'Z)^{-1}Z']z \Rightarrow \eta_{(i)}(\nu) \tag{A.7}$$

over $\nu \in (0,1)$. Lastly, the denominator in (A.3) is such that

$$\begin{aligned} & T^{-3}a_i'\Omega^{-1/2\prime}M(\lambda^0)\Omega^{-1/2}a_i && \text{(A.8)} \\ = \; & T^{-3}a_i'\Omega^{-1}a_i - T^{-2}a_i'\Omega^{-1}X^0S_T\left(T^{-1}S_TX^{0\prime}\Omega^{-1}X^0S_T\right)^{-1}T^{-2}S_TX^{0\prime}\Omega^{-1}a_i \\ \to \; & Q_{BB}^{(i)}(\nu) - Q_{BF}^{(i)}(\nu)Q_{FF}^{-1}Q_{FB}^{(i)}(\nu) = \xi_1^{(i)}(\nu) \quad \text{uniformly in } \nu. \end{aligned}$$

The result in part (i) of the theorem follows from (A.7) and (A.8). The result in part (ii) also follows since the above result holds jointly in i. □

Table 1. Probabilities of Rejecting the Null Hypothesis, LR test

(a) $\delta_i = 0.5$

		Asymptotic				Bootstrap			
α	ρ	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
0	-0.5	0.05	0.98	0.06	0.95	0.04	0.97	0.05	0.94
0	0	0.07	1.00	0.07	0.98	0.06	0.98	0.05	0.97
0	0.5	0.05	1.00	0.05	1.00	0.05	1.00	0.04	1.00
0.3	-0.5	0.17	0.99	0.21	0.99	0.05	0.98	0.06	0.95
0.3	0	0.17	1.00	0.18	1.00	0.05	0.99	0.06	0.97
0.3	0.5	0.18	1.00	0.15	1.00	0.07	1.00	0.05	1.00
0.7	-0.5	0.49	1.00	0.52	1.00	0.08	0.97	0.08	0.96
0.7	0	0.50	1.00	0.40	1.00	0.04	0.99	0.07	0.98
0.7	0.5	0.47	1.00	0.33	1.00	0.08	1.00	0.05	1.00

(b) $\delta_i = 1.0$

		Asymptotic				Bootstrap			
α	ρ	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
0	-0.5	0.04	1.00	0.09	1.00	0.06	1.00	0.04	1.00
0	0	0.04	1.00	0.07	1.00	0.05	1.00	0.06	1.00
0	0.5	0.05	1.00	0.05	1.00	0.07	1.00	0.06	1.00
0.3	-0.5	0.10	1.00	0.14	1.00	0.06	1.00	0.06	1.00
0.3	0	0.11	1.00	0.14	1.00	0.04	1.00	0.05	1.00
0.3	0.5	0.12	1.00	0.10	1.00	0.04	1.00	0.04	1.00
0.7	-0.5	0.14	1.00	0.17	1.00	0.07	1.00	0.05	1.00
0.7	0	0.20	1.00	0.22	1.00	0.06	1.00	0.08	1.00
0.7	0.5	0.14	1.00	0.11	1.00	0.05	1.00	0.05	1.00

(c) $\delta_i = 1.5$

		Asymptotic				Bootstrap			
α	ρ	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
0	-0.5	0.01	1.00	0.02	1.00	0.06	1.00	0.06	1.00
0	0	0.02	1.00	0.03	1.00	0.05	1.00	0.05	1.00
0	0.5	0.01	1.00	0.02	1.00	0.07	1.00	0.05	1.00
0.3	-0.5	0.03	1.00	0.04	1.00	0.07	1.00	0.07	1.00
0.3	0	0.03	1.00	0.04	1.00	0.03	1.00	0.04	1.00
0.3	0.5	0.02	1.00	0.02	1.00	0.04	1.00	0.03	1.00
0.7	-0.5	0.02	1.00	0.02	1.00	0.03	1.00	0.03	1.00
0.7	0	0.04	1.00	0.05	1.00	0.05	1.00	0.07	1.00
0.7	0.5	0.03	1.00	0.02	1.00	0.04	1.00	0.02	1.00

Note: T=100. The true break dates are T/2. The null hypotheses are as follows: (1) R=I and r=(0.5,0.5)', (2) R=I and r=(0.525,0.475)', (3) R=[1,-1] and r=0 and (4) R=[1,-1] and r=0.05. Hence, the null rejection probabilities for (1) and (3) stand for the finite sample sizes while those for (2) and (4) are powers.

Table 2. Probabilities of Rejecting the Null Hypothesis, GLS-Wald test

(a) $\delta_i = 0.5$

		Asymptotic				Bootstrap			
α	ρ	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
0	-0.5	0.13	0.99	0.08	0.96	0.05	0.94	0.04	0.90
0	0	0.13	1.00	0.08	0.98	0.07	0.98	0.04	0.97
0	0.5	0.15	1.00	0.07	1.00	0.05	1.00	0.04	1.00
0.3	-0.5	0.14	0.99	0.14	0.97	0.06	0.93	0.05	0.88
0.3	0	0.15	1.00	0.15	0.99	0.05	0.98	0.04	0.97
0.3	0.5	0.17	1.00	0.09	1.00	0.06	1.00	0.05	1.00
0.7	-0.5	0.43	1.00	0.22	0.99	0.06	0.67	0.05	0.82
0.7	0	0.37	1.00	0.21	1.00	0.03	0.87	0.05	0.92
0.7	0.5	0.40	1.00	0.27	1.00	0.06	0.99	0.05	1.00

(b) $\delta_i = 1.0$

		Asymptotic				Bootstrap			
α	ρ	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
0	-0.5	0.32	1.00	0.09	1.00	0.03	1.00	0.06	1.00
0	0	0.29	1.00	0.09	1.00	0.06	1.00	0.03	1.00
0	0.5	0.33	1.00	0.23	1.00	0.03	1.00	0.05	1.00
0.3	-0.5	0.28	1.00	0.13	1.00	0.03	1.00	0.06	1.00
0.3	0	0.35	1.00	0.23	1.00	0.04	1.00	0.03	1.00
0.3	0.5	0.34	1.00	0.23	1.00	0.04	1.00	0.04	1.00
0.7	-0.5	0.21	1.00	0.18	1.00	0.06	0.90	0.07	1.00
0.7	0	0.30	1.00	0.26	1.00	0.05	0.99	0.06	1.00
0.7	0.5	0.22	1.00	0.15	1.00	0.03	1.00	0.07	1.00

