Equilibrium Corporate Finance: Makowski meets Prescott and Townsend*

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Abstract

We study a general equilibrium model with production where financial markets are incomplete. At a competitive equilibrium firms take their production and financial decisions so as to maximize their value. If firms form perfectly competitive conjectures, as shown by Makowski (1983a,b), shareholders unanimously support value maximization and competitive equilibria are constrained Pareto optimal. We extend this result to allow for intermediated short sales of firms' equity and default. We also extend the analysis to encompass informational asymmetries. In this context we show that perfectly competitive conjectures implicitly support the equilibrium concept introduced by Prescott and Townsend (1984) and unanimity and constrained Pareto optimality are maintained. For all these economies the Modigliani-Miller theorem typically does not hold and the firms' corporate financing structure is determinate in equilibrium.

Keywords: capital structure, competitive equilibria, incomplete markets, asymmetric information

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1 Introduction

The notion of competitive equilibrium in incomplete market economies with production is considered problematic. Starting with the contributions of Dreze (1974), Grossman and Hart (1979) and Duffie and Shafer (1986), a large literature has dealt with the question of what is the appropriate objective function of the firm in these economies.[1] The issue arises because, when financial markets are incomplete and equity is traded in asset markets, firms' production decisions may affect the set of insurance possibilities available to consumers, the asset span of the economy.[2]

Furthermore, it is arguable that the study of the macroeconomic properties of incomplete market economies as well as the development of the integrated study of corporate finance with macroeconomics and asset pricing theory have been severely hindered by the recognition of the foundational issues associated to the objective function of the firm.[3]

In two important contributions Louis Makowski (1983a,b) addresses these foundational issues regarding the notion of competitive equilibrium in such economies.[4] Makowski's approach is based on the specification of a notion of *perfectly competitive conjectures* to guide firms' decisions when the value of production plans lying outside the span of the (incomplete) financial markets is considered. This notion of perfectly competitive conjectures, which we refer to here as the *Makowski criterion,* can be interpreted as a rationality condition on firms' out-of-equilibrium beliefs and relies on a no short-sales condition on agents' trades of firms' equity to guarantee perfect competition.[5] Under the condition that agents cannot short-sell equity, Makowski (1983a) shows that the *Makowski criterion* implies that value maximization is unanimously supported by shareholders as the firm's objective. Under the

[1]See, e.g., Bonnisseau and Lachiri (2004), DeMarzo (1993), Dierker et al. (2002), Dreze et al. (2007), Kelsey and Milne (1996) and many others.

[2]It is only in rather special environments, as pointed out by Diamond (1967) (see also the more recent contribution by Carceles-Poveda and Coen-Pirani (2009)), that the spanning condition holds and such issue does not arise.

[3]Macro models with production typically assume that firms' equity is not traded, or that firms have a backyard technology, often explicitly citing the problems with the objective function of the firm as a justification for this assumption. Corporate finance models, on the other hand, typically rely either on a partial equilibrium analysis or to complete markets for the same reason (see e.g., Parlour and Walden (2011)).

[4]At times competitive equilibria in financial market economies with production are called *stock market equilibria* in the literature.

[5]Makowski explicitly links it to Ostroy's no-surplus characterization of perfect competition (Ostroy (1980, 1984)).

same conditions, Makowski (1983b) shows that competitive equilibria are constrained Pareto optimal.

Unfortunately these two papers have been somewhat overlooked. The literature on General Equilibrium with Incomplete Markets (GEI) as well as the more specific literature on the objective function of the firm with incomplete markets seem unaware of Makowski's results.[6] This may be partly be due, in our opinion, to the fact that the no-short sales assumption in Makowski contrasts with the practice of GEI, where portfolio sets are unbounded, as in standard finance models. But unbounded portfolio sets are not compatible with perfect competition when the asset span is endogenous.[7] Moreover, in the presence of equity, long positions, which give rights not only to future payoffs but also to control over firms, are conceptually different from short positions.

In this paper we provide a systematic study of the properties of competitive equilibria when firms' conjectures satisfy the *Makowski criterion.* First of all, we re-formulate the contribution of Makowski (1983a,b) in the context of a simple two-period economy along the lines of classical GEI models and of macroeconomic models with production,[8] maintaining the assumption that agents cannot short-sell equity. In this case, i) value maximization is unanimously supported by shareholders as the firm's objective, and ii) competitive equilibria are constrained Pareto optimal. We also show that competitive equilibria exist. Most importantly, we extend the analysis and show that all these results obtain even if we allow for (bounded) short-sales on equity, modelled as the product of the financial intermediation of assets, as well as of firms' default on the debt issued. Furthermore, we show that in these economies the capital structure of firms at equilibrium is determinate in a precise and specific sense, that is, the Modigliani-Miller theorem does not hold.

Finally, we introduce informational asymmetries between the decision maker in the firm

[6]For instance, Makowski is not cited in the main surveys of the GEI literature, as Geanakoplos (1990) and Magill and Shafer (1991), nor in the most well-known contributions to the second literature, from Dreze (1985) to DeMarzo (1993), Kelsey and Milne (1996), Carceles-Poveda and Coen-Pirani (2009).

[7]Ironically, Duffie and Shafer (1986) do cite Makowski's papers to say they propose a 'strong notion of competition in which shareholders take both prices and the span of markets as given." On the other hand, the recent literature on financial innovation and optimal security design has extensively and explicitly adopted Makowski's notion of perfectly competitive conjectures; see Allen and Gale (1988, 1991) and Pesendorfer (1995).

[8]In a complementary paper, Bisin et al. (2009) we extend the analysis to Bewley economies with production, the main workhorse of heterogeneous agents macroeconomics. See Heathcote et al. (2009) for a recent survey of Bewley models.

(e.g., the manager) and equity holders or bondholders. While this is a natural extension in this context, as such informational asymmetries play a fundamental role in the theory and practice of corporate finance, there is little work on competitive equilibria in these economies.[9] This is important as it allows to study the interaction between corporate finance and asset pricing as well as risk sharing, an issue that has recently received some attention in the literature (see, e.g. Dow et al. (2005)). In this paper we show that the competitive equilibrium concept that results from the imposition of perfectly competitive conjectures as in the *Makowski criterion* is equivalent to the equilibrium concept introduced by Prescott and Townsend (1984) in the context of pure exchange economies with moral hazard, once extended to production economies. Furthermore, competitive equilibria with perfectly competitive conjectures exist and support unanimity of value maximization and constrained Pareto optimal allocations.[10]

Hence we conclude, on the basis of the findings reported in this paper, that the analysis of production economies with incomplete markets and possibly agency frictions rests on solid foundations.

In Section 2 we first introduce the baseline economy with riskless debt and no short sales. In this section, after showing that equilibria always exist, we also discuss and compare the equilibrium notion considered with the alternative ones adopted in the previous literature. We revisit in this context Makowski's results on unanimity and efficiency. We also study firms' capital structure and Modigliani-Miller Theorem. In Section 3 we extend the analysis to account for risky debt and short sales. Finally, in Section 4 we study economies with asymmetric information.

2 The economy

The economy lasts two periods, $t = 0, 1$ and at each date a single consumption good is available. The uncertainty is described by the fact that at $t = 1$ one state s out of the set $\mathcal{S} = \{1, ..., S\}$ realizes. We assume for simplicity that there is a single type of firm in

[9] Notable exceptions are Acharya and Bisin (2009), Magill and Quinzii (2002), Dreze et al. (2008), Zame (2007), Prescott and Townsend (2006).

[10] We do not discuss economies with adverse selection in this paper. While we conjecture that unanimity and existence can still be proved in such case, constrained Pareto optimality will not be typically maintained: Bisin and Gottardi (2006) identify in fact an externality in pure exchange insurance economies with adverse selection which has an obvious counterpart in production economies.

the economy which produces the good at date 1 using as only input the amount k of the commodity invested in capital at time 0.[11] The output depends on k as well as another technology choice ϕ, affecting the stochastic structure of the output at date 1,[12] according to the function $f(k, \phi; s)$, defined for $k \in K, \phi \in \Phi$, and $s \in \mathcal{S}$. We assume that $f(k, \phi; s)$ is continuously differentiable, increasing in k and concave in k, ϕ; moreover, Φ, K are closed, compact[13] subsets of $\mathbb{R}_+$ and $0 \in K$.

In addition to firms, there are I types of consumers. Consumer $i = 1, .., I$ has an endowment of w_0^i units of the good at date 0 and $w^i(s)$ units at date 1 in each state $s \in \mathcal{S}$, thus the agent's endowment is also subject to the shock affecting the economy at $t = 1$. He is also endowed with θ_0^i units of stock of the representative firm. Consumer i has preferences over consumption in the two dates, represented by $\mathbb{E}u^i\left(c_0^i, c^i(s)\right)$, where $u^i(\cdot)$ is also continuously differentiable, increasing and concave.

There is a continuum of firms, of unit mass, as well as a continuum of consumers of each type i, which for simplicity is also set to have unit mass.[14]

2.1 Competitive equilibrium

Firms take both production and financial decisions. For simplicity, their equity and debt are the only assets in the economy. Let the outstanding amount of equity be normalized to 1 (the initial distribution of equity among consumers satisfies $\sum_i \theta_0^i = 1$) and assume this is kept constant. Hence the choice of a firm's capital structure is only given by the decision concerning the amount B of bonds issued, which in turn also equals the firm's debt/equity ratio. The problem of the firm consists in the choice of its production plan k, ϕ and its financial structure B. To begin with, we assume all firms' debt is risk free.[15]

[11] It should be clear from the analysis which follows that our results hold unaltered if the firms' technology were described, more generally, by a production possibility set $Y \subset \mathbb{R}^{S+1}$.

[12] The parameter ϕ may describe, for instance, the loading on different factors affecting the firm's output. To illustrate this, consider the following instance of production function
$f(k, \phi; s) = [a(s) + \phi\epsilon(s)]\, k^\alpha$ where $\phi \in \{0, 1\}$ is the loading of the firm's cash-flow on the risk component given by $\epsilon(s)$. See also the example in Section 2.1.2.

[13] The condition that the set of admissible values of k is bounded above is by no means essential and is only introduced for simplicity.

[14] Makowski (1983a,b) deals with finite economies to highlight Ostroy (1980, 1984)'s no surplus condition for competition. We instead aim at minimal distance from the classic GEI formulation of competitive equilibrium.

[15] We shall allow for the possibility that firms' default on their debt in Section 4.1.

Firms are perfectly competitive and hence take prices as given. The notion of price taking behavior has no ambiguity when referred to the bond price p. For equity, however, a firm's cash flow, and hence the return on its equity, is $[f(k,\phi;s)-B]$ and varies with the firm's production and financing choices, k,ϕ,B. What should be its value, when all these different "products" are not traded in the market?[16] In this case, as pointed out by Grossman and Hart (1979), firms operate on the basis of a price conjecture[17] $q(k,\phi,B)$, which specifies the market valuation of the firm's cash flow for any possible choice k,ϕ,B. Firms choose then their production and financing plans k,ϕ,B so as to maximize their value, as determined by such pricing map and the bond price. The firm's problem is then:

$$V=\max_{k,\phi,B} -k+q(k,\phi,B)+pB \tag{1}$$

subject to the solvency constraint (ensuring that the bonds issued are risk free):

$$f(k,\phi;s)\geq B,\ \forall s\in\mathcal{S} \tag{2}$$

Let $\bar{k},\bar{\phi},\bar{B}$ denote the solutions to this problem.[18]

At $t=0$, each consumer i chooses his portfolio of equity and bonds, θ^i and b^i respectively, so as to maximize his utility, taking as given the price of bonds p and the price of equity available in the market q. In this section we follow Makowski (1983a,b) and assume that agents cannot short-sell the firm equity nor its debt:

$$b^i\geq 0,\ \theta^i\geq 0,\ \forall i. \tag{3}$$

The problem of agent i is then:

$$\max_{\theta^i,b^i,c^i}\mathbb{E}u^i(c_0^i,c^i(s)) \tag{4}$$

subject to (3) and

$$c_0^i\ =w_0^i\ +[-k\ +q\ +p\ B\ ]\,\theta_0^i-q\ \theta^i\ \ -p\ b^i \tag{5}$$

$$c^i(s)=w^i(s)+[f(k,\phi;s)-B\ ]\,\theta^i\ \ +b^i,\ \forall s\in\mathcal{S} \tag{6}$$

[16]When financial markets are complete, the present discounted value of any future payoff is uniquely pinned down by the price of the existing assets. This is no longer true when markets are incomplete.

