WEALTH EFFECTS, INCENTIVES AND PRODUCTIVITY*

Dilip Mookherjee

Forthcoming, Review of Development Economics

ABSTRACT

Comparative static effects of varying the wealth level of a risk averse agent in a moral hazard setting with limited liability constraints are investigated. There are two principal opposing effects of increasing wealth: the incentive effect, which allows stronger punishments for poor performance, thereby encouraging higher effort; and the preference effect, which reduces the agent's effort incentives owing to income effects in the demand for leisure. It is shown that optimal effort levels are initially constant, subsequently increasing and eventually decreasing in wealth. Hence agents with intermediate wealth levels are the most productive.

* Department of Economics, Boston University, 270 Bay State Road, Boston MA 02215; (617)3534392(tel), (617)3534143(fax), dilipm@bu.edu. I would like to thank the MacArthur Foundation for research support, and to Debraj Ray for useful discussions.
1 Introduction

Recent analyses of moral hazard with limited liability constraints have yielded predictions concerning the effect of wealth inequality or asset ownership on effort incentives. These models generate explanations for numerous agrarian institutions in developing countries: for instance, why the poor may be involuntarily unemployed, evicted from tenancy, or have limited access to credit. They have also generated theories concerning the effects of land reforms of different kinds.\(^1\) A key element of these models is the dependence of optimal contracts on wealth levels. Specifically, higher wealth owned by the agent may generate higher effort incentives because they permit greater punishment for poor performance, which correspondingly result in transfer of benefits to the principal. For instance, a wealthier agent can post larger collateral, thereby committing to a higher effort level, since he has more to lose in the event of a low income realization. Similarly, in a tenancy context, the landlord can extract a larger rent in bad times from a wealthier tenant. In contrast, the only way to get the poor to work hard is to increase rewards for good performance, which is costly to the principal. This effect can therefore be called the incentive effect.

Less attention has been devoted by the literature to a countervailing effect which arises if the agent's utility over consumption is concave. Increased wealth levels tend to diminish the marginal utility of consumption, thus generating an increased aversion to selecting high effort levels. This is essentially an income effect on the demand for leisure, which implies that it is more difficult to induce the agent to work hard. This direct preference effect would be strong whenever the agent exhibits a strong preference for consumption smoothing.

There is considerable evidence for preferences for consumption smoothing in the agrarian sectors of many developing countries.\(^2\) So it is natural to ask what the net result of the two effects will be in general. Despite the central role played by this model in diverse contexts, this question does not appear to have been addressed satisfactorily in existing literature.\(^3\)

The purpose of this paper is to pose this question in a simple context where there are two possible outcomes ('success' and 'failure') of the agent's effort on which transfers can be conditioned. What makes the problem somewhat non-trivial is that the optimal contracting
problem is not a concave maximization problem, in the absence of strong assumptions on
the curvature of marginal utility of consumption or marginal disutility of effort. Moreover,
the presence of the two contrasting effects of wealth variations implies that single-crossing
conditions required by monotone comparative static analyses (Milgrom and Shannon, 1994)
fail to apply.

**INSERT FIGURE 1**

Our main results are summarized in Figure 1. In general, effort is initially constant with
respect to wealth increases (upto \( w \)), then (in a suitable sense) tends to rise with wealth over
an intermediate range (from \( w \) to \( \tilde{w} \)), and then fall thereafter. More precisely, there exists
an increasing sequence of wealth levels starting at \( w \) along which effort initially increases
and then decreases monotonically. If the agent’s preferences satisfies Inada conditions then
they asymptotically approach the minimum conceivable effort level \( c \). This sequence is
depicted by the solid dots in Figure 1. With stronger assumptions, specifically that the
marginal disutility of effort is convex, it is shown that optimal effort levels are as depicted
by the dotted line: initially constant, then rising over the range \( w \) to \( \tilde{w} \), and falling away
thereafter to \( c \). It is notable that these results do not require any assumptions concerning
how risk aversion varies with wealth.

For very poor agents, therefore, wealth effects are locally absent. Along this range, the
agent receives rents owing to an ‘efficiency wage’ contract which provides incentives solely
via rewards for good performance. Since this contract awards the agent with an expected
payoff exceeding his reservation utility, the same contract continues to be optimal if wealth
increases slightly. At \( w \), the efficiency wage contract generates a utility exactly equal to
the agent’s reservation utility. Small wealth increases thereafter then render the efficiency
wage contract infeasible: the principal reacts by designing a contract which increases the
reward for good performance, as the ‘incentive’ effect dominates the ‘preference’ effect over
this range. For higher wealth levels, however, the preference effect dominates, thus causing
effort levels to decline.\(^4\)

These results can therefore be interpreted as asserting that the middle class are the most
productive, striking the right balance between the incentive and preference effects.
The poor are less productive because they cannot be punished for poor performance, as they have so little to lose. The rich, despite having much to lose, on the other hand, are also less productive because they are lazy. The middle class stand to lose a reasonable amount in case of poor performance, but are easier to motivate than the rich. So they end up selecting maximal levels of risk and work effort. While the model analyzed here is exceedingly simple, it forms an essential building block in more complicated dynamic models of growth and inequality. The results suggest some intriguing hypotheses that could be explored with such models: for instance TFP growth net of that contributed by technological innovations will tend to be highest at intermediate stages of development, and that the effects of wealth inequality depend on the importance of the middle class relative to the poor or the rich.