(c) $\delta_i = 1.5$

		Asymptotic				Bootstrap			
α	ρ	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
0	-0.5	0.08	1.00	0.08	1.00	0.08	1.00	0.05	1.00
0	0	0.13	1.00	0.13	1.00	0.04	1.00	0.06	1.00
0	0.5	0.09	1.00	0.06	1.00	0.08	1.00	0.06	1.00
0.3	-0.5	0.07	1.00	0.07	1.00	0.07	1.00	0.07	1.00
0.3	0	0.10	1.00	0.09	1.00	0.06	1.00	0.05	1.00
0.3	0.5	0.07	1.00	0.05	1.00	0.06	1.00	0.05	1.00
0.7	-0.5	0.03	1.00	0.03	1.00	0.03	0.78	0.03	1.00
0.7	0	0.07	1.00	0.07	1.00	0.07	0.99	0.07	1.00
0.7	0.5	0.04	1.00	0.02	1.00	0.04	1.00	0.02	1.00

Note: T=100. The true break dates are T/2. The null hypotheses are as follows: (1) R=I and r=(0.5,0.5)', (2) R=I and r=(0.525,0.475)', (3) R=[1,-1] and r=0 and (4) R=[1,-1] and r=0.05. Hence, the null rejection probabilities for (1) and (3) stand for the finite sample sizes while those for (2) and (4) are powers.

Table 3. Probabilities of Rejecting the Null Hypothesis, OLS-Wald test

(a) $\delta_i = 0.5$

		Asymptotic				Bootstrap			
α	ρ	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
0	-0.5	0.16	0.99	0.09	0.96	0.04	0.94	0.05	0.90
0	0	0.11	1.00	0.08	0.98	0.05	0.99	0.04	0.96
0	0.5	0.15	1.00	0.11	1.00	0.05	1.00	0.05	1.00
0.3	-0.5	0.15	0.99	0.13	0.97	0.06	0.94	0.05	0.91
0.3	0	0.15	1.00	0.15	0.99	0.05	0.98	0.06	0.97
0.3	0.5	0.15	1.00	0.09	1.00	0.05	1.00	0.05	1.00
0.7	-0.5	0.39	1.00	0.20	0.99	0.05	0.80	0.04	0.83
0.7	0	0.37	1.00	0.20	1.00	0.05	0.93	0.05	0.94
0.7	0.5	0.35	1.00	0.21	1.00	0.04	0.99	0.03	1.00

(b) $\delta_i = 1.0$

		Asymptotic				Bootstrap			
α	ρ	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
0	-0.5	0.35	1.00	0.07	1.00	0.02	1.00	0.06	1.00
0	0	0.30	1.00	0.09	1.00	0.05	1.00	0.02	1.00
0	0.5	0.37	1.00	0.31	1.00	0.02	1.00	0.01	1.00
0.3	-0.5	0.35	1.00	0.11	1.00	0.03	1.00	0.05	1.00
0.3	0	0.33	1.00	0.22	1.00	0.04	1.00	0.02	1.00
0.3	0.5	0.32	1.00	0.25	1.00	0.03	1.00	0.02	1.00
0.7	-0.5	0.26	1.00	0.20	1.00	0.05	0.96	0.05	0.99
0.7	0	0.28	1.00	0.25	1.00	0.06	1.00	0.04	1.00
0.7	0.5	0.26	1.00	0.20	1.00	0.05	1.00	0.06	1.00

(c) $\delta_i = 1.5$

		Asymptotic				Bootstrap			
α	ρ	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
0	-0.5	0.12	1.00	0.12	1.00	0.06	1.00	0.05	1.00
0	0	0.10	1.00	0.10	1.00	0.04	1.00	0.03	1.00
0	0.5	0.12	1.00	0.11	1.00	0.07	1.00	0.06	1.00
0.3	-0.5	0.09	1.00	0.09	1.00	0.06	1.00	0.06	1.00
0.3	0	0.09	1.00	0.09	1.00	0.04	1.00	0.05	1.00
0.3	0.5	0.09	1.00	0.09	1.00	0.06	1.00	0.05	1.00
0.7	-0.5	0.06	1.00	0.06	1.00	0.03	0.95	0.04	1.00
0.7	0	0.07	1.00	0.07	1.00	0.04	0.99	0.04	1.00
0.7	0.5	0.07	1.00	0.07	1.00	0.04	1.00	0.05	1.00

Note: T=100. The true break dates are T/2. The null hypotheses are as follows: (1) R=I and r=(0.5,0.5)', (2) R=I and r=(0.525,0.475)', (3) R=[1,-1] and r=0 and (4) R=[1,-1] and r=0.05. Hence, the null rejection probabilities for (1) and (3) stand for the finite sample sizes while those for (2) and (4) are powers.

Table 4. P-values of Common Break Date Tests, GLS-Wald/LR

(a) Original Temperature Series and Anthropogenic Forcing, 1900-1992

		Wald		LR		Break Dates		
forc.	temp.	asym.	boot.	asym.	boot.	forc.	temp.	comm.
W	G H	-	0.19	0.01	0.11	1963	1985	1962
	G N	-	0.36	-	0.21	1963	1976	1963
	G B	-	0.18	-	0.08	1963	1941	1963
	G K	0.01	0.52	0.01	0.30	1963	1976	1963
	Avg G	-	0.21	0.01	0.22	1963	1985	1963
	N H	-	0.19	-	0.06	1963	1985	1962
	N N	-	0.21	-	0.14	1962	1942	1962
	Avg N	-	0.31	0.01	0.15	1963	1985	1962
	S H	-	0.34	0.03	0.21	1963	1976	1963
	S N	-	0.26	0.20	0.65	1963	1967	1963
	Avg S	-	0.03	0.03	0.25	1963	1975	1963
TRF	G H	0.05	0.51	0.01	0.13	1966	1978	1966
	G N	0.24	0.63	0.03	0.43	1966	1975	1966
	G B	0.03	0.58	-	0.11	1966	1942	1965
	G K	-	0.04	0.04	0.51	1965	1905	1966
	Avg G	0.29	0.72	0.03	0.34	1966	1976	1966
	N H	-	0.25	0.01	0.07	1966	1985	1966
	N N	0.09	0.63	-	0.22	1965	1940	1965
	Avg N	-	0.38	0.01	0.20	1966	1985	1965
	S H	0.05	0.49	0.05	0.25	1966	1976	1966
	S N	1.00	1.00	1.00	1.00	1966	1966	1966
	Avg S	0.05	0.39	0.29	0.62	1966	1975	1966