[17]These conjectures are also referred to as *price perceptions* (see Grossman and Hart (1979), Kihlstrom and Matthews (1990) and Magill and Quinzii (1998)).

[18]We could allow the technology choice ϕ to entail a resource cost $W(\phi,k,B)$, which may also depend on the other production and financial choices made by the firm. We would only have to subtract this cost from the expression of the firm's valuation in (1), with no other change in the analysis which follows. The presence of this cost is made explicit in Section 4.1.

Let $\bar{\theta}^i, \bar{b}^i, \bar{c}_0^i, (\bar{c}^i(s))_{s\in\mathcal{S}}$ denote the solutions of this problem.

In equilibrium, the following market clearing conditions for the assets must hold:[19]

$$\begin{aligned} \textstyle\sum_i b^i &\leq B \\ \textstyle\sum_i \theta^i &\leq 1 \end{aligned} \tag{7}$$

In addition, the equity price conjecture entertained by firms must satisfy the following consistency condition:

C) $q(\bar{k}, \bar{\phi}, \bar{B}) = q$;

This condition requires that, in equilibrium, the price of equity conjectured by firms coincides with the price of equity faced by consumers in the market: firms' conjectures are "correct" in equilibrium.

We also restrict out of equilibrium conjectures by firms, requiring they satisfy:

M) $q(k, \phi, B) = \max_i \mathbb{E}\left[\overline{MRS}^i(s)(f(k, \phi; s) - B)\right]$, $\forall k, \phi, B$, where $\overline{MRS}^i(s)$ denotes the marginal rate of substitution between consumption at date 0 and at date 1 in state s for consumer i, evaluated at his equilibrium consumption level $\bar{c}^i$.

Condition M) is the *Makowski criterion.* It requires that for any k, ϕ, B the value of the equity price conjecture $q(k, \phi, B)$ equals the highest marginal valuation - across all consumers in the economy - of the cash flow associated to k, ϕ, B. The consumers with the highest marginal valuation for the firm's cash flow when the firm chooses k, ϕ, B are in fact those willing to pay the most for the firm's equity in that case and the only ones willing to buy equity - at the margin - at the price given by M). Under condition C), as we show in (8) below, such property is clearly satisfied for the firms' equilibrium choice $\bar{k}, \bar{\phi}, \bar{B}$. The *Makowski criterion* requires that the same is true for any other possible choice k, ϕ, B: the value attributed to equity equals the maximum any consumer is willing to pay for it.

Note that the consumers' marginal rates of substitutions $\overline{MRS}^i(s)$ used to determine the conjecture over the market valuation of the future cash flow of a firm are taken as given, evaluated at the equilibrium consumption values, unaffected by the individual firm's choice

[19] We state here the conditions for the case of symmetric equilibria, where all firms take the same production and financing decision, so that only one type of equity is available for trade to consumers. They can however be easily extended to the case of asymmetric equilibria as, for instance, the one considered in the example of Section 2.1.2.

of k, ϕ, B. This is the sense in which firms and their price conjectures are competitive: each firm is "small" relative to the mass of consumers and each consumer holds a negligible amount of shares of a firm.

Summarizing,

Definition 1 (Competitive equilibrium) *A competitive equilibrium of the economy is a collection* $\left(\bar{k}, \bar{\phi}, \bar{B}, \{\bar{c}^i, \bar{\theta}^i, \bar{b}^i\}_i, \bar{p}, \bar{q}, q(\cdot)\right)$ *such that:*
i) $\bar{k}, \bar{\phi}, \bar{B}$ *solve the firm's problem (1) s.t. (2) given* $\bar{p}, q(\cdot)$*; ii) for all* i*,* $\bar{c}^i, \bar{\theta}^i, \bar{b}^i$ *solve consumer* i*'s problem (4) s.t. (3), (5) and (6) for given* $\bar{p}, \bar{q}$*; iii) markets clear ((7) holds); iv) the equity price map* $q(\cdot)$ *is consistent, that is satisfies the consistency conditions C) and M).*

It readily follows from the consumers' first order conditions that in equilibrium the price of equity and the bond satisfy:

$$\begin{aligned} \bar{q} &= \max_i \mathbb{E}\left[\overline{MRS}^i(s)(f(\bar{k}, \bar{\phi}; s) - \bar{B})\right] \qquad (8) \\ \bar{p} &= \max_i \mathbb{E}\left[\overline{MRS}^i(s)\right], \end{aligned}$$

as implied by the consistency conditions C) and M).

Remark 1 *It is of interest to point out that, when price conjectures satisfy the Makowski criterion, the model is equivalent to one where markets for all possible 'types' of equity are open (that is, equity corresponding to any possible value of* k, ϕ, B *is available for trade to consumers) and, in equilibrium all such markets - except the one corresponding to the firms' equilibrium choice* $\bar{k}, \bar{\phi}, \bar{B}$ *- clear at zero trade. As a consequence,* $q(k, \phi, B)$ *corresponds to the equilibrium price of equity of a firm who were to "deviate" from the equilibrium choice* $\bar{k}, \bar{\phi}, \bar{B}$ *and choose* k, ϕ, B *instead. In this sense, we can say that the Makowski criterion imposes a consistency condition on the out of equilibrium values of the equity price conjectures, that corresponds to a "refinement" somewhat analogous to backward induction.*
To see this, suppose that consumers can trade any claim with payoff $[f(k, \phi; s) - B\ ]$*, at the price* $q(k, \phi, B)$*, for all* $(k, \phi) \in \Phi \times K$ *and* B *satisfying (2). The expressions of the budget constraints for type* i *consumers in (5) and (6) have then to be modified as follows:*

$$\begin{aligned} c_0^i &= w_0^i + \left[-\bar{k} + \bar{q} + p\ \bar{B}\ \right]\theta_0^i - \int_{\Phi \times K} \int_{\min_s f(k,\phi;s) \geq B} q(k, \phi, B)\ d\theta^i(k, \phi, B) - p\ b^i \\ c^i(s) &= w^i(s) + \int_{\Phi \times K} \int_{\min_s f(k,\phi;s) \geq B} [f(k, \phi; s) - B\ ]\, d\theta^i(k, \phi, B) + b^i,\ \forall s \in \mathcal{S} \qquad (9) \end{aligned}$$

Similarly, to the market clearing conditions in (7) we should add:

$$\sum_i \theta^i \ (k, \phi, B) \leq 0 \text{ for all } (k, \phi, B) \neq (\bar{k}, \bar{\phi}, \bar{B}).$$

It is immediate to verify that, when condition M) holds, if $\bar{c}^i, \bar{\theta}^i, \bar{b}^i$ solves consumer i's problem (4) subject to (3), (5) and (6), a solution to the problem of maximizing i's utility subject to (9) obtains again at $\bar{c}^i, \bar{b}^i$ and $\theta^i(\bar{k}, \bar{\phi}, \bar{B}) = \bar{\theta}^i$, $\theta^i(k, \phi, B) = 0$ for all other $(k, \phi, B) \neq (\bar{k}, \bar{\phi}, \bar{B})$. This follows from the fact that the utility of all consumers is continuously differentiable and concave in the holdings of any type of equity and, when $q(k, \phi, B)$ satisfies the Makowski criterion, their marginal utility of a trade in equity of any type $(k, \phi, B) \neq (\bar{k}, \bar{\phi}, \bar{B})$, evaluated at zero trade, is less or equal than its price.
Hence the equilibrium allocation is unchanged if consumers are allowed to trade all possible types of equity at these prices. Note that this argument crucially relies on the no short sale condition; see also Hart (1979) and Geanakoplos (2004).

Definition 1 of a competitive equilibrium is stated for simplicity for the case of symmetric equilibria, where all firms choose the same production plan. When the equity price map satisfies the consistency conditions C) and M), the firms' choice problem is not convex. Asymmetric equilibria might therefore exist, in which different firms choose different production plans. The proof of existence of equilibria indeed requires that we allow for such asymmetric equilibria, so as to exploit the presence of a continuum of firms of the same type to convexify the firms' choice problem. A standard argument allows then to show that firms' aggregate supply is convex valued and hence that the existence of (possibly asymmetric) competitive equilibria holds. We relegate a sketch of the proof in Appendix A.1.

Proposition 1 (Existence) *A competitive equilibrium always exist.*

2.1.1 Objective function of the firm

It is useful to compare the *Makoswki criterion* to other specifications of the price conjecture $q(k, \phi, B)$ we find in the literature. A minimal consistency condition on $q(k, \phi, B)$ is clearly given by condition C), which only requires the conjecture to be correct in correspondence to the firm's equilibrium choice. Duffie and Shafer (1986) indeed only impose such condition and consider as admissible any pricing kernel which satisfies it and induces prices with no arbitrage opportunities, that is lies in the same space where agents' marginal rates of substitution lie. Since when markets are incomplete these rates are typically different across consumers, they find a rather large indeterminacy of the set of competitive equilibria.

Consider then the criterion proposed by Dreze (1974) in an important early contribution to this literature. Stated in our environment, the *Dreze criterion* is:

$$q(k, \phi, B) = \mathbb{E} \sum_i \bar{\theta}^i \overline{MRS}^i(s) \left[f(k, \phi; s) - B \right], \ \forall k, B \tag{10}$$

It requires the price conjecture for any plan k, ϕ, B to equal - pro rata - the marginal valuation of the agents who in equilibrium are equity holders of the firm (that is, the agents who value the most the plan chosen by the firm in equilibrium and hence choose to buy equity). It does not however require that the firm's equity holders are those who value the most any possible plan of the firm. Intuitively, the choice of a plan which maximizes the firm's value with $q(k, \phi, B)$ as in (10) corresponds to a situation in which the firm's equity holders choose the plan which is optimal for them[20] without contemplating the possibility of selling the firm in the market, to allow the buyers of equity to operate the production plan they prefer. Equivalently, the value of equity for out of equilibrium production plans is determined using the - possibly incorrect - conjecture that the firms' equilibrium shareholders will still own the firm if it changes its production plan.

It is useful to compare directly the *Makowski* and the *Dreze criteria*. The first one requires that each plan is evaluated according to the marginal valuation of the agent who values it the most. It is then easy to see that any allocation constituting an equilibrium under this criterion (as in Definition 1) is also an equilibrium under the *Dreze criterion*: all shareholders have in fact the same valuation for the firm's production plan and their marginal utility for any other possible plan is lower, hence a fortiori the chosen plan maximizes the weighted average of the shareholders' valuations. But the reverse implication is not true, i.e., an equilibrium under the *Dreze criterion* is not in general an equilibrium under the *Makowski criterion*.

Grossman and Hart (1979) propose another specification of the consistency condition and hence a different equilibrium notion in a related environment. The *Grossman Hart criterion* (again, restated in our environment) is:

$$q(k, \phi, B) = \mathbb{E} \sum_i \theta_0^i \overline{MRS}^i(s) \left[f(k, \phi; s) - B \right], \ \forall k, B$$

[20] It is in fact immediate to verify that the plan which maximizes the firm's value with $q(k, \phi, B)$ as in (10) is also the plan which maximizes the welfare of the given set of shareholders of the firm.

We can interpret such notion as describing a situation where the firm's plan is chosen by the initial equity holders (i.e., those with some predetermined stock holdings at the beginning of date 0) so as to maximize their welfare, again without contemplating the possibility of selling the equity to other consumers who value it more. Equivalently, the value of equity for out of equilibrium production plans is derived using the conjecture that the firm's initial shareholders stay in control of the firm also out of equilibrium.

To summarize, according to the *Makowski criterion* the firm evaluates different production plans using possibly different marginal valuations (that is, possibly different pricing kernels, but all still consistent with the consumers' marginal rate of substitution at the equilibrium allocation). This is not the case of Dreze (1974) nor of Grossman and Hart (1979), both of whom rely on the use of a single pricing kernel. This is a fundamental distinguishing feature of the equilibrium notion based on the *Makowski criterion* with respect to the many others proposed in the GEI literature, including those which have applied theoretical constructs from the theory of social choice and voting to model the control of equity holders over the firm's decisions; see for instance DeMarzo (1993), Boyarchenko (2004), Cres and Tvede (2005).

But *the proof is in the pudding.* The *Makowski criterion*, besides being logically consistent as no *small* firm has *large* effects, also has some desirable properties: i) it delivers a unanimity result and ii) it produces equilibria which satisfy a constrained version of the First Welfare Theorem.

2.1.2 Unanimity, constrained Pareto optimality, and Modigliani-Miller

We turn to state and prove our main results for the baseline economy just described, with riskless debt and no short sales. As noted, a version of the unanimity result is in Makowski (1983a), while one of the the constrained efficiency result is in Makowski (1983b).