Section 2 introduces the model, and Section 3 the main results. Section 4 concludes with a discussion of possible extensions to contracts with eviction, and the effects of land reform. Proofs are collected in the Appendix.

2 The Model

It is most convenient to adapt the model in my earlier paper (Mookherjee, 1994) to accommodate concave utility over consumption for the agent. There is a single risk neutral principal (e.g., employer or landlord or lender, hereafter denoted L), and a single risk-averse agent (e.g., worker or tenant or borrower or farmer, denoted by F). L can distinguish between two possible outcomes of the work done by F, denoted success (s) and failure (f) respectively. Outcome $i = s, f$ generates a gross monetary benefit (e.g., sale value of the crop) to F of $i$, where $s > f$. A financial contract between L and F represents a transfer from $t_i$ following outcome $i$, $i = s, f$. If the farmer’s wealth level is $w$, then his net income or consumption following outcome $i$ is $c_i = w + i - t_i$. The probability of the successful outcome is increasing in the effort $e$ selected by F. We can therefore measure the effort level in terms of the induced probability of $i = s$. Effort is selected from some interval $[e, \tilde{e}] \subset [0, 1]$ and cannot be observed or contracted upon.

Preferences are as follows. F’s utility is given by $u(c) - D(e)$, where $c$ denotes con-
umption, and $U$ is strictly increasing, twice differentiable, strictly concave, satisfying the Inada conditions $u(\infty) = \infty$ and $u'(\infty) = 0$. In order to survive F’s consumption must exceed at least some lower bound, which we normalize to 0. Moreover, $u(0)$ is finite, so that we can normalize $u(0) = 0$. This ensures that the moral hazard problem cannot be solved via threats of forcing F to consume arbitrarily small amounts, which generate utility that approach negative infinity. The disutility of effort $D(\cdot)$ is strictly increasing, thrice differentiable, and strictly convex. We normalize $D(\xi) = 0$. L on the other hand is risk-neutral, seeking to maximize the expected value of the transfer $t_i \equiv w + i - c_i$.

The agent F has an exogenous level of wealth $w \geq 0$. If F and L fail to agree to a contract, F will be unemployed, in which case he will consume $w$ and exert effort $\xi$. We shall assume that L has all the bargaining power, and can offer a take-it-or-leave-it contract to F. As will be argued in Section 3, however, analysis of this problem will help identify the set of all Pareto optimal contracts, and therefore the outcome when F also has some bargaining power. The model applies equally well to tenancy, credit, insurance or hired labor contracts.

We shall also assume that an ‘efficiency wage’ phenomenon arises in this model for poor farmers. Specifically, with $h$ denoting the inverse of the utility function $u$:

(A1) $c(s - f) - ch(D'(\xi))$ is maximized over $[\xi, \bar{\xi}]$ at some $c^* > \xi$.

This condition implies that it pays L to induce F to select an effort strictly higher than the minimal level, when the latter’s participation constraint does not bite. A sufficient condition for this is that the incremental benefit L derives from the successful outcome is large enough that

$$s - f > h(D'(\xi)) + cD''(\xi)h'(D'(\xi)).$$

3 Analysis and Results

Following Grossman and Hart (1983), it will be convenient to represent a contract by the induced levels of outcome-contingent utility from consumption for F: $v_s = u(c_s), v_f = u(c_f)$. 
The contract \((v_s, v_f)\) induces \(F\) to select effort \(e \in [\underline{e}, \bar{e}]\) to maximize

\[ ev_s + (1 - e)v_f - D(e) \]

It is easy to check that it never pays \(L\) to offer a contract with \(c_s < c_f\), since it is improved upon by a full insurance contract with \(c_s = c_f\) which induces the same effort \(e\) but imposes less risk on \(L\). Hence without loss of generality we shall impose \(v_s \geq v_f\). This implies that the optimal effort choice of \(L\) will satisfy the first-order condition

\[ v_s - v_f = D'(e). \tag{1} \]

The limited liability constraint is

\[ v_i \geq 0 \tag{2} \]

while the participation constraint for the agent is (upon using (1) above)

\[ v_f + eD'(e) - D(e) \geq u(w). \tag{3} \]

The contract \(v_s, v_f\) generates consumption \(h(v_i)\) for \(F\) following outcome \(i\), and a net transfer of \(w + i - h(v_i)\) to \(L\). The optimal contract will therefore involve \(v_s, v_f, e\) which maximizes the expected transfer to \(L\):

\[ w + e(s - h(v_s)) + (1 - e)(f - h(v_f)) \tag{4} \]

subject to constraints (1), (2) and (3). It is easy to check that optimal contracts exist for any \(w\). The main question to be addressed is the way that effort in an optimal contract varies with \(w\).