Note: Each temperature series is denoted by two letters. For the first one, G, N and S stand for global, northern and southern hemispheric temperatures, respectively. For the second one, H, N, B and K stand for HadCRUT4, NASA, Berkeley and Karl. Avg stands for average. W and TRF stand for well-mixed green-house gases and total radiative forcing, respectively. Entries less than 0.01 are marked as "-".

Table 4. P-values of Common Break Date Tests, GLS-Wald/LR (cont.)

(b) Filtered Temperature Series and Anthropogenic Forcing, 1900-1992

		Wald		LR		Break Dates		
forc.	temp.	asym.	boot.	asym.	boot.	forc.	temp.	comm.
W	$\tilde{G}$ H	0.23	0.48	0.54	0.72	1963	1964	1963
	$\tilde{G}$ N	-	0.01	0.25	0.50	1963	1957	1963
	$\tilde{G}$ B	-	0.15	0.34	0.67	1963	1956	1963
	$\tilde{G}$ K	-	0.04	0.17	0.45	1963	1956	1963
	Avg $\tilde{G}$	-	0.09	0.40	0.62	1963	1960	1963
	$\tilde{N}$ H	0.05	0.29	0.38	0.56	1963	1965	1963
	$\tilde{N}$ N	0.01	0.29	0.54	0.77	1963	1965	1963
	Avg $\tilde{N}$	0.02	0.28	0.43	0.66	1963	1965	1963
	$\tilde{S}$ H	-	0.10	0.18	0.41	1963	1956	1963
	$\tilde{S}$ N	-	-	0.02	0.22	1963	1954	1963
	Avg $\tilde{S}$	-	-	0.05	0.22	1963	1955	1963
TRF	$\tilde{G}$ H	0.79	0.86	0.66	0.84	1966	1965	1966
	$\tilde{G}$ N	0.13	0.39	0.18	0.51	1966	1960	1966
	$\tilde{G}$ B	0.33	0.61	0.34	0.66	1966	1960	1966
	$\tilde{G}$ K	0.20	0.48	0.19	0.53	1966	1960	1966
	Avg $\tilde{G}$	0.15	0.41	0.34	0.64	1966	1960	1966
	$\tilde{N}$ H	1.00	1.00	1.00	1.00	1966	1966	1966
	$\tilde{N}$ N	0.83	0.89	0.69	0.87	1966	1965	1966
	Avg $\tilde{N}$	0.80	0.86	0.83	0.91	1966	1965	1966
	$\tilde{S}$ H	0.04	0.29	0.08	0.31	1966	1956	1966
	$\tilde{S}$ N	0.24	0.49	0.04	0.31	1966	1960	1966
	Avg $\tilde{S}$	0.03	0.23	0.05	0.26	1966	1956	1966

Note: Each temperature series is denoted by two letters. For the first one, G, N and S stand for global, northern and southern hemispheric temperatures, respectively. For the second one, H, N, B and K stand for HadCRUT4, NASA, Berkeley and Karl. Avg stands for average. The filtered temperature series are denoted with a tilde. W and TRF stand for well-mixed green-house gases and total radiative forcing, respectively. Entries less than 0.01 are marked as "-".

Table 4. P-values of Common Break Date Tests, GLS-Wald/LR (cont.)

(c) Original Temperature Series and Anthropogenic Forcing, 1963-2014

		Wald		LR		Break Dates		
forc.	temp.	asym.	boot.	asym.	boot.	forc.	temp.	comm.
W	G H	0.01	0.48	0.37	0.64	1992	2007	1992
	G N	0.01	0.65	0.22	0.53	1992	2006	1992
	G B	-	0.28	0.27	0.57	1992	1975	1992
	G K	0.08	0.73	0.34	0.69	1992	2005	1992
	Avg G	0.03	0.59	0.47	0.82	1992	2007	1992
	N H	0.35	0.67	0.33	0.61	1992	1985	1992
	N N	0.31	0.65	0.41	0.72	1992	1985	1992
	Avg N	0.33	0.66	0.35	0.65	1992	1985	1992
	S H	0.08	0.70	0.27	0.62	1992	2005	1992
	S N	-	0.09	0.01	0.06	1992	1972	1992
	Avg S	-	0.20	0.19	0.46	1992	1969	1992
TRF	G H	-	0.25	0.11	0.35	1990	1971	1990
	G N	-	0.34	0.10	0.28	1991	2007	1991
	G B	-	0.34	0.18	0.50	1990	1976	1990
	G K	-	0.01	0.18	0.43	1991	1966	1991
	Avg G	-	0.26	0.22	0.54	1991	1971	1990
	N H	-	0.14	0.09	0.29	1990	1969	1990
	N N	-	0.10	0.03	0.22	1991	1972	1990
	Avg N	-	0.14	0.05	0.26	1991	1971	1990
	S H	-	0.33	0.23	0.55	1991	2006	1990
	S N	-	0.04	0.01	0.08	1991	1980	1990
	Avg S	-	0.44	0.38	0.70	1991	1983	1991

Note: Each temperature series is denoted by two letters. For the first one, G, N and S stand for global, northern and southern hemispheric temperatures, respectively. For the second one, H, N, B and K stand for HadCRUT4, NASA, Berkeley and Karl. Avg stands for average. W and TRF stand for well-mixed green-house gases and total radiative forcing, respectively. Entries less than 0.01 are marked as "-".