Unanimity Equity holders unanimously support their firm's choice of the production and financial decisions which maximize its value (or profits), as in (1). This follows from the fact that, when the equity price conjectures satisfy conditions C) and M), as we already noticed in Remark 1, the model is equivalent to one where a continuum of types of equity is available for trade to consumers, corresponding to any possible choice of k, ϕ, B the representative firm can make, at the price $q(k, \phi, B)$.[21]

Unanimity then holds by the same argument as the one used to establish this property for Arrow-Debreu economies. More formally, notice that we can always consider a situation where, in equilibrium, each consumer holds at most a negligible fraction of each firm. The effect on the consumers' utility of alternative choices by a firm can then be evaluated using the agents' marginal utility. For any possible choice k, ϕ, B of a firm, the (marginal) utility of a type i agent if he holds the firm's equity,

$$\mathbb{E}\left[\overline{MRS}^i(s)\left(f(k, \phi; s) - B\right)\right],$$

is always less or at most equal to his utility if he sells the firm's equity at the market price, given by

$$\max_i \mathbb{E}\left[\overline{MRS}^i(s)\left(f(k, \phi; s) - B\right)\right].$$

Hence the firm's choice which maximizes the latter also maximizes the equity holders' utility:

Proposition 2 (Unanimity) *At a competitive equilibrium, equity holders unanimously support the production and financial decisions $\bar{k}, \bar{\phi}, \bar{B}$ of the firms; that is, every agent i holding a positive initial amount θ_0^i of equity of the representative firm will be made - weakly - worse off by any other choice k', ϕ', B' of the firm.*

Constrained Pareto optimality We show next that all competitive equilibria of the economy described exhibit desirable welfare properties. Evidently, since the hedging possibilities available to consumers are limited by the presence of the equity of firms and risk free bonds as the only assets, we cannot expect competitive equilibrium allocations to be fully Pareto optimal, but only to make the best possible use of the existing markets, that is to be constrained Pareto optimal in the sense of Diamond (1967).

To this end, we say a consumption allocation $(c^i)_{i=1}^2$ is *admissible* if:[22]

[21] As we said earlier, this property depends on the fact that consumers face a no short sale condition. In Section 3 we will show that the unanimity, as well as the constrained efficiency, results extend to the case where limited short sales are allowed, under an appropriate specification of the markets for selling short assets.

[22] To keep the notation simple we state here the definition of admissible allocations for symmetric allocations, as we did for competitive equilibria. Our analysis and the efficiency result hold however in the more general case where asymmetric allocations are allowed; see also the next section.

1. it is *feasible*: there exists a production plan k, ϕ of firms such that

$$\begin{array}{c} \sum_i c_0^i \ + k \leq \sum_i w_0^i \\ \sum_i c^i(s) \leq \sum_i w^i(s) + f(k, \phi; s), \ \forall s \in \mathcal{S} \end{array} \tag{11}$$

2. it is *attainable with the existing asset structure*: there exists B and, for each consumer's type i, a pair θ^i, b^i such that:

$$c^i(s) = w^i(s) + [f(k, \phi; s) - B\ ]\,\theta^i \ + b^i, \ \forall s \in \mathcal{S} \tag{12}$$

Next we present the notion of *optimality* restricted by the *admissibility* constraints:

Definition 2 (Constrained Pareto optimality) *A competitive equilibrium allocation is constrained Pareto optimal if we cannot find another admissible allocation which is Pareto improving.*

The validity of the First Welfare Theorem with respect to such notion can then be established by an argument essentially analogous to the one used to establish the Pareto optimality of competitive equilibria in Arrow-Debreu economies.[23]

Proposition 3 (Constrained Pareto optimality) *Competitive equilibria are constrained Pareto optimal.*

Remark 2 *Dierker et al. (2002) present an economy with the property that all equilibria according to the Dreze criterion (Dreze equilibrium) are not constrained optimal. This appears to contradict the results in this paper. According to our equilibrium notion, in fact, all equilibria are constrained Pareto optimal, an equilibrium exist and any equilibrium is also a Dreze equilibrium. The apparent contradiction is due, however, to Dierker et al. (2002)'s restriction to symmetric equilibria. We will show that, in their economy, a unique competitive equilibrium exists which is asymmetric and constrained efficient. This equilibrium only is selected by our definition, according to the Makowski criterion.*
Let $\mathcal{S} = \{s', s''\}$. There are two types of consumers, with type 2 having twice the mass of type 1, and (non VNM) preferences, respectively, $u^1(c_0^1, c^1(s'), c^1(s'')) = c^1(s') / \left(1 - (c_0^1)^{\frac{9}{10}}\right)^{\frac{10}{9}}$ and

[23]The proof is in Appendix A.2. See also Allen and Gale (1988) for a constrained efficiency result in a related environment.

$u^2(c_0^2, c^2(s'), c^2(s'')) = c_0^2 + (c^2(s''))^{1/2}$, *endowments* $w_0^1 = .95$, $w_0^2 = 1$ *and* $w^1(s) = w^2(s) = 0$ *for all* $s \in \mathcal{S}$. *The technology of the representative firm is described by* $f(k, \phi; s) = \phi k$ *for* $s = s'$ *and* $(1-\phi)k$ *for* $s = s''$, *where* $\phi \in \Phi = [2/3, 0.99]$. *We abstract from the firms' financial decisions and set* $B = 0$. *The problem faced by firms in this environment is then* $\max_{\phi,k} -k + q(k,\phi)$, *where* $q(k,\phi) = \max\left\{\frac{\partial u^1/\partial c^1(s')}{\partial u^1/\partial c_0^1}\phi k; \frac{\partial u^2/\partial c^2(s'')}{\partial u^2/\partial c_0^2}(1-\phi)k\right\}$.
In this economy, Dierker et al. (2002) find a unique Dreze equilibrium where all firms choose a production plan with $\phi \approx 0.7$.[24]
According to our equilibrium concept, however, a symmetric equilibrium, where all firms choose the same value of k *and* ϕ, *does not exist. Given the agents' endowments and preferences, both types of consumers buy equity in equilibrium. It is then easy to see that the firms' optimality condition with respect to* ϕ *can never hold for an interior value of* ϕ *nor for a corner solution.*[25] *On the other hand, an asymmetric equilibrium exists, where a fraction* $1/3$ *of the firms choose* $\phi^1 = 0.99$ *and* $k^1 = 0.3513$ *and the remaining fraction chooses* $\phi^2 = 2/3$ *and* $k^2 = 0.1667$, *type* 1 *consumers hold only equity of the firms choosing* ϕ^1, k^1 *and type* 2 *consumers only equity of the other firms. At this allocation, we have* $\frac{\partial u^1/\partial c^1(s')}{\partial u^1/\partial c_0^1} = 1.0101$, $\frac{\partial u^2/\partial c^2(s'')}{\partial u^2/\partial c_0^2} = 3$. *Also, the marginal valuation of type* 1 *agents for the equity of firms choosing* ϕ^2, k^2 *is* 0.1122, *thus smaller than the market value of these firms' equity, equal to* 0.1667, *while the marginal valuation of type* 2 *agents for the equity of the firms choosing* ϕ^1, k^1 *is* 0.0105, *smaller than the market value of these firms' equity, equal to* 0.3513. *Therefore, at these values the firms' optimality conditions are satisfied. It can then be easily verified that this constitutes a competitive equilibrium according to our definition and that the equilibrium allocation is constrained optimal.*

Modigliani-Miller In this section we study the properties of the firms' corporate finance and investment decisions at an equilibrium. To this end, it is convenient to introduce the

[24]The definition of Dreze equilibrium in Dierker et al. (2002) uses a specification of the firms' conjecture over their market value for out of equilibrium production plans that differs from the map $q(\phi, k)$ satisfying the consistency conditions imposed here in two important respects. The market value is computed i) by considering only the set of equilibrium shareholders rather than all consumers, and ii) by taking into account the effect of each plan on the marginal rate of substitution of shareholders rather than taking such rates as given.

[25]Consider for instance $\phi = 0.99$. To have an equilibrium at this value the marginal valuation of equity for both consumers must be the same at $\phi = 0.99$ and higher than at any other values of ϕ, but this second property clearly cannot hold for type 2 consumers.

notation I^e to denote the collection of all agents i such that

$$\bar{q} = \mathbb{E}\left[\overline{MRS}^i(s)\left(f(\bar{k}, \bar{\phi}; s) - \bar{B}\right)\right]$$

that is, the collection of all agents that in equilibrium either hold equity or are indifferent between holding and not holding equity. We can similarly define the collection I^d of all agents i such that $\bar{p} = \mathbb{E}\overline{MRS}^i(s)$, that is, the collection of all agents that in equilibrium either hold bonds or are indifferent between holding and not holding bonds. With a slight abuse of language we denote the agents in I^e as equity holders and those in I^d bond holders.

On this basis we can state the following useful implication of the firm's optimality conditions[26]: at a solution of the first order conditions of the firm's choice problem (1), where the no default constraint (2) does not bind, all equity holders are also bondholders. When such constraint binds, on the other hand, it is possible that no equityholder is also a bondholder. More importantly, we can study the implications of these conditions for the firm's optimal financing choice, described by B. Is such choice indeterminate? Equivalently, does the Modigliani-Miller irrelevance result hold in our setup? The answer clearly depends on whether the no default constraint is slack or binds. We consider each of these two cases in turn.

Let $\underline{s}$ denote the lowest output state[27]. When $f(\bar{k}, \bar{\phi}; \underline{s}) > \bar{B}$ (the no default constraint is slack) the value of the firm V is locally invariant with respect to any change in B. Furthermore, this invariance result extends to any admissible[28] change in B: all equity holders are in fact indifferent with respect to any admissible, discrete change ΔB, whether positive or negative. The other agents might not be indifferent, but the optimality of $\bar{B}, \bar{k}, \bar{\phi}$ implies their valuation of the firm is always lower.

When the optimum obtains at a corner, $f(\bar{k}, \bar{\phi}; \underline{s}) = \bar{B}$, either the same property still holds (V is invariant with respect to any admissible change in B), or V is strictly increasing in B. The latter property occurs when no equity holder is also a bond holder (in fact each shareholder would like to short the bond), in which case the firm's problem has a unique solution for B.

To sum up, except in the case in which no equity holder is also a bond holder, at a competitive equilibrium the value of the firm V is invariant with respect to any admissible

[26] See Appendix A.3 for a complete characterization of these conditions.

[27] This may clearly depend on k, ϕ, but we omit to make it explicit for simplicity of the notation.

[28] An upper bound on the admissible levels of B is obviously given by the value at which the no default constraint binds, while the lower bound is 0.

change in B. It is important to note however that, while in such situation the capital structure is indeterminate for any individual firm, this does not mean that the capital structure of the economy, that is of all firms in the economy, is also indeterminate. In particular, the equilibrium is invariant only to changes in the aggregate stock of bonds in the economy ΔB such that all equity holders remain also bond holders and this imposes a lower bound on the aggregate value of ΔB consistent with the given equilibrium (given by $-\min_{i\in I^e} \bar{b}^i/\bar{\theta}^i$). We have thus established the following:

Proposition 4 (Modigliani-Miller) *At a competitive equilibrium, the capital structure choice of each individual firm is indeterminate, except when the firm's no default constraint binds and no equity holder is also a bond holder (in which case there is a unique optimal level of B, at $f(\bar{k}, \bar{\phi}; \underline{s})$). On the other hand, the equilibrium capital structure of all firms in the economy is, at least partly, determinate: for any equilibrium value $\bar{B}$ only the values of the capital structure for all firms in the economy given by $\bar{B}+\Delta B$ such that $\Delta B \geq -\min_{i\in I^e} \bar{b}^i/\bar{\theta}^i$ are consistent with such equilibrium.*

Thus the Modigliani-Miller irrelevance result does not fully hold in equilibrium. The reason for this result is the presence of borrowing constraints, which restrict the set of equilibrium values of the capital structures to an interval; see Stiglitz (1969) for a first result along these lines.