Note that this problem can be decomposed in the manner of Grossman and Hart (1983) as follows: (i) given an effort level \(e\), find the contract that will implement it at minimum expected cost \(C(e; w)\) to \(L\); and then (ii) select the optimal effort level \(e(w)\) to maximize
L’s expected net income \( es + (1 - e)f - C(e; w) \). The stage (i) problem reduces to selecting \( v_s, v_f \) to minimize

\[ ch(v_s) + (1 - e)h(v_f) \]

subject to (1), (2), and (3). And the stage (ii) problem reduces to selecting \( e(w) \) to maximize \( e(s - f) - C(e; w) \).

A key step in the analysis is to characterize the minimum cost function \( C(e; w) \) explicitly. For this we need the following notation. Let \( \alpha(e) \equiv eD'(e) - D(e) \), \( W(e) = h(\alpha(e)) \), and \( \bar{W} \equiv W(\bar{e}), \underline{W} \equiv W(\underline{e}) \). Note that \( \alpha(.) \) and \( W(.) \) are both strictly increasing. Finally, define \( E(w) \) to be the inverse of \( W(.) \), so \( E(.) \) maps \([\underline{W}, \bar{W}]\) into \([\underline{e}, \bar{e}]\).

**Lemma 1** Define the functions

\[ C_I(e) = ch(D'(e)) \]

and

\[ C_P(e; w) = eh(u(w) - \alpha'(e) + D'(e)) + (1 - e)h(u(w) - \alpha'(e)). \]

Then

\[ C(e; w) = \begin{cases} C_I(e) & \text{if } w < \bar{W} \text{ and } e \geq E(w) \\ C_P(e; w) & \text{otherwise} \end{cases} \]

**Proof:** In the problem of implementing effort \( e \) at minimum cost, initially ignore the participation constraint (3). Then the solution involves \( v_f = 0, v_s = D'(e) \), resulting in expected cost \( C_I(e) \). This contract satisfies (3) if \( \alpha'(e) = eD'(e) - D(e) \geq u(w) \), which reduces to \( e \geq E(w) \). Hence if this condition is satisfied (which requires that \( w < \bar{W} \)) this contract is optimal. Otherwise the participation constraint will necessarily bind, whence \( v_f = u(w) - \alpha'(e) \). Given (1) it follows that \( v_s = u(w) - \alpha'(e) + D'(e) \). Hence if either \( w > \bar{W} \) or \( e < E(w) \), \( C(e; w) = C_P(e; w) \).

**INSERT FIGURE 2**
Figure 2 depicts the nature of the second-best cost function. \( C_I \) depicts the cost incurred when the limited liability constraint is binding, where F receives the minimum consumption level of 0 following unsuccessful performance. Along this function higher effort levels are induced solely by allowing F to retain more in the successful state, so \( v_s = D'(e) \). Since the successful state arises with probability \( e \), the expected cost \( C_I = eh(D'(e)) \). Moreover, F obtains expected utility of \( \alpha'(e) \) whenever the limited liability constraint binds in this fashion. If his wealth is low enough that \( u(w) < \alpha'(e) \), F obtains more than what he could get elsewhere. Then the participation constraint does not bind, and it is optimal for L to ensure that only the limited liability constraint binds. This happens whenever the effort level to be induced is large enough relative to the wealth level (specifically \( e \geq E(w) \)): the agent earns rents for incentive reasons, owing to the existence of the limited liability constraint.

On the other hand, when the effort to be induced is low relative to the wealth level (\( e < E(w) \)), F obtains a bonus in the successful state that is insufficient by itself (in combination with minimum subsistence in the unsuccessful state) to ensure that he has an incentive to participate. Hence the agent has to be allowed to consume more than the minimum subsistence level in the unsuccessful state (so \( v_f = u(w) - \alpha'(e) \)), with consumption in the successful state raised correspondingly (\( v_s = D'(e) + u(w) - \alpha'(e) \)) to preserve effort incentives. The cost of implementing low effort levels is then given by \( C_P \) instead of \( C_I \).

In general, therefore, one can view the cost of implementing effort levels as the upper envelope of the two functions \( C_I \) and \( C_P \). Given the wealth level, for low effort levels (\( e < E(w) \)) the \( C_P \) function operates, and over this range the participation constraint binds exactly, while the limited liability constraint does not bind. For high effort levels, on the other hand, the cost is \( C_I \): here the limited liability constraint binds, while the participation constraint does not.

Note moreover that at any point where the two functions intersect, the \( C_I \) function has a steeper slope. Given wealth \( w \), the two costs are identical at effort \( E(w) \), and

\[
\frac{\partial C_I(E(w))}{\partial e} = h(D'(E(w)) + E(w)h'(D'(E(w)))D''(E(w))
\]
\[ > h(D'(E(w)) + E(w)h'(D'(E(w))))D''(E(w)) \]
\[ - \alpha'(E(w))[E(w)h'(D'(E(w))) + (1 - E(w))h'(0)] \]
\[ = \partial C_P(E(w); w)/\partial e. \]  

(5)

In words, the marginal cost from L's standpoint of inducing the agent to work harder is lower when the limited liability constraint does not bind. The reason is obvious: the increased effort can be induced partially by increasing the penalty for poor performance. In contrast when the limited liability constraint does bind, penalties for poor performance are already maximal, so L has to induce higher effort solely by increasing the reward for good performance.