Table 4. P-values of Common Break Date Tests, GLS-Wald/LR (cont.)

(d) Filtered Temperature Series and Anthropogenic Forcing, 1963-2014

		Wald		LR		Break Dates		
forc.	temp.	asym.	boot.	asym.	boot.	forc.	temp.	comm.
W	$\tilde{G}$ H	1.00	1.00	1.00	1.00	1992	1992	1992
	$\tilde{G}$ N	0.78	0.88	0.46	0.68	1992	1991	1992
	$\tilde{G}$ B	1.00	1.00	1.00	1.00	1992	1992	1992
	$\tilde{G}$ K	0.77	0.87	0.55	0.75	1992	1991	1992
	Avg $\tilde{G}$	0.76	0.87	0.74	0.87	1992	1991	1992
	$\tilde{N}$ H	-	0.28	0.17	0.37	1992	2007	1992
	$\tilde{N}$ N	0.82	0.91	0.88	0.94	1992	1991	1992
	Avg $\tilde{N}$	1.00	1.00	1.00	1.00	1992	1992	1992
	$\tilde{S}$ H	0.75	0.84	0.79	0.88	1992	1991	1992
	$\tilde{S}$ N	-	0.14	0.06	0.19	1992	1984	1992
	Avg $\tilde{S}$	0.01	0.23	0.28	0.47	1992	1986	1992
TRF	$\tilde{G}$ H	0.18	0.63	0.75	0.90	1990	1992	1991
	$\tilde{G}$ N	0.71	0.87	0.83	0.92	1990	1991	1991
	$\tilde{G}$ B	0.55	0.83	0.82	0.94	1991	1992	1991
	$\tilde{G}$ K	0.70	0.87	0.85	0.91	1990	1991	1991
	Avg $\tilde{G}$	0.58	0.82	0.95	0.94	1990	1991	1991
	$\tilde{N}$ H	-	0.07	0.21	0.47	1990	2007	1991
	$\tilde{N}$ N	0.24	0.66	0.80	0.94	1990	1991	1991
	Avg $\tilde{N}$	0.18	0.67	0.74	0.92	1990	1992	1991
	$\tilde{S}$ H	0.48	0.78	0.90	0.94	1990	1991	1991
	$\tilde{S}$ N	-	0.16	0.06	0.34	1990	1984	1990
	Avg $\tilde{S}$	-	0.26	0.34	0.64	1990	1986	1990

Note: Each temperature series is denoted by two letters. For the first one, G, N and S stand for global, northern and southern hemispheric temperatures, respectively. For the second one, H, N, B and K stand for HadCRUT4, NASA, Berkeley and Karl. Avg stands for average. The filtered temperature series are denoted with a tilde. W and TRF stand for well-mixed green-house gases and total radiative forcing, respectively. Entries less than 0.01 are marked as "-".

Table 5. P-values of Common Break Date Tests, OLS-Wald

(a) Original Temperature Series and Anthropogenic Forcing, 1900-1992

		W (1963)		TRF (1966)	
		asym.	boot.	asym.	boot.
G H	(1985)	-	0.13	-	0.16
G N	(1976)	-	0.33	0.16	0.55
G B	(1941)	-	0.14	0.02	0.54
G K	(1976)	0.01	0.51	0.26	0.69
Avg G	(1985)	-	0.18	-	0.34
N H	(1985)	-	0.17	-	0.18
N N	(1939)	-	0.39	0.06	0.58
Avg N	(1985)	-	0.28	-	0.31
S H	(1976)	-	0.33	0.05	0.47
S N	(1966)	0.01	0.32	1.00	1.00
Avg S	(1975)	-	0.02	0.05	0.37

(b) Filtered Temperature Series and Anthropogenic Forcing, 1900-1992

		W (1963)		TRF (1966)	
		asym.	boot.	asym.	boot.
$\tilde{G}$ H	(1965)	0.01	0.19	0.79	0.86
$\tilde{G}$ N	(1960)	-	-	0.13	0.40
$\tilde{G}$ B	(1960)	-	0.16	0.33	0.61
$\tilde{G}$ K	(1960)	-	0.03	0.20	0.50
Avg $\tilde{G}$	(1960)	-	0.07	0.15	0.41
$\tilde{N}$ H	(1966)	-	0.13	1.00	1.00
$\tilde{N}$ N	(1965)	0.01	0.20	0.83	0.89
Avg $\tilde{N}$	(1965)	0.02	0.22	0.80	0.86
$\tilde{S}$ H	(1956)	-	0.06	0.04	0.34
$\tilde{S}$ N	(1960)	-	0.15	0.25	0.53
Avg $\tilde{S}$	(1956)	-	-	0.03	0.24

Note: Each temperature series is denoted by two letters. For the first one, G, N and S stand for global, northern and southern hemispheric temperatures, respectively. For the second one, H, N, B and K stand for HadCRUT4, NASA, Berkeley and Karl. Avg stands for average. The filtered temperature series are denoted with a tilde. W and TRF stand for well-mixed green-house gases and total radiative forcing, respectively. Entries less than 0.01 are marked as "-". The numbers in parentheses are break dates.

Table 5. P-values of Common Break Date Tests, OLS-Wald (cont.)