An example. It is useful to illustrate the properties of the equilibrium and the firms' production and financial decisions by considering a simple example, with two types of consumers, $I = 2$. Suppose both consumers have initial equity holdings $\theta_0 = .5$ and preferences described by $\mathbb{E}u^i(c_0^i, c^i(s)) = u(c_0^i) + \beta\mathbb{E}u(c^i(s))$, $i = 1, 2$; with $u = \frac{c^{1-\gamma}}{1-\gamma}$, $\gamma = 2$ and $\beta = 1$. The production technology exhibits two factors and multiplicative shocks affecting each of them: $f(k, \phi; s) = \phi a_1(s)k^\alpha + (1-\phi)a_2(s)k^\alpha$, where $a_h(s)$ is the aggregate productivity shock affecting factor $h = 1, 2$ and $\phi \in \Phi = \{0, 1\}$ describes the choice of one of the two factors. We assume $\alpha = .75$. The structure of endowment and productivity shocks is reported in Table 1, for $\mathcal{S} = \{s_1, s_2, s_3\}$.

In addition, the date 0 endowment is $w_0^i = w^i(s_2)$ for all i and $\pi(s_1) = \pi(s_2) = \pi(s_3) = \frac{1}{3}$.

We find that for this specification there is a unique equilibrium allocation where the factor loadings and the investment levels are $\bar{\phi} = 0$ and $\bar{k} = .4888$ for all firms, the capital structure

	s_1	s_2	s_3
w^1	1	2	3
w^2	1.1	2	2.9
a_1	1	2	3
a_2	1.1	2	2.9

Table 1: Example with risk free debt: stochastic structure.

of all firms in the economy is given by any level of $\bar{B}$ lying in the interval $[.1828, .6431]$, while the financial decision of each individual firm is indeterminate, given by $\bar{B} \in [0, .6431]$.

In order to better illustrate the determinants of the firms' equilibrium capital structure, set $\phi = 0$ and treat parametrically the level of debt issued by each firm. For any given value B^{ex} of such debt we find the investment level k which maximizes firms' value, the individual consumption and portfolio holdings $\{c^i, \theta^i, b^i\}_{i=1}^2$ solving (4) and the prices $\{q, p\}$ such that markets clear and the consistency conditions for q hold. In 1 we plot, as B^{ex} is varied from 0 to .6431, the values obtained for the consumers' asset holdings, on the first line, and their marginal valuations for the assets, on the second line. We can then use this figure to determine when we have an equilibrium, which happens when the optimality condition for the firms' financing decisions is satisfied.

At $B^{ex} = 0$ the default constraint does not bind. From the top left panel we see that both consumers hold equity and from the bottom right panel that consumer 1 has a higher marginal valuation for the bond than consumer 2. At $B^{ex} = 0$ any firm can so increase its value[29] by issuing debt, thus $B = 0$ is not an equilibrium value. As B^{ex} is progressively increased from 0 to .1828, it remains true that consumer 1 has a higher marginal valuation for the bond. As for equity, the two consumers' valuations coincide so that both hold equity. Thus for all values of B^{ex} from 0 to .1828 it is not true that all equity holders are also bond holders; since the default constraint never binds in this region, any firm can increase its value by issuing debt.

At $B^{ex} = .1828$, on the other hand, the two consumers have the same marginal valuation for the bond (see the bottom right panel) and they both hold equity. Thus, all equity

[29]The firms' value is determined using the specification of the equity price conjecture obtained, as stated in the consistency condition M of Section 2.1, from the consumers' marginal rate of substitution at the equilibrium allocation associated to $B^{ex} = 0$.

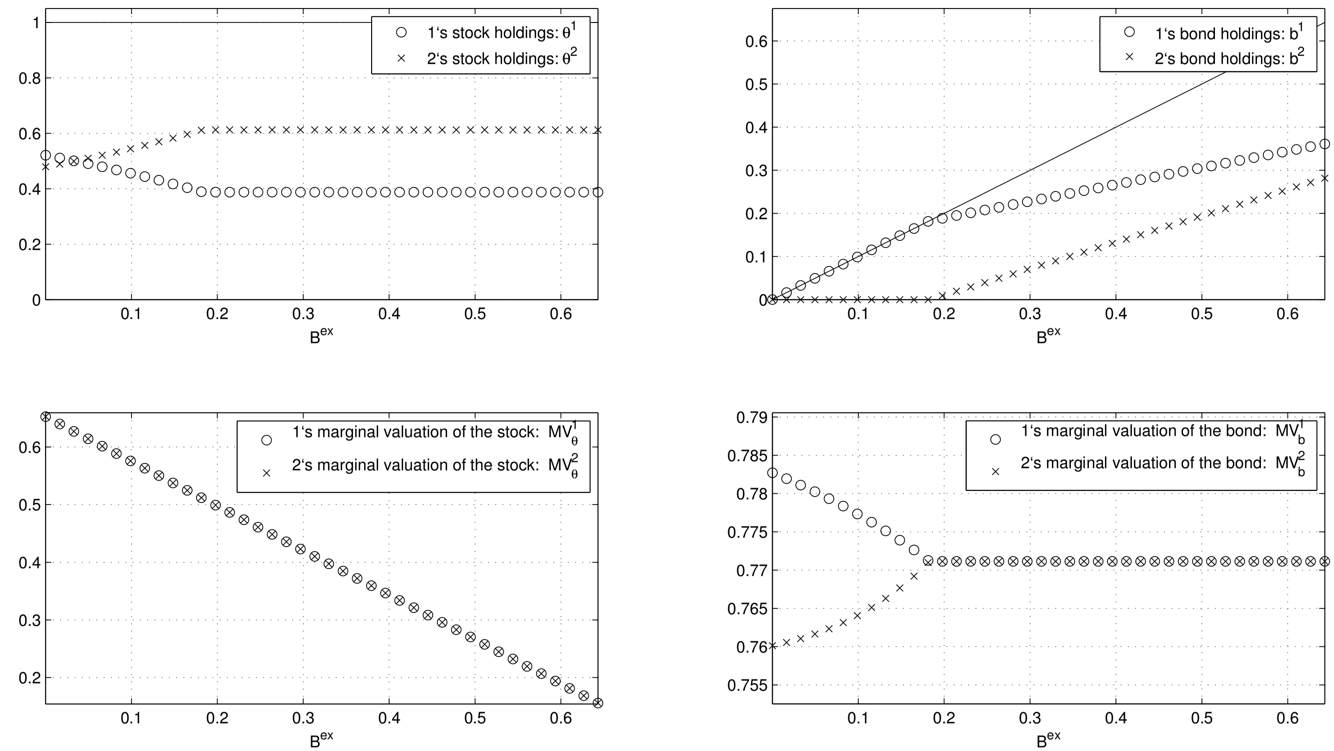

Figure 1: Parametric exercise: market clearing values, for given $B^{ex} \in [0, .6431]$, $\phi = 2$. i) First row: consumers' asset holdings. ii) Second row: consumers' willingness to pay for equity $\mathbb{E}MRS^i(s)\,[a_1(s)k^\alpha - B^{ex}]$ and bonds $\mathbb{E}MRS^i(s)$, $i = 1, 2$.

holders are also bond holders and the prices and allocations obtained when $B^{ex} = .1828$ (with $k = .4888$) constitute an equilibrium of our model. As B^{ex} is increased beyond .1828, up to its maximal level such that the no default condition is satisfied (.6431), the allocation and bond prices remain the same and still constitute an equilibrium. Values of $B^{ex} > .6431$ can only be sustained if the firm's investment k is increased so as to satisfy the no default constraint: we find however that this never happens at an equilibrium.

To sum up, the equilibrium consumption and investment levels are uniquely determined while the capital structure of all firms in the economy is only partly determinate, given by any $\bar{B} \in [.1828, .6431]$. This is in accord with our findings in Proposition 4 for the case in which the default constraint does not bind (as it is here).

Figure 2 then shows that, also in accord with Proposition 4, the financial decision of each individual firm is indeterminate. It plots the value of an arbitrary firm, $-k+q(k, \phi, B)+pB$, for $\bar{\phi} = 0$ and different levels of k and B: we see that the firm's maximal level is attained at $\bar{k} = .4888$ and all $\bar{B} \in [0, .6431]$.

3 Intermediated short sales

If agents are allowed infinite short sales of the equity of firms, as is the case for traded assets in the standard GEI model, a *small* firm can have a *large* effect on the economy by choosing a production plan with cash flows which, when traded as equity, changes the asset span. It is clear that the price taking assumption is hard to justify in such context, since changes in the firm's production plan have non-negligible effects on consumers' admissible trades and hence on allocations and equilibrium prices. This problem does not arise when consumers face a constraint preventing short sales, as (3) in the environment considered in the previous section and in Makowski (1983a,b). In this case the production decisions of any small firm have a small effect on attainable allocations and, as argued by Hart (1979), price taking behavior is justified when the number of firms is large.

Evidently, for price taking behavior to be justified a no short sale constraint is more restrictive than necessary and a bound on short sales of equity would suffice. Given the importance of short sales in asset markets, it is of interest to extend the analysis to the case where consumers can sell short the firm's equity.[30] A short position on equity is, both conceptually and in the practice of financial markets, different from a simple negative holding

[30] We could allow for short sales of the bond as well, at only a notational cost.

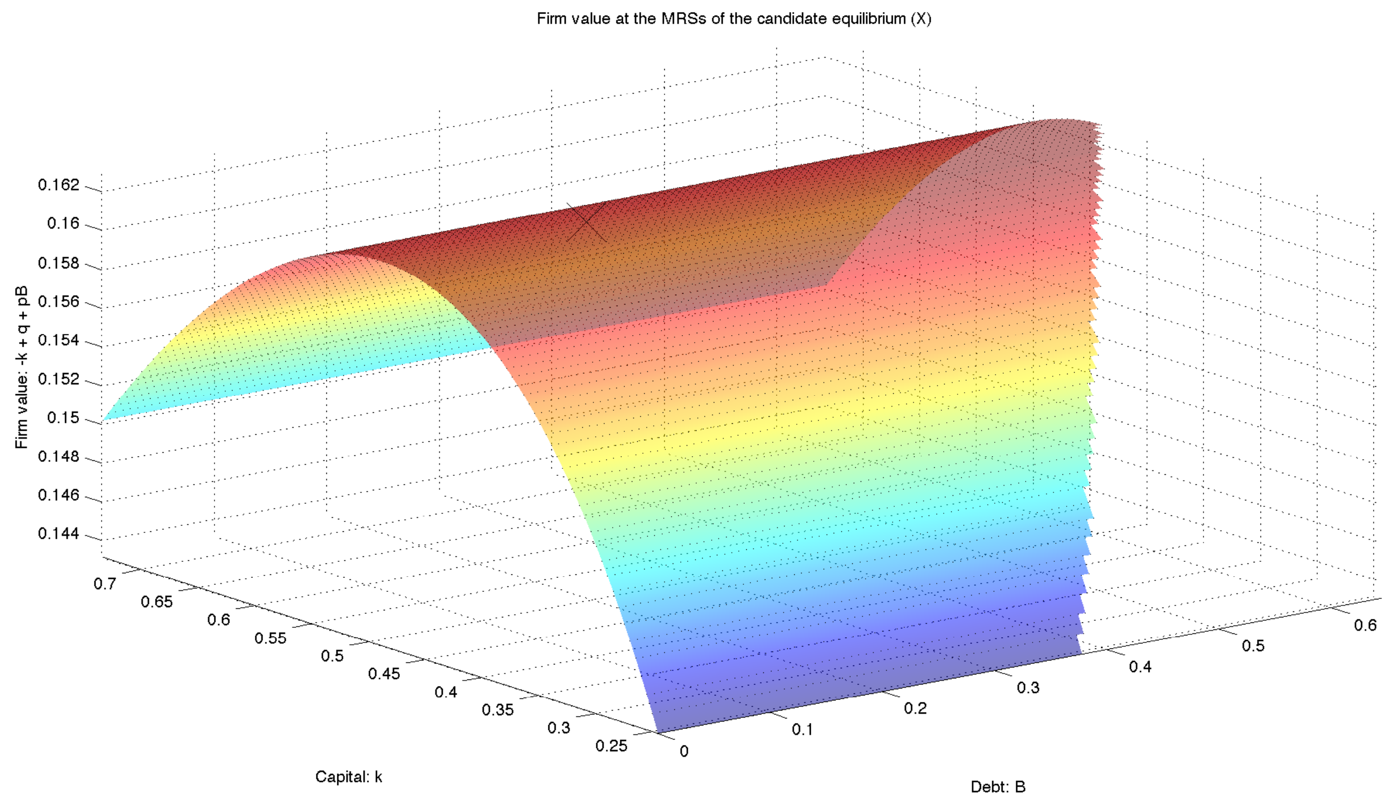

Figure 2: Value of an arbitrary firm, $-k + q(k, \bar{\phi}, B) + pB$, as a function of k and B (for $\bar{\phi} = 0$), where $q(k, \bar{\phi}, B)$ is computed using the consumers' MRSs at the equilibrium allocation. The $\times$ in the plot represents the lower bound of the Modigliani-Miller region, i.e. $\bar{B} = .1828$.

of equity. A short sale is not a simple sale; it is a loan contract with a promise to repay an amount equal to the future value of equity. To model short sales it is then natural to introduce financial intermediaries, who can issue claims corresponding to both short and long positions (more generally, derivatives) on the firm's equity, subject to frictions, e.g. default or transaction costs. This ensures that the notion of competitive equilibrium is well-defined, even if such frictions are arbitrarily small.