To select an optimal effort level, L can proceed as follows. Given wealth \( w < \bar{W} \), split the set of possible efforts \([\bar{e}, \bar{c}]\) into the two regions \([\bar{e}, E(w)]\) and \([E(w), \bar{c}]\). Over the former region, the participation constraint will bind, and the best effort from L's standpoint is

\[ e_P(w) \equiv \max_{c \in [\bar{e}, E(w)]} V_P(c; w) \]  

(6)

where \( V_P(w) \equiv c(s - f) - C_P(c; w) \). Over the region consisting of effort levels above \( E(w) \), the participation constraint does not bind and the relevant cost function is \( C_I \), so the best effort in this region is

\[ e_I(w) \equiv \max_{c \in [E(w), \bar{c}]} V_I(c) \]  

(7)

where \( V_I(e) \equiv c(s - f) - C_I(e) \). If \( w > \bar{W} \) then \( E(w) \) is not defined: in this case the participation constraint always binds and \( C = C_P \). Then \( e(w) = e_P(w) \) which maximizes \( V_P(c; w) \) over \([\bar{e}, \bar{c}]\). Note that if the maximizing effort level is not unique, we have postulated that L will select the highest optimal effort. This is the natural assumption to make because the agent always (weakly) prefers a higher effort level to a lower one, should L be indifferent between them. Let \( V_P^* \) and \( V_I^* \) denote the maximum values achieved in problems (6) and (7) respectively.

It follows that the optimal effort choice will have the following property.
Lemma 2

\[ e(w) = \begin{cases} 
  c_I(w) & \text{if } w < \bar{W} \text{ and } V_I^* \geq V_P^* \\
  c_P(w) & \text{otherwise}
\end{cases} \]

We first consider the case of low wealth levels.

Proposition 1

\[ e(w) = e^* > \epsilon \text{ for all } w \leq w \equiv W(e^*) > 0. \]

Proof: Assumption (A1) ensures that it pays L to induce an effort level \( e^* \) higher than \( \epsilon \) in this situation, implying that \( W(e^*) > 0 \). The participation constraint is then automatically satisfied for all \( w \leq W(e^*) \), so over this range of wealth levels this is the optimal contract.

This is the familiar efficiency wage argument: for very low wealth levels it pays the principal to induce more than the minimal level of effort, given assumption (A1). This can be done by paying a reward for good performance, in view of the floor to the agent’s utility in the unsuccessful state. Hence the agent earns rents, and it pays to offer the same contract for all wealth levels below \( w \equiv W(e^*) \), where the rents just disappear. So the optimal effort is locally independent of wealth for very poor agents.

Our first main result is that there exist wealth levels slightly exceeding \( w \), where the effort level is higher than at \( w \), and effort is locally increasing in wealth. Hence the incentive effect dominates over this region.

Proposition 2 There exists a decreasing sequence of wealth levels \( w_n \to w^+ \) satisfying:

(i) \( c(w_n) = c_I(w_n) \text{ for all } n \)

(ii) \( c(w_{n+1}) < c(w_n), \text{ all } n, \text{ and} \)
(iii) \( \lim_n e(w_n) = e^* \).

The proof of this is presented in the Appendix. The heuristic reasoning is that when F's wealth rises beyond \( w \), the efficiency wage contract is no longer feasible since it violates the participation constraint. L has to react by making the contract more attractive to F. Doing this by letting F consume more following an unsuccessful outcome has the adverse effect of reducing F's effort incentives. So L prefers instead to make the contract more attractive by increasing the amount F can retain in the successful state, while keeping the transfers the same in the failure state. This increases L's effort, as well as the spread in consumption between the two states at the same time. The limited liability constraint continues to bind, so the optimal effort coincides with that corresponding to the region where the cost function is \( C_I \).

**INSERT FIGURE 3**

Figure 3 illustrates the argument why the limited liability constraint continues to bind at wealth levels slightly above \( w \). Since \( e^* \) maximizes \( V_I(e) = e(s - f) - C_I(e) \) over \([g, \bar{e}]\), the marginal cost of effort at \( e^* \) according to the \( C_I \) function must exactly equal \( s - f \), the marginal benefit to L. Now as argued above, the marginal cost of effort according to the \( C_P(e; W(e^*)) \) function where the limited liability constraint does not bite, is lower than that according to the \( C_I \) function at \( e^* \) (see (5)). Hence by continuity the same is true at wealth level \( w' \) slightly above \( W(e^*) \): see Figure 3. So if the effort level is slightly less than \( E(w') \) at this wealth level, the participation constraint rather than the limited liability constraint will bind. Then the relevant cost function will be \( C_P \), along which the marginal cost will be strictly lower than the marginal benefit \( s - f \). It will then pay L to induce F to increase effort to \( E(w') \), where the limited liability constraint will begin to bind again.

It is evident that the policy of reacting to an increase in F's wealth by increasing the payment in the successful state alone will, beyond some point, impose significant risk on F. This too will begin to exert a cost on L in order to compensate F adequately with the appropriate risk premium. Hence if wealth increases sufficiently beyond \( w \), at some stage it will pay L to begin to increase the amount consumed by F in the unsuccessful state as well.
Then the limited liability constraint will cease to bind, causing $F$ to react with a reduction in effort. This intuition is correct in the following sense.