(c) Original Temperature Series and Anthropogenic Forcing, 1963-2014

		W (1992)		TRF (1990)	
		asym.	boot.	asym.	boot.
G H	(1974)	-	0.29	-	0.26
G N	(1966)	-	0.16	-	0.01
G B	(1975)	-	0.26	-	0.21
G K	(1966)	-	0.10	-	0.00
Avg G	(1974)	-	0.38	-	0.24
N H	(1972)	-	0.15	-	0.18
N N	(1972)	-	0.12	-	0.07
Avg N	(1972)	-	0.13	-	0.11
S H	(2005)	0.14	0.71	-	0.43
S N	(1980)	-	0.14	-	0.05
Avg S	(1987)	0.44	0.75	0.12	0.61

(d) Filtered Temperature Series and Anthropogenic Forcing, 1963-2014

		W (1992)		TRF (1990)	
		asym.	boot.	asym.	boot.
$\tilde{G}$ H	(1992)	1.00	1.00	0.18	0.64
$\tilde{G}$ N	(1991)	0.78	0.89	0.71	0.88
$\tilde{G}$ B	(1992)	1.00	1.00	0.28	0.68
$\tilde{G}$ K	(1991)	0.77	0.89	0.70	0.89
Avg $\tilde{G}$	(1991)	0.77	0.87	0.59	0.83
$\tilde{N}$ H	(2007)	-	0.21	0.00	0.07
$\tilde{N}$ N	(1991)	0.82	0.89	0.25	0.67
Avg $\tilde{N}$	(1992)	1.00	1.00	0.18	0.65
$\tilde{S}$ H	(1991)	0.75	0.86	0.48	0.78
$\tilde{S}$ N	(1984)	-	0.16	-	0.15
Avg $\tilde{S}$	(1985)	-	0.15	-	0.14

Note: Each temperature series is denoted by two letters. For the first one, G, N and S stand for global, northern and southern hemispheric temperatures, respectively. For the second one, H, N, B and K stand for HadCRUT4, NASA, Berkeley and Karl. Avg stands for average. The filtered temperature series are denoted with a tilde. W and TRF stand for well-mixed green-house gases and total radiative forcing, respectively. Entries less than 0.01 are marked as "-". The numbers in parentheses are break dates.

Table 6. P-values of Common Break Date Tests, OLS-Wald (cont.)

(a) Multivariate Systems, 1900-1992

		HadCRUT4			NASA			Average		
		p-value	Break Dates		p-value	Break Dates		p-value	Break Dates	
Forc.	Temp.	asymp./boot.	Forcing	Temp.	asymp./boot.	Forcing	Temp.	asymp./boot.	Forcing	Temp.
W	G,N,S	- / 0.22	1963		- / 0.58	1963		- / 0.10	1963	
	G	- / -		1985	- / 0.44		1976	- / 0.26		1985
	N	- / 0.53		1985	- / 0.46		1939	- / 0.30		1985
	S	- / 0.28		1976	0.13 / 0.52		1966	- / 0.06		1975
TRF	G,N,S	- / 0.16	1966		- / 0.57	1966		- / 0.13	1966	
	G	- / 0.43		1985	0.04 / 0.46		1976	- / 0.41		1985
	N	- / 0.34		1985	- / 0.41		1939	- / 0.30		1985
	S	0.01 / 0.47		1976	1.00 / 0.95		1966	- / 0.05		1975
W	$\tilde{G}$,$\tilde{N}$,$\tilde{S}$	- / 0.10	1963		- / -	1963		- / -	1963	
	$\tilde{G}$	0.06 / 0.35		1965	- / -		1960	- / 0.12		1960
	$\tilde{N}$	- / 0.34		1966	0.04 / 0.40		1965	0.05 / 0.32		1965
	$\tilde{S}$	- / 0.13		1956	0.02 / 0.48		1960	- / -		1956
TRF	$\tilde{G}$,$\tilde{N}$,$\tilde{S}$	- / 0.08	1966		- / -	1966		- / -	1966	
	$\tilde{G}$	0.35 / 0.68		1965	- / 0.03		1960	- / 0.02		1960
	$\tilde{N}$	1.00 / 0.94		1966	0.17 / 0.57		1965	0.22 / 0.47		1965
	$\tilde{S}$	- / 0.11		1956	- / 0.35		1960	- / -		1956

Note: G, N and S stand for global, northern and southern hemispheric temperatures, respectively. The filtered temperature series are denoted with a tilde. W and TRF stand for well-mixed green-house gases and total radiative forcing, respectively. Entries less than 0.01 are marked as "-".

Table 6. P-values of Common Break Date Tests, OLS-Wald (cont.)

(b) Multivariate Systems, 1963-2014

		HadCRUT4			NASA			Average		
		p-value	Break Dates		p-value	Break Dates		p-value	Break Dates	
Forc.	Temp.	asymp./boot.	Forcing	Temp.	asymp./boot.	Forcing	Temp.	asymp./boot.	Forcing	Temp.
W	G,N,S	- / 0.36	1992		- / 0.10	1992		- / 0.49	1992	
	G	- / 0.29		1974	- / 0.07		1966	- / 0.29		1974
	N	- / 0.19		1972	- / 0.13		1972	- / 0.16		1972
	S	0.01/ 0.55		2005	- / 0.13		1980	0.07 / 0.60		1987
TRF	G,N,S	- / 0.36	1990		- / 0.05	1990		- / 0.35	1990	
	G	- / 0.29		1974	- / 0.01		1966	- / 0.30		1974
	N	- / 0.18		1972	- / 0.08		1972	- / 0.12		1972
	S	- / 0.48		2005	- / 0.05		1980	0.22 / 0.62		1987
W	$\tilde{G}$,$\tilde{N}$,$\tilde{S}$	- / 0.13	1992		- / 0.21	1992		- / 0.15	1992	
	$\tilde{G}$	1.00 / 0.93		1992	0.77 / 0.86		1991	0.70 / 0.82		1991
	$\tilde{N}$	- / 0.19		2007	0.48 / 0.78		1991	1.00 / 0.95		1992
	$\tilde{S}$	0.63 / 0.77		1991	- / 0.20		1984	- / 0.14		1985
TRF	$\tilde{G}$,$\tilde{N}$,$\tilde{S}$	- / 0.10	1990		- / 0.18	1990		- / 0.14	1990	
	$\tilde{G}$	0.23 / 0.57		1992	0.72 / 0.88		1991	0.61 / 0.83		1991
	$\tilde{N}$	- / 0.08		2007	0.27 / 0.71		1991	0.21 / 0.65		1992
	$\tilde{S}$	0.50 / 0.70		1991	- / 0.17		1984	- / 0.13		1985

Note: G, N and S stand for global, northern and southern hemispheric temperatures, respectively. The filtered temperature series are denoted with a tilde. W and TRF stand for well-mixed green-house gases and total radiative forcing, respectively. Entries less than 0.01 are marked as "-".