In this section we consider a specific form of friction, whereby intermediaries bear no cost to issue claims, but face the possibility of default on the short positions they issue (e.g., on the loans induced by the sale of such positions). We show that the results of the previous section, including unanimity and constrained optimality, extend to the case where short sales are allowed.[31] To allow a clearer understanding of the argument, it is convenient to consider first a reduced form version of the model where the default rate on short positions is exogenously given and equal to $\delta > 0$ in every state, for all consumers. In Appendix B.1 we then show how the analysis and results extend to the general case where default rates are endogenously chosen by consumers.

An intermediary who is intermediating m units of the derivative on the firm's equity (that is, issuing m long and short positions) is repaid only a fraction $(1-\delta)$ of the amount due on each short position issued. To ensure its own solvency, the intermediary must hold an appropriate portfolio of claims, as a form of collateral, whose yield can cover the shortfalls in the revenue from its intermediation activity due to consumers' defaults. The best hedge against consumers' default risk on short positions on equity is clearly equity itself. The intermediary must hold then an amount γ of equity of the firm satisfying the following constraint

$$m \leq m(1-\delta) + \gamma, \tag{13}$$

to ensure its ability to meet all its future obligations.

To cover this collateral cost intermediaries may charge a different price for long and short positions in the derivative issued. Let q^+ (resp. q^-) be the price at which long (resp. short) positions in the derivative issued by the intermediary are traded, while q is still the price faced by consumers and intermediaries when acquiring a unit of equity from the firm. The intermediary chooses then the amount of long and short positions in the derivative

[31] We could also allow intermediaries to issue different types of derivatives on the firm's equity, again at only notational cost.

intermediated, $m \in R_+$, and the amount γ of equity held as a hedge, so as to maximize its total revenue at date 0:

$$\max_{m,\gamma} \left[(q^+ - q^-)m - q\gamma\right] \tag{14}$$

subject to the solvency constraint (13).

The intermediation technology is characterized by constant returns to scale. Thus, a solution to the intermediary's choice problem exists provided

$$q \geq \frac{q^+ - q^-}{\delta};$$

and is characterized by $\gamma = \delta m$ and $m > 0$ only if $q = \frac{q^+ - q^-}{\delta}$.

In this set-up derivatives are thus "backed" by equity in two ways: (i) the yield of each derivative is "pegged" to the yield of equity of the firm;[32] (ii) to issue any short position in the derivative, the intermediary has to hold an appropriate amount of equity of the same firm to whose return the derivative is pegged to cover the intermediation costs (insure against the risk of its customers' default).

Let $\lambda_+^i \in R_+$ denote consumer i's holdings of long positions in the derivative, and $\lambda_-^i \in R_+$ his holdings of short positions. The consumer's budget constraints in this environment are then as follows:[33]

$$c_0^i \ = w_0^i \ + [-k \ + q \ + p \ B\ ]\, \theta_0^i - q\ \theta^i \ - p\ b^i - q^+ \lambda_+^i + q^- \lambda_-^i \tag{15}$$

$$c^i(s) = w^i(s) + [f(k, \phi; s) - B\ ]\, (\theta^i + \lambda_+^i) \ - [f(k, \phi; s) - B\ ]\, \lambda_-^i (1 - \delta) + b^i, \ \forall s \in \mathcal{S} \tag{16}$$

The consumer's choice problem consists in maximizing his expected utility subject to the above constraints and $\left(\theta^i, b^i, \lambda_+^i, \lambda_-^i\right) \geq 0$.

The asset market clearing conditions are now, for equity

$$\gamma + \sum_{i \in I} \theta^i = 1,$$

and for the derivative security

$$\sum_{i \in I} \lambda_+^i = \sum_{i \in I} \lambda_-^i = m.$$

[32]The role of equity as a benchmark to which the return on derivatives can be pegged can be justified on the basis of the fact that asset returns cannot be written as a direct function of future states of nature.

[33]In the expression of the date 1 budget constraint we see that the consumer repays only a fraction $(1 - \delta)$ of the amount due on his λ_-^i short positions, and defaults on the rest.

The firm's choice problem is unchanged, still given by (1) subject to (2). However, the specification of the equity price conjectures $q(k,\phi,B)$ has to be properly adjusted to reflect the fact that intermediaries, in addition to consumers, may demand equity in the market. The *Makowski criterion* has then to be replaced by the following condition:

$$M^{SS})\ \ q(k,\phi,B) = \max\left\{ \begin{array}{c} \max_i \mathbb{E}\left[\overline{MRS}^i(s)\left(f(k,\phi;s)-B\right)\right], \\ \frac{\max_i \mathbb{E}\left[\overline{MRS}^i(s)(f(k,\phi;s)-B)\right]-\min_i \mathbb{E}\left[\overline{MRS}^i(s)(f(k,\phi;s)-B)(1-\delta)\right]}{\delta} \end{array} \right\}, \forall k,\phi,B.$$

The above expression states that the conjecture of a firm over the price of its equity when its choices are k,ϕ,B, $q(k,\phi,B)$, equals the *maximal valuation, at the margin, among consumers and intermediaries, of the equity's cash flow* corresponding to k,ϕ,B. The second term on the right hand side of the expression in condition M^{SS}) is in fact the intermediaries' marginal valuation for equity and can be interpreted as the *value of intermediation*. Since an appropriate amount of equity is needed, to be retained as collateral, in order to issue the corresponding derivative claims, the intermediary's willingness to pay for any type of equity is determined by the consumers' marginal valuation for the corresponding derivative claims which can be issued. Hence the above specification of the firms' equity price conjectures allows firms to take into account the effects of their decisions on the value of intermediation.

A competitive equilibrium of the economy with short sales is then defined along the same lines of Definition 1 in Section 2.1. Two possible situations can arise in equilibrium with regards to the level of intermediation:

Full: $q = (q^+ - q^-)/\delta > q^+$, which is in turn equivalent to $q^+ > q^-/(1-\delta)$. In this case equity sells at a premium over the long positions on the derivative claim issued by the intermediary (because of its additional value as input in the intermediation technology). Thus all the amount of equity outstanding is purchased by the intermediary, who can bear the additional cost of equity thanks to the presence of a sufficiently high spread $q^+ - q^-$ between the cost of long and short positions on the derivative.

Partial: $q = q^+$. In this case there is a single price at which equity and long positions in the derivative can be traded. Consumers are then indifferent between buying long positions in equity and the derivative and some if not all the outstanding amount of equity is held by consumers. When consumers hold all the outstanding amount of equity, intermediaries are non active at equilibrium and the bid ask spread $q^+ - q^-$ is low (in particular, less or equal than δq).[34]

For the economy with intermediation we described, the same unanimity and efficiency properties of competitive equilibria as in the economy with no short sales hold:

Proposition 5 *At a competitive equilibrium of an economy with intermediated short-sales, equity holders unanimously support the production and financial decisions of the firms. Moreover, the equilibrium allocation is constrained Pareto optimal.*

The argument of the proof of such claims is essentially the same as the one for Propositions 2 and 3, and again relies on the fact that a competitive equilibrium of the model described above is equivalent to one where all markets, that is not only the markets for all possible types of equity (associated to any possible choice k, ϕ, B of firms), but also the markets for all types of corresponding derivatives are open for trade to consumers. For all $(k', \phi', B') \neq \left(\bar{k}, \bar{\phi}, \bar{B}\right)$ the price for long and short positions are, respectively:

$$\begin{aligned} q^+(k', \phi', B') &= \max_i \mathbb{E}\left[\overline{MRS}^i(s)\left(f(k', \phi'; s) - B'\right)\right] \\ q^-(k', \phi', B') &= \min_i \mathbb{E}\left[\overline{MRS}^i(s)\left(f(k', \phi'; s) - B'\right)(1-\delta)\right] \end{aligned}$$

and at these prices both the market for long and short positions clear with a zero level of trade. This follows from the specification of the consistency condition M^{SS}) imposed on the firms' price conjectures, hence the efficiency result.

Note that in the present economy with intermediated short-sales consumers face no upper bound on their short sales of equity, but the presence of a bid ask spread still limits their hedging possibilities.[35] It is interesting to compare our optimality result with Theorem 5 in Allen and Gale (1991), where it is shown that the competitive equilibria of an economy

[34]In this case consumers face effectively a zero spread, as $q^+ - q^-$ reflects only the difference in returns between long and short positions.

[35]In the case of partial intermediation, as we said in the previous footnote, the spread is effectively zero, but it is immediate to see from M^{SS}) that this requires that $\mathbb{E}\left[\overline{MRS}^i(s)\left(f(k, \phi; s) - B\right)\right]$ is equalized across all consumers. This condition is generically not satisfied for non traded assets (that is corresponding to out of equilibrium firms' choices). As a consequence, for these assets we have full intermediation, so that the spread is positive, which limits the consumers' hedging possibilities, as argued.

with finite, exogenous bounds $\bar{K}$ on short sales are constrained suboptimal.[36] In their set-up, long and short positions trade at the same price, i.e., the bid ask spread is zero. The inefficiency result in Allen and Gale (1991) then follows from the fact that the expression of their market value which firms maximize ignores the effect of their decisions on the value of intermediation. In other words, a firm does not take into account the possible gains arising from the demand for short positions in the firm's equity. In contrast in our economy, where equity is an input in the intermediation process that allows short sales positions to be traded in the market, the firm considers the value of its equity not only for the consumers but also for the intermediaries when making its production and financial decisions. The gains from trade due to intermediation are so taken into account by firms.[37]

It is also useful to contrast our findings with the inefficiency result in Pesendorfer (1995). Example 2 in Pesendorfer (1995) shows that a competitive economy where financial intermediaries can introduce complementary innovations in the market may get stuck at an equilibrium in which no intermediary innovates, even though welfare would be higher if all innovations were traded in the market. The result in this example is related to similar findings obtained in competitive equilibrium models with differentiated goods; notably, Hart (1980) and Makowski (1980). In fact the inefficiency arising in the economy considered by Pesendorfer (1995) is conceptually similar to that of Allen and Gale (1991) just discussed: each intermediary is implicitly restricted not to trade with other intermediaries. Equivalently, equilibrium prices for non-traded innovations are restricted not to include at the margin their effect on the value of intermediation. If instead prices for non-traded innovations were specified so as to equal the maximum between the consumers' and the intermediaries' marginal valuation, as in our equation M^{ss}), constrained optimality would obtain at equilibrium.

[36] Though firms' decisions in Allen and Gale (1991) concern primarily which securities to issue, their analysis could be easily reformulated in a set-up where firms have to choose their level of output and take financial decisions, as in this paper.

[37] Another way to understand the difference between the present set-up and the one in Allen and Gale (1991) is by comparing the degree of completeness of the market in the two cases. Here, as argued above, the situation is effectively one where the markets for all possible derivative claims (corresponding to any plan k, ϕ, B) are open and clear at the equilibrium prices. Hence if no firms chooses a particular plan k', ϕ', B', the market for the associated derivatives is cleared at no trade, possibly with a large spread between the price for buying and selling positions. This is not the case in Allen and Gale (1991). To have an equilibrium in their set-up, where long and short positions are restricted to trade at the same price, the bound on short sales $\bar{K}$ must be 0 for the claims corresponding to values of k, ϕ, B different from those chosen by firms. Effectively, then, these markets are closed and an inefficiency may so arise.

4 Asymmetric information

In this section we will study economies in which an additional link between production and financing decisions is due to the presence of asymmetric information between debt holders, equity holders and the firm's management (the agents who manage the firm and choose its production plans).

In corporate finance models with such informational asymmetries have been studied for decades, at least since the work of Jensen and Meckling (1976). In fact, these models are workhorses for much of corporate finance and, in particular, for the study of the determinants of firms' capital structure and managerial incentive compensation.[38] It is thus important to extend our analysis to allow for the consideration of these issues, as the work mentioned above is typically cast in a partial equilibrium framework. A general equilibrium model allows to study the interaction between managerial incentive contracts, the equilibrium property of the firms' capital structure, and the general equilibrium effects of these agency problems, like the endogenous determination of aggregate risk in the economy and its implications for asset pricing.