**Proposition 3** There exists an increasing sequence of wealth levels $w_n \to \infty$ with $w_n > w$ along which:

(i) $c(w_n) = e_P(w_n)$

(ii) $c(w_{n+1}) \leq c(w_n)$, with strict inequality if $c(w_n) > e$.

(iii) $e(w_n) \to e$.

Effort levels therefore decline along some increasing sequence of (large) wealth levels, eventually approaching the minimum effort level $e$ (see Figure 1 again). In particular, along this sequence the limited liability constraint does not bite ($e = e_P$). Effort levels fall for two reasons: the need to reduce the risk borne by $F$, and the higher cost of inducing extra effort owing to income effects in $F$’s demand for leisure. The lowest levels of productivity are therefore displayed by the rich.

Stronger results on the monotonicity and continuity of $c(w)$ can be obtained with stronger assumptions on degree of preference for consumption smoothing and effort disutility.

**Proposition 4** If $h(D'(e))$ is convex, $D'(e) = 0$ and $D'(\bar{e}) = \infty$, there exist wealth levels $\hat{w}$ and $w'$ (satisfying $w' > \hat{w} > w$) with the following properties:

(i) over the range $(w, \hat{w})$, $c(w) = E(w) = e_I(w)$ is continuous and strictly increasing

(ii) over the range $(w', \infty)$, $c(w) = e_P(w) \in (e, E(w) = e_I(w))$ is continuous and strictly decreasing.

(iii) $\lim_{w \to \infty} e(w) = e$. 

11
This says that if $F$ is sufficiently risk averse (i.e., the composition of $h$ the inverse utility function, and the effort marginal disutility function $D'$ is convex), and the latter satisfies the usual boundary conditions that ensure interior effort solutions, then effort initially rises smoothly as wealth rises from $w$. Moreover, beyond some wealth level sufficiently larger, effort falls smoothly, asymptotically approaching the minimum level $c$ as the wealth level become arbitrarily large. It does not say anything about intermediate levels of wealth (between $\hat{w}$ and $w'$).

A complete description of optimal effort levels is provided in our final result, which concerns the case where the agent’s marginal utility of effort is convex. Without making any further assumptions on the nature of risk aversion, the optimal effort selection problem becomes a concave maximization problem. This ensuring that the optimal effort is throughout continuous in the wealth level. Moreover, it changes slope only once: initially constant, then rising and falling thereafter, as depicted by the dotted line in Figure 1.

**Proposition 5** If $D'(.)$ is convex, then there exists $\hat{w} > w$ such that

(i) $c(w)$ is continuous

(ii) $c(w) = c^*$, i.e., constant, over the range $[0, w]$

(iii) $c(w) = E(w) = c_I(w)$ is strictly increasing over $(w, \hat{w})$

(iv) $c(w) = c_P(w)$ is strictly decreasing over $(\hat{w}, \infty)$, with $\lim_{w \to \infty} c(w) = c$.

**INSERT FIGURE 4**

See Figure 4 for a geometric illustration of the argument. Both cost functions $C_I$ and $C_P$ are convex. As wealth increases, both the level and slope of the cost $C_P$ function increases: at any given effort level, $F$ has to be paid more on average to compensate for his higher outside option, and the marginal cost of inducing additional effort increases owing to the income effect in the demand for leisure. The slope of the $C_P$ function at the point $E(w)$ where the two functions meet, progressively increases. For wealths between $w$ and $\hat{w}$ it is
nevertheless smaller than \( s - f \), the marginal benefit to \( L \) from higher effort. So the optimal effort level equals \( E(w) \) over this range. At \( \hat{w} \) the slope of \( C_P \) at \( E(\hat{w}) \) exactly equals \( s - f \). From this point onwards, further increases in wealth serve to increase the marginal cost of effort from \( L \)'s point of view (i.e., the slope of \( C_P \)) further, thus lowering the optimal effort thereafter.

4 Extensions

In order to focus sharply on the interplay between the incentive and preference effects, the model of this paper was kept as simple as possible. Dutta, Ray and Sengupta (1989), Banerjee and Ghatak (1996) and Mookherjee and Ray (1996) study extensions of this model to a multiperiod setting. The third paper considers implications for dynamic wealth accumulation via saving, while the first two examine the possible effectiveness of eviction threats as a supplementary incentive device. In the presence of eviction threats, our analysis will be modified as follows. Provided the convexity conditions of Propositions 4 or 5 are valid, it is easily verified that the tenant enjoys rents in the optimal static contract if and only if his wealth falls below \( w \). Hence eviction threats are effective only if the agent has wealth below \( w \), in the ‘best’ stationary equilibrium in the infinite horizon setting from \( L \)'s point of view. In such an equilibrium, optimal effort levels will initially decrease with \( w \) upto the wealth level \( w \) as eviction threats become gradually less effective. Following this, they will follow the pattern described in Propositions 4 and 5. Figure 5 illustrates this. Hence the slope of effort with respect to wealth levels will change sign more than once. It may no longer true then that the most productive agents have intermediate wealth levels, as it is possible that the threat of eviction makes the poorest agents the most productive.