Table 7: P-values of the LR test for no break in temperature series

Filtered Temperature Series and Anthropogenic Forcing

		1900-1992 ($T = 93$)		1963-2014 ($T = 52$)	
		p-value	Break date	p-value	Break date
Forc.	Temp.	(LR test)	under H_1	(LR test)	under H_1
W	$\widetilde{G}$ H	0.00	1964	0.08	1992
	$\widetilde{G}$ N	0.00	1957	0.01	1991
	$\widetilde{G}$ B	0.04	1956	0.12	1992
	$\widetilde{G}$ K	0.00	1956	0.02	1991
	Avg $\widetilde{G}$	0.01	1960	0.04	1991
	$\widetilde{N}$ H	0.01	1965	0.16	2007
	$\widetilde{N}$ N	0.01	1965	0.19	1991
	Avg $\widetilde{N}$	0.00	1965	0.21	1992
	$\widetilde{S}$ H	0.00	1956	0.03	1991
	$\widetilde{S}$ N	0.01	1954	0.00	1984
	Avg $\widetilde{S}$	0.00	1955	0.01	1986
TRF	$\widetilde{G}$ H	0.00	1965	0.06	1992
	$\widetilde{G}$ N	0.00	1960	0.01	1991
	$\widetilde{G}$ B	0.04	1960	0.09	1992
	$\widetilde{G}$ K	0.01	1960	0.02	1991
	Avg $\widetilde{G}$	0.00	1960	0.03	1991
	$\widetilde{N}$ H	0.00	1966	0.15	2007
	$\widetilde{N}$ N	0.01	1965	0.14	1991
	Avg $\widetilde{N}$	0.00	1965	0.19	1992
	$\widetilde{S}$ H	0.00	1956	0.02	1991
	$\widetilde{S}$ N	0.00	1960	0.00	1984
	Avg $\widetilde{S}$	0.00	1956	0.01	1986

Note: Each temperature series is denoted by two letters. For the first one, G, N and S stand for global, northern and southern hemispheric temperature respectively. For the second one, H, N, B and K stand for HadCRUT4, NASA, Berkeley and Karl. Avg stands for average. W and TRF stand for well-mixed green-house gases and total radiative forcing, repectively. The third and fifth columns report p-values for the LR test obtained from a bootstrap procedure and the fourth and sixth columns present the break date estimate in the temperature series under the alternative hypothesis given an estimated break in the forcing series.