Once again we shall mostly stress foundational issues, from the specification of the objective function of the firm to the analysis of the effects of its financial decisions and the efficiency properties of equilibria, rather than applications. This is necessary because, while general equilibrium theory has been extended to the study of economies with asymmetric information, from the seminal work of Prescott and Townsend (1984) to, e.g., the more recent work of Dubey et al. (2005), Bisin and Gottardi (1999) and Bisin and Gottardi (2006), most of this work concerns asymmetric information on the consumption side.

More specifically, we shall introduce asymmetric information regarding the observability of the firms' production and financial decisions k, ϕ, B. An implicit assumption in the analysis of the economy considered in the previous sections is that these decisions are observed by all the agents, investors in particular, so they can correctly anticipate what the payoff in each state will be when they choose their trades in the asset markets at date 0. In this section we study the case where the firm's choice of ϕ, unlike that of k and B, is not observed by investors in financial markets at time 0; we can refer so to such a situation as one with *unobservable risk composition.* This generates an informational asymmetry of the moral hazard type. We also allow for the possibility that the firm defaults on its debt; corporate

[38] See Tirole (2006).

debt is then a risky asset also the bondholders' return depend on the firm's choice of k, ϕ, B.

We first show that a competitive equilibrium notion based on Makowski's criterion, as a natural extension of our analysis in the previous sections, is equivalent to the one introduced by Prescott and Townsend (1984)) for a moral hazard exchange economy (with no production). This is shown for the simpler case where the informational asymmetry is between the firm and its outside investors (that is the agents buying debt and equity of the firm). We extend next the analysis to the more natural environment where the decisions within the firm are more explicitly modelled and hence the asymmetry is between the manager of the firm and the investors in the firm. In this case the choice of the management and its compensation is also required. We show that existence, unanimity and efficiency are still valid for such economies and illustrate the effects of asymmetric information on the properties of competitive equilibria for the same economy of the example considered in Section 2.1.2.

4.1 Makowski meets Prescott and Townsend

Consider the simpler case where the firm's choice of ϕ is not observed by the firm's equity-holders and bond-holders. We can think of this choice as taken by the initial shareholders. We make explicit now the cost $W(\phi; k, B)$ entailed by the choice of ϕ, as discussed in footnote 18, to highlight the agency problem. For instance, ϕ could represent a costly action to limit the firm's downside risk. Its cost may depend also on the size of the firm, k, and on its capital structure, B, as size and capital structure in turn affect the firm's corporate governance and its exposure to risk.

In addition, we allow for the possibility that a firm defaults on its debt in some states. Hence corporate debt is now a risky asset and its return, $\min\left\{1, \frac{f(k,\phi;s)}{B}\right\}$, varies, like equity's, with the state as well as the firm's production, k, ϕ, and financial decisions, B. As a consequence the firm operates on the basis of a price conjecture also for its debt.

In this environment a competitive equilibrium can be defined and studied by directly extending the equilibrium concept used in the previous sections with symmetric information. The consumer's problem is unchanged with respect to the one in the previous sections, given by (4), except for the fact that the return on debt is now given by $\min\left\{1, \frac{f(k,\phi;s)}{B}\right\}$ instead of 1.[39]

Similarly firms' choices result from value maximization, as with symmetric information,

[39] We assume that agents cannot short-sell the firm equity nor its debt for simplicity.

though now the price conjectures only depend on k, B, since these are only the only choices of the firm observable by the agents buying debt and equity of the firm. Letting then price conjectures for equity and debt, respectively, be denoted $q(k, B)$ and $p(k, B)$, the problem of the firm is:

$$V = \max_{k,\phi,B} -k - W(\phi, k, B) + q(k, B) + p(k, B)B \tag{17}$$

Note also that the solvency constraint (2) is no longer imposed on the firm's choice problem since we allow for the possibility of default on corporate debt.

Where asymmetric information displays its main effects is in the formulation of the consistency conditions for the equilibrium price conjectures. The Makowski criterion requires that the firm rationally anticipates its value, that is the market value of its equity and bonds, for any of its possible choices. With symmetric information these conjectures equal, as we saw, the highest marginal valuation across consumers for the cashflow of equity, and now also of bonds, associated to any possible choice of (k, ϕ, B). With asymmetric information, the price conjectures do not depend on ϕ as outside investors do not observe ϕ, even though the yield of equity and bonds depends on ϕ. But the specification of rational price conjectures must reflect the correct anticipation of the level of ϕ chosen by the firm, given (k, B). More formally, investors expect ϕ to satisfy:

$$\phi = \phi(k, B) \in \arg\max_{\phi} -k - W(\phi, k, B) \tag{18}$$

We are now ready to specify the consistency conditions imposed on firms' price conjectures in the environment under consideration:

C^{AI}) $q(\bar{k}, \bar{B}) = q$, $p(\bar{k}, \bar{B}) = p$;

M^{AI}) $q(k, B) = \max_i \mathbb{E}\left[\overline{MRS}^i(s) \max\{f(k, \phi(k, B); s) - B, 0\}\right]$, and
$p(k, B) = \max_i \mathbb{E}\left[\overline{MRS}^i(s) \min\left\{1, \frac{f(k,\phi(k,B);s)}{B}\right\}\right]$, $\forall k, B$, where $\phi(k, B)$ satisfies (18).

Finally, the market clearing conditions are unchanged, given by (7).

Summarizing, a competitive equilibrium with asymmetric information is defined as in Definition 1, simply replacing (1) with (17) and the consistency conditions C), M) with C^{AI}), M^{AI}).

Prescott and Townsend (1984)'s approach to competitive equilibrium in a related class of economies with asymmetric information is different. Their equilibrium concept does not rely, as above, on prices that only depend on observable choices and reflect the correct anticipation of the unobservable choices made, as implicitly done in condition M^{AI}) above. In contrast, in Prescott and Townsend's equilibrium concept, prices depend both on unobservable as well as observable choices and this is sustained by restricting admissible choices to those which are incentive compatible. Though they only study pure exchange economies we can still repropose the main features of their approach here. More formally, in a Prescott-Townsend's equilibrium for the present environment, price conjectures are given by $q(k, \phi, B)$ and $p(k, \phi, B)$ and the firm's choice problem becomes

$$V = \max_{k,\phi,B} -k - W(\phi, k, B) + q(k, \phi, B) + p(k, \phi, B)B \tag{19}$$

s.t.

$$\phi \in \arg\max -k - W(\phi, k, B) \tag{20}$$

Thus the admissible choices of the firm are restricted to satisfy (20). Only when (k, ϕ, B) satisfies (20) we can say that a firm choosing k, B will indeed also choose ϕ.

The consistency conditions at a Prescott and Townsend equilibrium need also to be modified accordingly with respect to C^{AI}) and M^{AI}):

$\mathbf{C}^{PT}$) $q(\bar{k}, \bar{\phi}, \bar{B}) = q,\ p(\bar{k}, \bar{\phi}, \bar{B}) = p$;

$\mathbf{M}^{PT}$) $q(k, \phi, B) = \max_i \mathbb{E}\left[\overline{MRS}^i(s) \max\left\{f(k, \phi; s) - B, 0\right\}\right]$ and
$p(k, \phi, B) = \max_i \mathbb{E}\left[\overline{MRS}^i(s) \min\left\{1, \frac{f(k,\phi);s)}{B}\right\}\right]$, $\forall k, \phi, B$.

It is straightforward to extend to this simple economy the results Prescott and Townsend (1984) obtained for the pure exchange economy with moral hazard they considered, showing in particular that equilibrium allocations are constrained (now also by incentives) Pareto optimal. Most importantly it is straightforward to show, essentially by substituting the incentive constraint (20) into the firm's problem (19), the following:[40]

Proposition 6 *Any competitive equilibrium with asymmetric information is a Prescott and Townsend equilibrium and viceversa. Furthermore competitive equilibria are constrained Pareto optimal.*

[40] See Jerez (2005) for a similar result in a pure exchange environment.

4.2 Competitive Equilibria with Managerial Choice

The simple environment with asymmetric information considered in the previous section allowed to clearly see the equivalence between Prescott and Townsend equilibria and the competitive equilibrium concept developed in this paper, based on the Makowski criterion. On the other hand this environment relies on the specification of an agency problem between the firm, or its initial shareholders, and the agents buying debt and equity of the firm. This is effectively a reduced-form specification, since the same consumer may be at the same time initial shareholder of the firm and debt and/or equity holder at the end of the period. Furthermore, the agency costs are exogenously specified.

In this section we show how to overcome these difficulties by modelling explicitly the decision process within the firm. The production plan k, B, ϕ is chosen by the manager of the firm who is in charge of the firm's production and financial decisions. While in the case of k, B there is no discretion as these choices are observable, ϕ is only privately observed by the manager, the characteristics of the agent appointed as manager matter. We postulate that the firm's shareholders select the one among the different type of consumers to serve as manager of the firm and choose the form of his compensation, so as to satisfy appropriate participation constraints taking also into account the manager's incentives. We will still use the notation W to indicate the cost of inducing the manager to choose a certain ϕ when the other firm's choice are given by k, B. However now W is endogenously specified, as the agency cost of incentivizing the manager to act in the interest of shareholders. Managers and their compensation are then endogenously chosen, in the environment studied in this section, illustrating how, in equilibrium models of corporate finance, corporate governance affects the pricing of equity and bonds.

More formally, a manager's compensation package consists of a gross payment x_0, in units of the consumption good at date 0, together with a portfolio of θ^m units of equity and b^m units of bonds. The firm's problem is then now the choice of the level of its physical capital k, its financial structure, described by B, as well as the type i of agent serving as its manager and his compensation package, θ^m, b^m, x_0, so as to maximize, as in the previous sections, the firm's market valuation. Each firm is still perfectly competitive and hence evaluates the effect of alternative production, financial and compensation choices on its market value on the basis of some given price conjectures. These conjectures specify the market valuation of the return on equity $q(k, B, i, \theta^m, b^m, x_0)$ and debt $p(k, B, i, \theta^m, b^m, x_0)$ associated to any possible choice of k, B and i, θ^m, b^m, x_0. As argued in the previous section, the price conjectures only depend on the observable decisions of the firm, hence not on ϕ. But, as we will see later, they correctly reflect the anticipation of the level of ϕ chosen by the type i agent selected as manager given k, B and θ^m, b^m, x_0.

We turn then our attention to the decision problem of the manager. An agent, if chosen as manager of a firm, will pick ϕ so as to maximize his utility, since the choice of ϕ is not observable. The choice of ϕ affects this agent's utility both because the agent may hold a portfolio whose return is affected by ϕ but also because the agent may incur some disutility cost associated to different choices of ϕ. Let this disutility costs be $v^i(\phi)$ for a type i consumer We will assume that the manager's portfolio is observable. In fact, without loss of generality, we assume that managers cannot trade their way out of the compensation package chosen by the equity holders, the compensation contract is then exclusive.[41] Hence the utility level attained by a type i agent hired as manager is given by

$$\max_{\phi} \mathbb{E}u^i\left(w_0^i \; + x_0, w^i(s) + \max\{f(k,\phi;s) - B, 0\}\theta^m \; + \min\left\{1, \frac{f(k,\phi;s)}{B}\right\} b^m\right) - v^i(\phi) \tag{21}$$

and the value of ϕ maximizing (21) constitutes the manager's optimal choice of ϕ. To be able to hire a type i agent as manager, an appropriate participation constraint must be satisfied: the compensation offered must be such that the value of (21) is not lower than i's reservation utility $\bar{U}^i$, which is endogenously determined in equilibrium (see below).

In the present environment the firm's market value is given by $-k + q + pB - W$, that is includes also the cost W of the manager's compensation package which needs then to be determined. The cost $W(k, B, i, \theta^m, b^m, x_0)$ of a compensation package θ^m, b^m, x_0 offered to a manager of type i when the other firm's choices are given by k, B is equal to:

a) the net payment made to this agent at date 0, equal to the gross payment x_0 minus the amount of the dividends due to this agent on account of his initial endowment θ_0 of equity, $\theta_0\left[-k + p(k, B, i, \theta^m, b^m, x_0)B - W(k, B, i, \theta^m, b^m, x_0)\right]$

b) plus the value of the portfolio $q(k, B, i, \theta^m, b^m, x_0)\left(\theta^{i,m} - \theta_0^i\right) + p(k, B, i, \theta^m, b^m, x_0)b^{i,m}$ attributed to him.