INSERT FIGURES 5,6

If the set of potential tenants have heterogenous wealth levels, the landlord will typically not be indifferent about who to lease out land to. In a setting where potential tenants have no preference for consumption smoothing, Shetty (1988) argued that landlords would prefer
wealthier tenants, thus creating a ‘tenancy ladder’ phenomenon. This essentially follows from the assumption concerning absence of the preference effect. When tenants are risk averse, our analysis implies that landlords will typically prefer tenants with intermediate rather than the largest wealth levels, since for the latter the preference effect outweighs the incentive effect to generate the lowest productivity levels. Hence the tenancy ladder phenomenon will operate only up to some wealth level; beyond this there will be a ‘reverse ladder’ phenomenon. In a dynamic setting the effect of this may be to reduce inequality between rich and middle class tenants, while at the same time increasing inequality between the middle class and the poor.

Such heterogeneity in wealth levels for different potential tenants significantly complicates the Dutta-Ray-Sengupta or Banerjee-Ghatak models of eviction. The landlord will then want to only hire tenants belonging to intermediate wealth levels. Provided Propositions 4 or 5 apply, such tenants earn no rents, so threats to evict them serve no incentive role. Nor are they credible if there are no other potential tenants available with exactly the same wealth level that they can be replaced with. It is then difficult to explain the phenomenon of eviction, without incorporating reputational considerations explicitly.

Finally, our results imply the following effects of land redistribution programs, using an extension of this model along the lines of my previous paper (Mookherjee, 1994) that introduces a need for production or consumption credit for the farmer. If the farmer is awarded the land, he acquires the right to cultivate the land without the landlord’s permission, but is still reliant on the latter for credit. The outcome of the switch to owner-cultivation from tenancy then depends on the nature of the resulting credit contract. The effect is to increase the bargaining power of the farmer, to an extent which depends on his wealth level. For very poor or very rich farmers, there is no effect on the allocation of bargaining power: very poor farmers are excessively reliant on the landlord for credit, while very rich farmers require no credit at all. Those with intermediate wealth levels enjoy an increase in bargaining power, whose effect is similar to an increase in wealth level. It follows from our results here that the effect will be to leave productivity unchanged for very poor or very rich farmers, increase productivity for the ‘lower middle class’, and decrease productivity for the ‘upper middle class’, as illustrated in Figure 6.
Appendix: Proofs

We start with the following lemma.

**Lemma 3** Over the range \( w \in [0, \bar{W}] \):

(i) \( e_I(w) \) is nondecreasing.

(ii) \( e_I(w) > E(w) \) implies that \( e_I(w) \) is locally constant, i.e., there exists \( \delta > 0 \) such that 
\[
e_I(w + \delta) = e_I(w).\]

(iii) \( e_I(0) = e^* < \bar{e}. \)

(iv) There exists a decreasing sequence \( w_n \to w^+ \) along which the corresponding effort 
levels are also decreasing: \( e_I(w_n) > e_I(w_{n+1}) \) for all \( n \), and \( e_I(w_n) \to w^+ \).

**Proof:** (i) and (ii) follow directly from the fact that increased \( w \) shrinks the feasible set of 
effort levels, and the convention that \( L \) selects the higher effort level whenever indifferent 
between two effort levels. (iii) follows from our assumption (A1).

To show (iv), we first argue that along every sequence \( w_n \to w^+ \), it must be the case 
that along every convergent subsequence \( c_n \equiv e_I(w_n) \to e^* \). Otherwise there would be a 
subsequence along which \( e_n \to \hat{e} > e^* \), since \( e_n \geq E(w_n) \geq e^* \). Since \( E(\cdot) \) is continuous, we 
have \( E(w_n) \to e^* \), and by definition of \( e_n \):
\[
e_n(s - f) - C_I(c_n) \geq E(w_n)(s - f) - C_I(E(w_n)).
\]

Taking limits
\[
\hat{e}(s - f) - C_I(\hat{e}) \geq e^*(s - f) - C_I(e^*).
\]

On the other hand since \( e^* \) is the effort level that maximizes \( V_I \) over \([\underline{e}, \bar{e}]\), and given our 
tie-breaking convention:
\[
e^*(s - f) - C_I(e^*) > \hat{e}(s - f) - C_I(\hat{e})
\]

15
since $\hat{c} > c^*$, and we have a contradiction.

Now $w_n > w$ implies that $E(w_n) > c^*$, and therefore that $c_n \geq E(w_n) > c^*$. So given any decreasing sequence $w_n \rightarrow w^+$, we can extract a subsequence along which the corresponding efforts are also decreasing, and converging to $c^*$ from above.

Proof of Proposition 2: Given part (iv) of Lemma 3, it suffices to show that there exists $w_2 > w$ such that $c(w) = c_I(w)$ over the range $(w, w_2)$. If this were not true, then we can find a sequence $w_n \rightarrow w^+$ such that $c_n \equiv e(w_n) = e_P(w_n) \neq e_I(w_n)$ for all $n$. Without loss of generality, let $c_n \rightarrow \hat{c}$. Now by the argument used in part (iv) of Lemma 3, \(\lim c_I(w_n) = c^*\). Since $c_P(w_n) \leq c_I(w_n)$ we have $\hat{c} \leq c^*$.