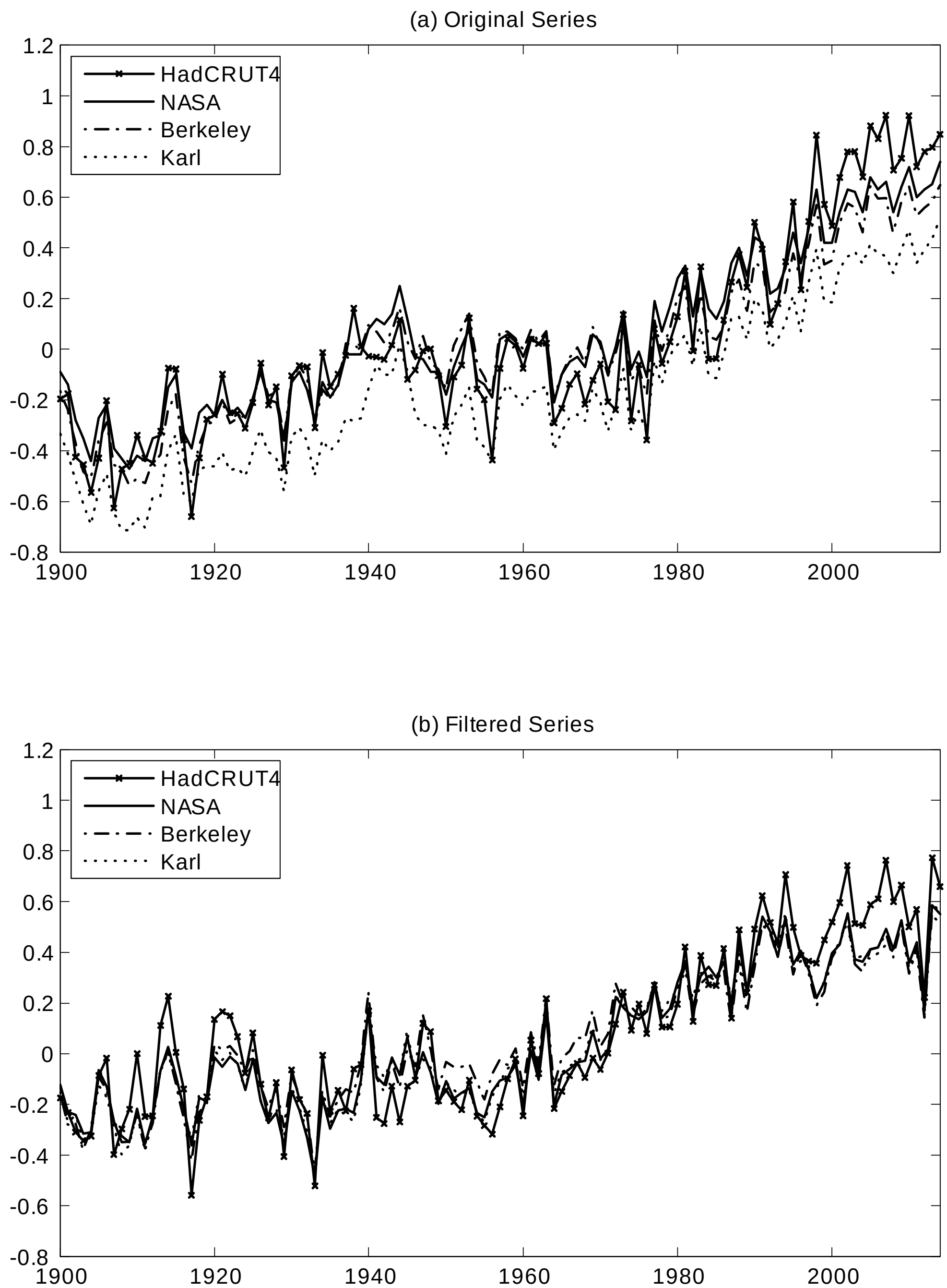

Figure 1: Global Temperature Series

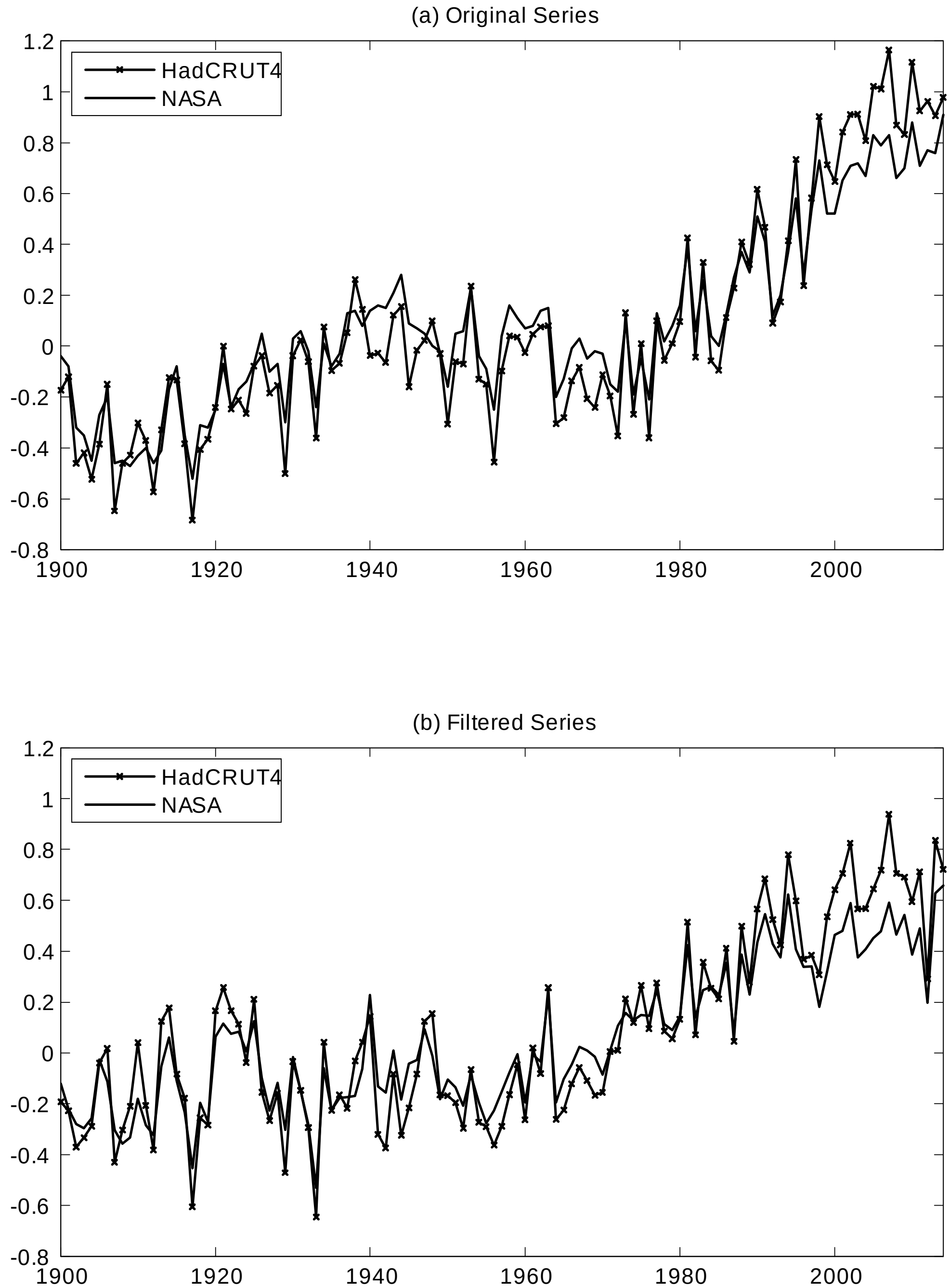

Figure 2: Northern Hemispheric Temperature Series