[41] See Acharya and Bisin (2009) and Bisin et al. (2008) for economies where much is made of the opposite assumption.

After collecting terms and simplifying, we obtain so the following expression for W:

$$W(k,B,i,\theta^m,b^m,x_0)=\frac{1}{1-\theta_0^i}\left[\begin{array}{l} x_0+q(k,B,i,\theta^m,b^m,x_0)\left(\theta^m-\theta_0^i\right)+p(k,B,i,\theta^m,b^m,x_0)b^m \\ -\theta_0^i\left[p(k,B,i,\theta^m,b^m,x_0)B-k\right]\end{array}\right] \tag{22}$$

We are now ready to formally state the firm's choice problem, which is conveniently divided in two steps. We first state the optimal choice problem of a firm who has hired as manager a type i consumer:

$$V^i=\max_{k,B,\theta^m,b^m,x_0} -k+q(k,B,i,\theta^m,b^m,x_0)+p(k,B,i,\theta^m,b^m,x_0)B-W(k,B,i,\theta^m,b^m,x_0) \tag{23}$$

s.t. (22) and:

$$\max_{\phi}\mathbb{E}u^i\left(w_0^i+x_0,w^i(s)+\max\{f(k,\phi;s)-B,0\}\theta^m+\min\left\{1,\frac{f(k,\phi;s)}{B}\right\}b^m\right)-v^i(\phi)\geq\bar{U}^i \tag{24}$$

The firm maximizes its market value determined on the basis of its price conjectures for any possible choice k,B,θ^m,b^m,x_0 under the incentive and participation constraints (24).

Next, the type $\bar{\imath}\in I$ of agent to be hired as manager is chosen by selecting the type which maximizes the firm's value:

$$\max_{i\in I}V^i \tag{25}$$

for V^i indicating the solution of problem (23).

Each consumer of a given type j, if not hired as manager, has to choose his portfolio of equity and bonds, θ^j and b^j, taking as given the price of bonds p and the price of equity q, as well as the dividends paid on equity at the two dates and the bonds' yield, so as to maximize his utility.[42]. The problem of such an agent is then as in Section 2, with the only difference that corporate debt is now a risky asset:

$$\max_{\theta^j,b^j,c^j}\mathbb{E}u^j(c_0^j,c^j(s)) \tag{26}$$

subject to

$$c_0^j=w_0^j+\left[-k+q+pB-W\right]\theta_0^j-q\,\theta^j-p\,b^j \tag{27}$$

$$c^j(s)=w^j(s)+\max\{f(k,\phi;s)-B,0\}\theta^j+\min\left\{1,\frac{f(k,\phi;s)}{B}\right\}b^j,\ \forall s\in\mathcal{S} \tag{28}$$

and

$$b^j\geq 0,\ \theta^j\geq 0,\ \forall j \tag{29}$$

[42]We maintain here the assumption that agents cannot sell short the firm's equity nor its debt. No conceptual difficulty is involved in allowing for intermediated short sales as in Section 3.

Let once again $\bar{\theta}^j, \bar{b}^j, \bar{c}^j$ denote the solutions of this problem and $\bar{U}^j$ the corresponding level of the agent's expected utility. It represents the reservation utility for a type j agent if hired as a manager.

In equilibrium, the bond and equity price conjectures faced by the firms must satisfy the following consistency conditions:

C^{AIM}**)** $p = p(\bar{k}, \bar{B}, \bar{\imath}, \bar{\theta}^m, \bar{b}^m, \bar{x}_0)$ and $q = q(\bar{k}, \bar{B}, \bar{\imath}, \bar{\theta}^m, \bar{b}^m, \bar{x}_0)$;

M^{AIM}**)** $p(k, B, i, \theta^m, b^m, x_0) = \max_i \mathbb{E}\left[\overline{MRS}^i(s) \min\left\{1, \frac{f(k,\phi(k,B,i,\theta^m,b^m,x_0);s)}{B}\right\}\right]$ and

$q(k, B, i, \theta^m, b^m, x_0) = \max_i \mathbb{E}\left[\overline{MRS}^i(s) \max\left\{f(k, \phi(k, B, i, \theta^m, b^m, x_0); s) - B, 0\right\}\right]$

for all k, B, where $\phi(k, B, i, \theta^m, b^m, x_0)$ satisfies:

$$\phi(k, B, i, \theta^m, b^m, x_0) = \tag{30}$$
$$\arg\max_{\phi\in\Phi} \mathbb{E}u^i\left(w_0^i \ + x_0, w^i(s) + \max\{f(k, \phi; s) - B, 0\}\theta^m \ + \min\left\{1, \frac{f(k,\phi;s)}{B}\right\} b^m\right) - v^i(\phi)$$

Condition C^{AIM}) requires that in equilibrium the prices faced by consumers in the market equal the prices conjectured by the firms for their equilibrium choices. Condition M^{AIM}) is the formulation of the Makowski criterion in the present environment. The complication caused by moral hazard, as already noticed in the previous section, is that the return on bond and equity depends on ϕ, but this is not observable by investors. The specification of the price conjecture reflects then the anticipation of the level of ϕ that will be chosen by the manager given his type, his compensation and the other firm's choices k, B, as described by the incentive compatibility constraint (30). This ensures that the firm's conjecture concerning the market value of its bond and equity for each of its possible choices regarding $k, B, i, \theta^m, b^m, x_0$ still equals the highest marginal valuation across all consumers, evaluated at their equilibrium consumption levels, for the return on debt and equity induced by these choices.

Finally, the following market clearing conditions must hold:[43]

$$\sum_{i\neq\bar{\imath}} \theta^i \ + \bar{\theta}^m \leq 1,$$
$$\sum_{i\neq\bar{\imath}} b^i \ + \bar{b}^m \leq B \tag{31}$$

[43]Recall that we have assumed for simplicity that the mass of agents of any given type i is equal to the mass of existing firms. This is obviously by no means essential.

Summarizing,

Definition 3 *A competitive equilibrium of an economy with moral hazard is a collection*

$$\left\{ \left(\bar{k}, \bar{B}, \bar{\phi}, \bar{\imath}, \bar{x}_0, \bar{\theta}^m, \bar{b}^m, \bar{W}\right), \left(\bar{c}^i, \bar{\theta}^i, \bar{b}^i, \bar{U}^i\right)_{i=1}^I, \bar{p}, \bar{q}, p(\cdot), q(\cdot) \right\}$$

such that: i) $\bar{k}, \bar{B}, \bar{\phi}, \bar{\imath}, \bar{x}_0, \bar{\theta}^m$ *and* $\bar{b}^m$ *solve the firm problem (25) given* $p(\cdot), q(\cdot)$ *and* $\{\bar{U}^i\}_{i=1}^I$*; ii)* $p(\cdot), q(\cdot)$ *satisfy the consistency conditions* C^{AIM}*) and* M^{AIM}*); iii) for all i,* $\bar{c}^i, \bar{\theta}^i, \bar{b}^i$ *solve consumer i's problem (26) s.t. (27), (28) and (29) for given* $\bar{p}, \bar{q}, \bar{k}, \bar{B}, \bar{\phi}$ *and* $\bar{W}$*; iv)* $\bar{W} = W(\bar{k}, \bar{B}, \bar{\phi}, \bar{\imath}, \bar{x}_0, \bar{\theta}^m, \bar{b}^m)$ *and* $\bar{U}^i = \mathbb{E}u^i(\bar{c}_0^i, \bar{c}^i(s))$ *and v) markets clear, (31).*

4.2.1 Unanimity and efficiency with moral hazard

In the economy with moral hazard just described each firm chooses the production and financing plan which maximizes its value. The firm takes fully into account the effects that its production and financing plan as well as its choice of management and associated compensation package have on its value and, in equilibrium, the model is equivalent to one where the markets for all types of equity and bonds are open. Consequently, by a very similar argument to the one developed in Section 2.1.2, equity holders' unanimity holds regarding the firm's production and financing decisions as well as the choice of management; that is the choice of k and B, as well as the decision over the manager and its compensation which in turn induces the choice of ϕ.

Proposition 7 *At a competitive equilibrium of the economy with moral hazard, equity holders unanimously support the production and financial decisions of firms as well as the choice of management,* $\bar{k}, \bar{B}, \bar{\phi}, \bar{\imath}, \bar{x}_0, \bar{\theta}^m, \bar{b}^m$*; that is, every agent i holding a positive initial amount* θ_0^i *of equity of the representative firm will be made - weakly - worse off by any other possible choice of the firm* $(k', B', \phi', i', x_0', \theta'^m, b'^m$ *satisfying (24)).*

We show next that all competitive equilibria of the economy described exhibit desirable welfare properties. Attainable allocations are now restricted not only by the limited set of financial assets that is available but also by the presence of moral hazard: the risk composition of the firms' cash-flow is chosen by the firms' managers and is not observable by the other agents. More formally, a consumption allocation $(c^i)_{i=1}^I$ is *admissible* in the presence of moral hazard if:

1. it is *feasible*: there exists a production plan k and a risk composition choice ϕ of firms such that (11) holds;

2. it is *attainable with the existing asset structure*: that is, there exists B and, for each consumer's type i, a pair θ^i, b^i such that

$$c^i(s) = w^i(s) + \max\{0, f(k,\phi;s) - B\}\theta^i \ + \min\left\{1, \frac{f(k,\phi;s)}{B}\right\} b^i, \ \forall s \in \mathcal{S}; \qquad (32)$$

3. It is *incentive compatible*: given the production plan k, ϕ and the financing plan B, there exists $\bar{\imath}$ such that:

$$\phi \in \arg\max_{\phi\in\Phi} \mathbb{E}u^i(c_0^{\bar{\imath}}, w^{\bar{\imath}}(s) + \max\{f(k,\phi;s) - B, 0\}\theta^{\bar{\imath}} \ + \min\left\{1, \frac{f(k,\phi;s)}{B}\right\} b^{\bar{\imath}}) - v^{\bar{\imath}}(\phi).$$

Constrained Pareto optimality is now straightforwardly defined as in Definition 2, with respect to the stronger notion of admissibility described above. The First Welfare theorem can then be established by an argument very similar to the one used earlier, for Proposition 3.

Proposition 8 *Competitive equilibria of the economy with moral hazard are constrained Pareto efficient.*

4.2.2 Capital structure with moral hazard

In equilibrium the financing plans of the firm are determined now not only by the demand of investors but also by managers' incentives. As in the economy considered in Section 2.1.2, investors' demand for bonds and equity gives the firm the incentive to leverage its position and finance production also with bonds. With riskless debt, as we noted in Section 2.1.2, this implies a lower bound on the quantity of corporate bonds issued by firms in equilibrium (while the upper bound is just given by feasibility, that is the no default constraint). When the firms' debt is risky, since the return on equity is a nonlinear function of B, both the aggregate and the individual firm's level of B are more precisely determined in equilibrium.[44]

[44] If risky debt is allowed in the setup of Section 2.1.2 (without moral hazard), an optimal choice for the firms obtains when all equity holders have the same valuation - and the same as bond holders - for bonds' payoff in the no default states. Differently from the case where debt is riskless this does not imply that all equity holders are also bond holders, since there is a second component of bonds' payoff, in the default states. Moreover, all bond holders have the same valuation for each of the two components of bonds' payoffs, taken separately. If

In the presence of moral hazard the capital structure of the firm also plays a role in determining the unobservable choice of ϕ and hence the returns on the firm's bonds and equity. For instance, a manager of a leveraged firm with a large amount of the firm's equity in his portfolio has the incentive to choose values of ϕ that induce a higher loading on riskier factors. This is because in this economy debt is risky and equity holders primarily benefit from the upside risk. Bond holders will therefore pay a premium for corporate bonds of less leveraged firms, whose managers also hold a larger proportion of debt than equity.

Thus both the capital structure and the portfolio composition of its manager can be used to align the manager's incentives with those of the firm's shareholders and hence to increase a firm's value. This contributes to further determine the capital structure of individual firms. As a consequence, the Modigliani-Miller's irrelevance region not only of aggregate but also of individual firms' financial decisions is considerably reduced in the presence of moral hazard. We illustrate these issues by means of the following example.

An example. Consider the same specification of the economy considered in the example of Section 2.1.2. As before, at date 0 the state is s_2, $w_0^i = w^i(s_1)$ for all i, and $\pi(s_1) = \pi(s_2) = \pi(s_3) = \frac{1}{3}$. There is a utility benefit for the agent who becomes manager and implements $\phi = 1$: $v^i(1) = -.006$, for all i; on the other hand, $v^i(0) = 0$ for all i.