Consider first the case where $\hat{c} = c^*$. Then

$$\partial C_P(c^*, w)/\partial e \geq s - f = \partial C_I(c^*, w)/\partial e$$

and since $w = W(c^*)$ we contradict Lemma 1.

So suppose instead that $\hat{c} < c^*$. Then since $V_P(c_n, w_n) \geq V_P(E(w_n), w_n)$ we obtain upon taking limits that

$$(s - f)\hat{c} - C_P(\hat{c}; w) \geq (s - f)c^* - C_P(c^*; w)$$

$$= (s - f)c^* - C_I(c^*; w).$$

But $c^*$ is the unconstrained maximizer of $V_I(e)$, so

$$(s - f)c^* - C_I(c^*) \leq (s - f)c^* - C_I(c^*)$$

and $\hat{c} < c^* = E(w)$ implies that $C_I(\hat{c}) < C_P(\hat{c}; w)$, so we obtain a contradiction.
**Lemma 4** For any \( w \) there exists \( w' > w \) such that \( c(w') = c_P(w') \).

**Proof:** If \( D'(\bar{e}) < \infty \) we have \( \bar{W} < \infty \), and the result is obvious in this case. So suppose that \( \bar{W} = D'(\bar{e}) = \infty \).

We first claim that for any \( w \) we can find \( w'' \geq w \) such that \( c_I(w'') = E(w'') \). To see this note that if \( c_I(w) > E_I(w) \) then by part (i) of Lemma 2, \( c_I \) is locally constant at \( w \). Since \( E(.) \) is continuous and strictly increasing, there exists \( w'' \) such that \( E(w'') = c_I(w) = c_I(w'') \).

If the lemma were false, there would be \( w \) such that \( c(w') = c_I(w') \) for every \( w' > w \). So the claim established above ensures that we can construct a sequence \( w_n \to \infty \) with \( c_n = c(w_n) = c_I(w_n) = E(w_n) \neq c_P(w_n) \) for all \( n \). Hence

\[
\begin{align*}
    s - f & \geq \frac{\partial C_P(c_n; w_n)}{\partial e} \\
    & = \frac{\partial C_P(c_n; W(c_n))}{\partial e} \\
    & = h(D'(c_n)) + e_n D''(c_n)(1 - e_n)[h'(D'(c_n)) - h'(0)] \\
    & \geq h'(D'(c_n)).
\end{align*}
\]

Since \( w_n \to \infty, e_n \to \bar{e} \) and so the Inada condition implies \( h'(D'(c_n)) \to \infty \), and we obtain a contradiction.  

**Lemma 5** \( \lim w_n = \infty \) implies \( \lim c_P(w_n) = c \).

**Proof:** Suppose otherwise and \( \lim c_P(w_n) = \epsilon > c \), despite \( w_n \to \infty \). Then it is feasible to reduce effort below \( c_P(w_n) \) for large \( n \) and

\[
\begin{align*}
    s - f & \geq \frac{\partial C_P(c_P(w_n); w_n)}{\partial c} \\
    & \geq h(u(w_n) - \alpha(c_P(w_n))) + D'(c_P(w_n)) - h(u(w_n) - \alpha(c_P(w_n))).
\end{align*}
\]
So if \( \pi_n = u(w_n) - \alpha(e_P(w_n)) \to \infty \) we obtain a contradiction since \( \epsilon_3 > \epsilon \) implies that \( D'(e_P(w_n)) \to D'(\epsilon_3) > 0 \). On the other hand if \( \pi_n \) is bounded we also obtain a contradiction because \( u(w_n) - \alpha(e_P(w_n)) + D'(e_P(w_n)) = u(w_n) + (1 - e_n)D'(e_n) + D(e_n) \geq u(w_n) \to \infty. \]

**Proof of Proposition 3**: Use Lemmas 4 and 5 to construct a sequence \( w_n \to \infty \) such that \( c(w_n) = e_P(w_n) \to \epsilon \). Hence properties (i) and (iii) are met. Further, by virtue of (iii) we can extract a subsequence for which (ii) holds.

**Proof of Proposition 4**: If \( h(D'(\epsilon)) \) is convex, the function \( C_f \) is convex. Hence the problem of selecting \( e_I(w) \) is a concave problem. Since \( e^* \) is the unconstrained maximizer of \( V_I \), it follows that for any \( w > w \), \( e_I(e) = E(w) \). Part (i) now follows upon using the argument used in the proof of Proposition 2.