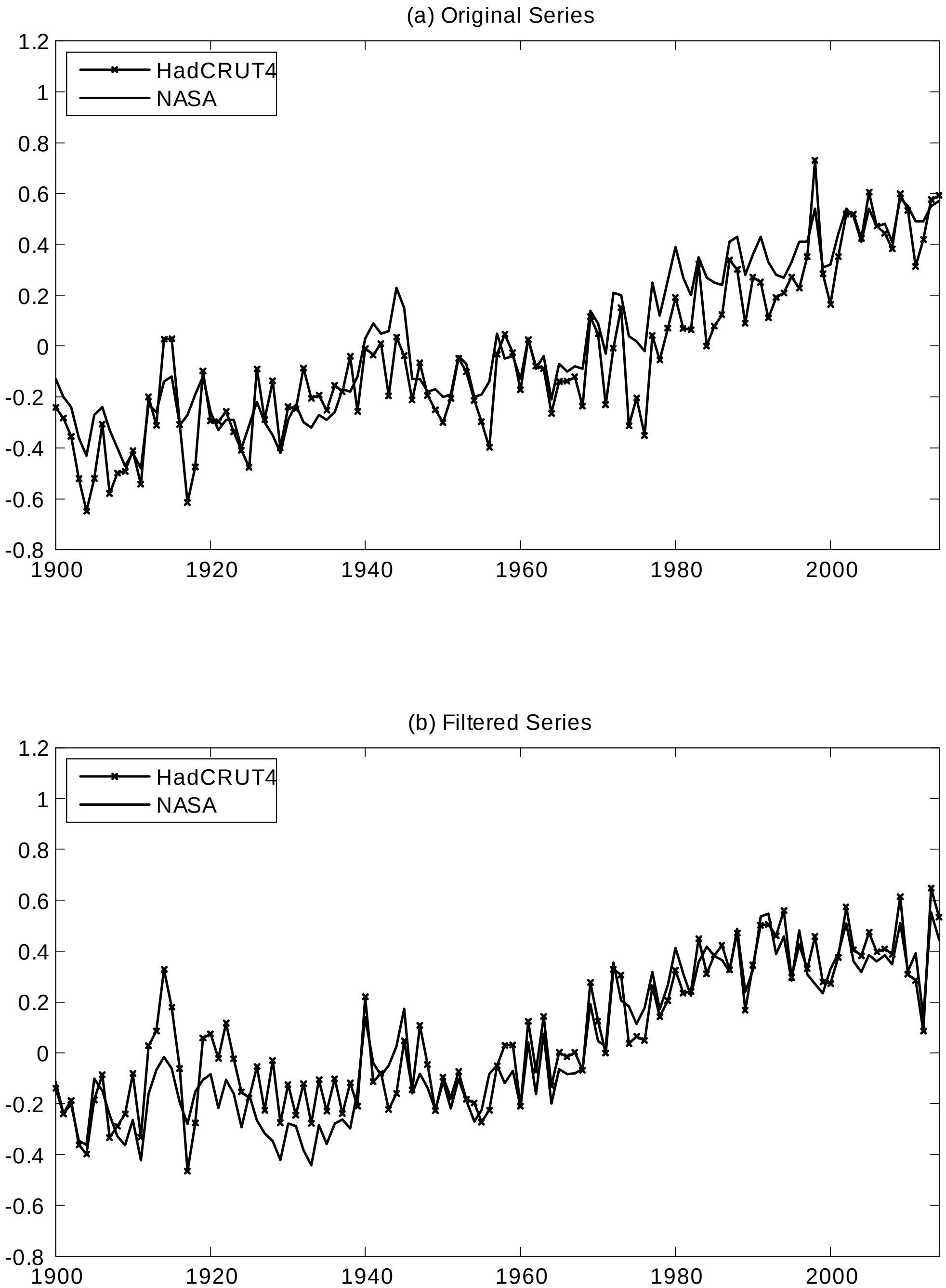

Figure 3: Southern Hemispheric Temperature Series

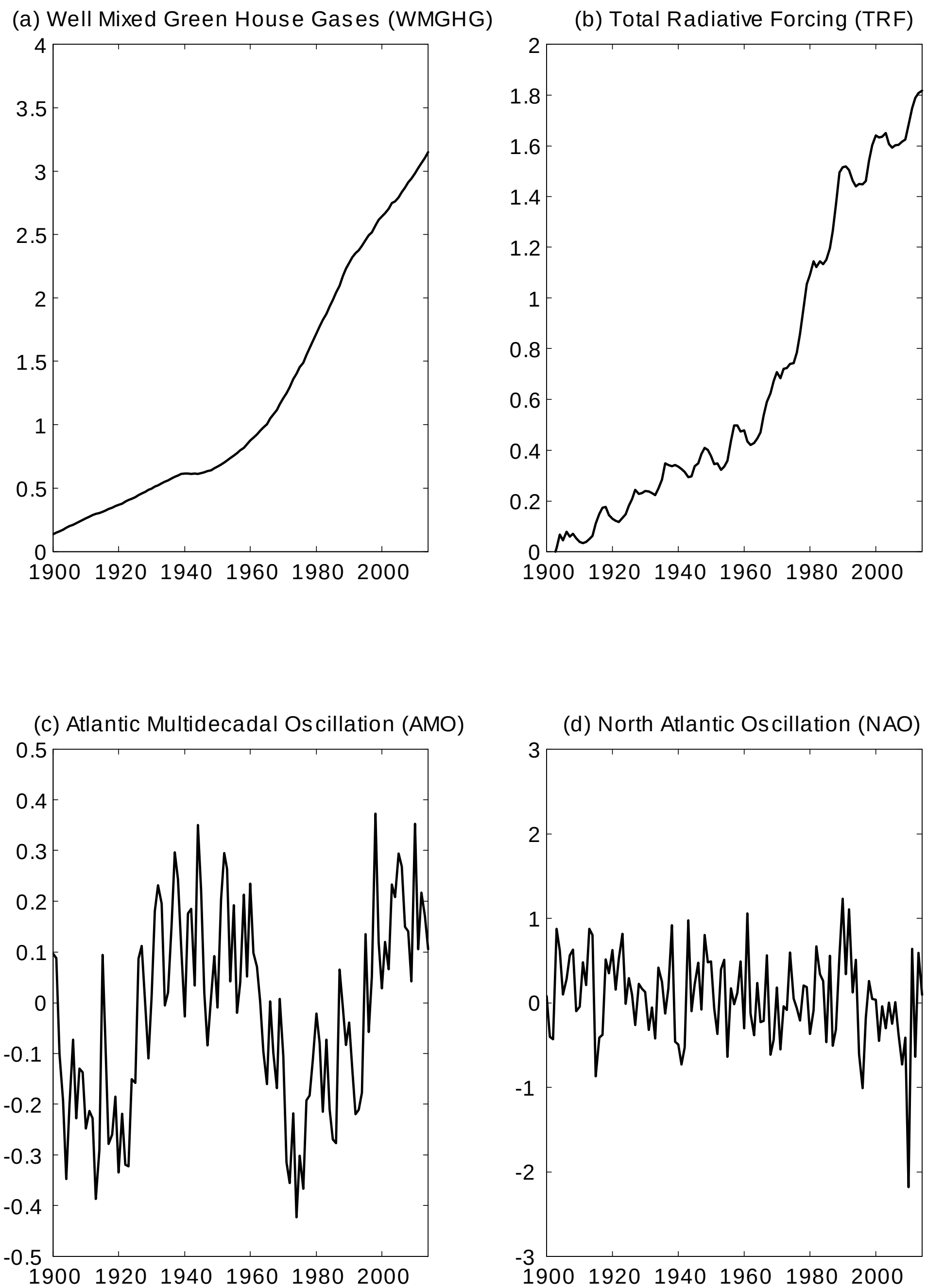

Figure 4: Forcing Variables and Modes of Variablility