To prove parts (ii) and (iii), note that

\[
\partial C_P(e; w)/\partial e = \frac{[h(u(w) - \alpha'(e) + D'(e)) - h(u(w) - \alpha'(e))]}{e(1 - e)D''(e)[h'(u(w) - \alpha'(e) + D'(e)) - h'(u(w) - \alpha'(e))]}.
\]

(8)

So the boundary condition \( D'(\epsilon) = 0 = \alpha(\epsilon) \) implies that for all \( w \):

\[
\partial C_P(\epsilon, w)/\partial e = 0.
\]

On the other hand, using an argument analogous to that in the proof of Lemma 5, \( D'(e) = \infty \) implies that for any \( w \):

\[
\partial C_P(\epsilon, w)/\partial e = \infty.
\]

Hence there exists \( w' \) such that for all \( w > w' \):

\[
\partial C_P(E(w), w))/\partial e > s - f.
\]

It follows that for all \( w > w' \), \( c(w) = e_P(w) \in (\epsilon, E(w)) \). From (8) it is evident that for any \( e \) with \( D'(e) > 0 \):

\[
\partial^2 C_P(e; w)/\partial w \partial e > 0.
\]
Hence by a standard monotone comparative static argument \( c(w) \) is strictly decreasing. Finally, (iii) follows from an argument analogous to that in Lemma 5. 

**Proof of Proposition 5:** First we shall show that \( D' \) convex implies that \( C_P \) is convex in \( c \) for any \( w \). Let \( v_f \equiv u(w) + D(e) - eD'(e), v_s \equiv v_f + D'(e) \). Then their derivatives with respect to effort are \( v'_f = -eD''(e) < 0, v'_s = (1 - e)D''(e) > 0, v'_f' = -eD'''(e) - D''(e) < 0 \), \( v'_s' = (1 - e)D'''(e) - D''(e) \). Moreover, use subscripts \( s \) and \( f \) to denote state contingent utilities where derivatives of \( h \) are taken, i.e., \( h'_s \) denotes \( h'(v_s), h'_s' \) denotes \( h''(v_s) \) etc.

Then

\[
\partial^2 C_P(c; w)/\partial c^2 = 2[h'_s v'_s - h'_f v'_f] + [(1 - e)h'_f v'_f' + ch'_s v'_s']
+ [(1 - c)h'_f(v_f')^2 + ch'_s(v_s')^2]
\geq [h'_s v'_s - h'_f v'_f] + [ch'_s v'_s' + (1 - e)h'_f v'_f']. \tag{9}
\]

Now \( ch'_s v'_s' + (1 - e)h'_f v'_f' = eD'''(e)(h'_s - h'_f) - h'_f D'''(e) \), while \( h'_s v'_s - h'_f v'_f = h'_s D'' + (h'_s - h'_f) v'_f. \) Hence expression (9) equals

\[
(h'_s - h'_f)(eD'''(e) + (1 - e)D''(e))
\]

which is nonnegative by virtue of the convexity of \( h \), and the assumption that \( D'' \geq 0. \)

Clearly \( C_I \) is also convex, so for any \( w > w \), we have \( c_I(w) = E(w) \). Moreover, since \( C_P \) is convex, \( E(w) \) is increasing, and \( \partial^2 C_P/\partial w \partial c \geq 0 \), it follows that \( \partial C_P(E(w), w)/\partial c \) is nondecreasing in \( w \). It is also continuous, and

\[
\lim_{w \to \infty} \partial C_P(E(w), w)/\partial c = \infty
\]

by virtue of an argument similar to that in Lemma 5. We know from (5) that

\[
\partial C_P(c^*, W(c^*))/\partial c \equiv \partial C_P(E(w), w))/\partial c < s - f = \partial C_I(c^*)/\partial c.
\]

Hence there must exist \( \hat{w} > w \) such that \( \partial C_P(E(\hat{w}), \hat{w})/\partial c = s - f. \) The rest of the result now follows. 

19
References


Banerjee Abhijit and Maitreesh Ghatak (1996), “Empowerment and Efficiency: The Economics of Tenancy Reform,” manuscript, Department of Economics, MIT.


Endnotes

1. See, for instance, Banerjee and Ghatak, 1996; Dutta, Ray and Sengupta, 1989; Laffont and Matoussi, 1995; and Mookherjee, 1994. Other applications of these models include the dynamics of inequality and productivity (Aghion and Bolton, 1994; Banerjee and Newman, 1994; Mookherjee and Ray, 1996; Piketty, 1996), the organization of firms (Bowles and Gintis 1994, 1995; Legros and Newman, 1994), and the importance of internal vis-a-vis external finance (Hoff, 1994; Holmstrom and Tirole, 1994).

2. See, for instance, Alderman and Paxson, 1993; Paxson, 1992; Rosenzweig and Stark, 1989; Rosenzweig and Wolpin, 1993; and Townsend, 1994.

3. For instance, most authors have either not posed this question, or have assumed that the agent has no preference for smoothing consumption. Banerjee-Ghatak (1996) is an exception: they show that effort is locally increasing in wealth whenever the limited liability constraint binds. But they do not provide a characterization of wealth ranges where the limited liability constraint binds.

4. Note that the model excludes the possibility of financial expenditures privately incurred by the tenant in order to raise farm productivity, such as on fertilizers, seeds or implements. Such forms of moral hazard are distinct from that associated with work effort, and are better modelled by a utility function of the form \( u(c - E) \) where \( E \) denotes farm expenditures. When \( u \) exhibits constant absolute risk aversion, wealth effects are totally absent: in such a case only the incentive effect operates, implying that productivity is always higher the wealthier the tenant.
FIGURE 2: EFFORT COSTS
FIGURE 3: INCENTIVE EFFECT
FIGURE 4: PREFERENCE EFFECT
FIGURE 5: EVICTION