In Table 2 we report the equilibrium values respectively for the case in which there is no moral hazard (the choice of ϕ is observable, hence only the manager's participation constraint must be satisfied) and the case in which there is moral hazard (the choice of ϕ is not observable, hence both the manager's incentive and participation constraints must be satisfied). Note that we allow firms to issue risky debt here and so to default on debt in some states.

In the case without moral hazard (the choice of ϕ is observable), the equilibrium values are the same as in the example of Section 2.1.2, where there was no utility benefit associated with $\phi = 1$ and no possibility for firms to default on their debt obligations [45]. Each firm is

in equilibrium default occurs in some states, the firms' aggregate capital structure is fully determinate, while individual capital structure is only partially determinate (the optimum is given by an interval of values of B). Here we omit the formal statement of the firms' optimality conditions with risky debt and leave it to Appendix C available at http://www.eui.eu/Personal/Gottardi/EqmMakowski_October2011_AppendixC.pdf

[45] In particular, $\bar{\phi} = 0$, $\bar{k} = .4888$, the capital structure of all firms in the economy is $\bar{B} \in [.1828, .6431]$ and the capital structure of each individual firm is indeterminate. In other words, firms are allowed to issue risky debt but find it optimal to issue risk free bonds.

	No Moral Hazard	Moral Hazard
$\bar{\phi}$	0	0
$\bar{\imath}$	1 or 2	2
$\bar{k}$	.4888	.4896
$\bar{B}$	[.1828,.6431]	.2160
$\bar{x}_0$	-.2576 or -.2313	-.2326
$\bar{\theta}^1$	.3877	.3544
$\bar{b}^1$	[.1828,.3613]	.2160
$\bar{q}$	[.5108,.1559]	.4870
$\bar{p}$	.7712	.7689
$-\bar{k}+\bar{q}+\bar{p}\bar{B}-\bar{W}$	.1629	.1633
$\bar{U}^1$	-1.0372	-1.0371
$\bar{U}^2$	-1.0217	-1.0219

Table 2: Equilibrium values with and without moral hazard.

indifferent between hiring an agent of type 1 and an agent of type 2 ($\bar{x}_0$ changes depending on who is hired).

However, under moral hazard (when the choice of ϕ is not observable), this equilibrium allocation is not incentive compatible: both agents, if hired as managers and given the same compensation package as the first column of Table 2, would choose $\phi = 1$ (when k and B are also at the equilibrium level found above) and trade in the asset markets. To address this incentive problem it is not enough that the firm prevents the manager from trading in the markets, but it must also appropriately modify its financial and production decisions in conjunction with the manager's portfolio. In this specific case, the firm still implements $\bar{\phi} = 0$ and does that by hiring an agent of type 2 as manager (the cost of providing incentives to type 2 is strictly lower than for type 1), and by increasing its investment level to $k = .4896$ while setting its debt level at $B = .2160$. The manager's compensation is then also different and exhibits a higher amount of equity $(1 - .3544)$ and a lower one of debt (0), together with a lower payment at date 0[46]. It is interesting to note that, even if the debt issued remains risk free, the level of debt of the firm is now uniquely determined (at $B = .2160$): the capital

[46]Note that in this situation the manager, if left free to trade, would buy less shares of the firm (0.5777) and some bonds (0.0408).

structure of the firm is uniquely determined by the manager's incentive problem.

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Appendix A

We collect here some proofs and one result.

A.1 Proof of Proposition 1

We only provide here an outline of the main steps. Since short sales are not allowed, the consumers' budget set is non empty, compact and convex for all[47] $(\phi, k) \in \Phi \times K$, all $B \geq 0$ satisfying (2), and all $(p, q) \gg 0$. Under the assumptions made on individual preferences, consumers' net demand functions (for bonds, equity and the consumption good) are then well behaved. Using condition Cii), the pricing map $q(\phi, k, B)$ in the firm's problem (1) can be written as a function of the agents' consumption $(c_0^i, (c^i(s)_{s\in\mathcal{S}})$. The convex hull of the correspondence describing the firms' net supply of bonds and of the consumption good as well as their choice of the other technology parameter ϕ, is then also well behaved, for all $p \geq 0$ and $c_0^i \in (0, \max\{\sum_i w_0^i\}]$, $c^i(s) \in (0, \max \sum_i w^i(s)]$ $\forall s \in \mathcal{S}$. By a standard fixed point argument there exists so a value of $\bar{\phi}, \bar{k}, \bar{B}, \bar{p}, \bar{q}, (\bar{c}_0^i, (\bar{c}^i(s)_{s\in\mathcal{S}})_{i=1}^I$ such that: (a) $\bar{q}$ equals the value of the price map specified in condition Cii) evaluated at $\bar{\phi}, \bar{k}, \bar{p}$ and $(\bar{c}_0^i, (\bar{c}^i(s)_{s\in\mathcal{S}})_{i=1}^I$, (b) $\bar{\phi}, \bar{k}, \bar{B}$ belong to the convex hull of the firms' optimal choice correspondence when $p = \bar{p}$ and the terms $\overline{MRS}^i$ appearing in the equity price map specified in condition Cii) are evaluated at $(\bar{c}_0^i, (\bar{c}^i(s)_{s\in\mathcal{S}})_{i=1}^I$, (c) for each $i = 1, .., I$, $(\bar{c}_0^i, (\bar{c}^i(s)_{s\in\mathcal{S}})$ is a solution of the choice problem of type i consumers at $\bar{q}, \bar{p}$, (d) the market clearing conditions hold. Finally, by Caratheodory's Theorem, $\bar{\phi}, \bar{k}, \bar{B}$ can be written as a convex combination of finitely many points belonging to the firms' optimal choice correspondence.■

A.2 Proof of Proposition 3

Suppose $(\hat{c}^i)_{i=1}^I$ is admissible and Pareto dominates the competitive equilibrium allocation $(\bar{c}^i)_{i=1}^I$. By the definition of admissibility a collection $\hat{k}, \hat{\phi}, \hat{B}$ and $\left(\hat{\theta}^i, \hat{b}^i\right)_{i=1}^I$ exists such that (11) and (12) are satisfied. Since $\bar{c}^i$ is the optimal choice of a type i consumer at the equilibrium prices $\bar{q}, \bar{p}$ and, as argued in Remark 1, the consumer's choice problem is analogous to one where any possible type of equity is available for trade, at a price $q(k, \phi, B)$ satisfying the consistency condition Cii), we get

$$\hat{c}_0^i + \hat{q}\hat{\theta}^i \quad + \bar{p}\ \hat{b}^i - w_0^i \geq \bar{c}_0^i + \bar{q}\ \bar{\theta}^i + \bar{p}\ \bar{b}^i - w_0^i\ ,$$

[47]Strictly speaking, the nonemptiness of the budget set is ensured for all $k \in K$ provided the maximal element of $k \in K$, $k_{\max}$, is such that $w_0^i \geq \theta_0^i k_{\max}$ for all i.

where $\hat{q} = \max_i \mathbb{E}\overline{MRS}^i(s) \left[f(\hat{k}, \hat{\phi}; s) - \hat{B}\right]$. Or, equivalently,

$$\left[-\hat{k} \ + \hat{q} \ + \bar{p}\ \hat{B}\ \right] \theta_0^i + \tau^i \geq \left[-\bar{k} \ + \bar{q} \ + \bar{p}\ \bar{B}\ \right] \theta_0^i, \tag{33}$$

for $\tau^i \equiv \hat{c}_0^i + \hat{q}\hat{\theta}^i \quad + p\ \hat{b}^i - \left[-\hat{k} \ + \hat{q} \ + \bar{p}\ \hat{B}\ \right] \theta_0^i - w_0^i$. Since (33) holds for all i, strictly for some i, summing over i yields:

$$\left[-\hat{k} \ + \hat{q} \ + \bar{p}\ \hat{B}\ \right] + \sum_i \tau^i > \left[-\bar{k} \ + \bar{q} \ + \bar{p}\ \bar{B}\ \right] \tag{34}$$

The fact that $\bar{k}, \bar{\phi}, \bar{B}$ solves the firms' optimization problem (1) in turn implies that:

$$-\bar{k} \ + \bar{q} \ + \bar{p}\ \bar{B} \geq -\hat{k} \ + \hat{q} \ + \bar{p}\ \hat{B},$$

which, together with (34), yields:

$$\sum_i \tau^i > 0,$$

or equivalently:

$$\sum_i \hat{c}_0^i + \hat{k} > \sum_i w_0^i,$$

a contradiction to (11) at date 0. ■

A.3 Characterization of the firms' optimality conditions

Proposition 9 *The optimal production and financing decisions of a firm are obtained:*[48]
(i) either at an interior solution, $f(k, \phi; \underline{s}) > B$, where all equity holders are also bond holders (while the reverse may not be true: $I^e \subseteq I^d$):

$$\max_{i \in I^e} \mathbb{E}\overline{MRS}^i(s) = \min_{i \in I^e} \mathbb{E}\overline{MRS}^i(s) = p = \max_i \mathbb{E}\overline{MRS}^i(s) \tag{35}$$

and

$$\max_{i \in I^e} \mathbb{E}\left[\overline{MRS}^i(s) f_k(s)\right] = \min_{i \in I^e} \mathbb{E}\left[\overline{MRS}^i(s) f_k(s)\right] = 1; \tag{36}$$

(ii) or at a corner solution, $f(k, \phi; \underline{s}) = B$, where all equity holders have again the same marginal valuation for the bond, but such valuation may now be strictly less than its price p (hence no equity holder is a bond holder):

$$p \geq \max_{i \in I^e} \mathbb{E}\overline{MRS}^i(s) = \min_{i \in I^e} \mathbb{E}\overline{MRS}^i(s)\ , \tag{37}$$

[48] We focus here on the conditions concerning the investment level k and capital structure B, ignoring those regarding ϕ, which are straightforward. The proof of Proposition 9 can be found in Appendix C, available online at http://www.eui.eu/Personal/Gottardi/EqmMakowski_October2011_AppendixC.pdf

$$1 \geq \max_{i \in I^e} \mathbb{E}\left[\overline{MRS}^i(s)\ f_k(s)\right] = \min_{i \in I^e}\left[\mathbb{E}\overline{MRS}^i(s)\ f_k(s)\right], \tag{38}$$

and

$$f_k(\underline{s})\left(p - \max_{i \in I^e} \mathbb{E}\overline{MRS}^i(s)\right) = 1 - \max_{i \in I^e} \mathbb{E}\left[\overline{MRS}^i(s)\ f_k(s)\right]. \tag{39}$$

Thus in both cases all shareholders value equally the effect on the payoff of equity of an infinitesimal increase in the investment level k. In addition, at an interior solution such value is always equal to the marginal cost of the investment. In contrast, at a corner solution we have to take into account the possibility of joint deviations in k, B and ϕ and this value may be strictly smaller. This happens whenever all equity holders value the bond less than p (that is, no equity holder is a bond holder), in which case the "gap" in the two expressions is exactly equal.

Appendix B

B.1 A model of short sales with consumers' default

We extend here the analysis of Section 3 by examining the case where the consumers' default rate, rather than being exogenous and state and type invariant, is optimally chosen by consumers, and may depend therefore on the state s as well as the type i of the consumer. We show in what follows the required changes in the model. The specification of the intermediation activity and the structure of markets is clearly more complicated, still the main results on unanimity and optimality remain valid.

Since consumers' loans are non-collateralized, we follow Dubey et al. (2005) in introducing a utility penalty ξ^i for a type i consumer per unit defaulted in any state s, for all i, s. It is convenient to assume here that preferences are additively separable over time, so that they take the following form:

$$u_0^i(c_0^i) + \mathbb{E}\left[u_1^i\left(c^i(s)\right) - \xi^i \delta_s^i\left[\lambda_-^i\left(f(k,\phi;s) - B\right)\right]\right] \tag{40}$$

where δ_s^i is the default rate of consumer i in state s. Given this feature of consumers' preferences, the optimal default level in each state s for consumer i is obtained by maximizing (40) with respect to $(\delta_s^i)_s$ subject to the date 1 budget constraint (16), where δ is replaced by δ_s^i. It is immediate to see that the solution is a well defined map $\delta_s^i(\theta^i, \lambda_+^i, b^i, \lambda_-^i)$ for all s and $\theta^i, \lambda_+^i, b^i, \lambda_-^i$, and for any given k, ϕ, B.

Thus the default rate in any state s on the loans granted to consumers via the sale of short positions depends not only on the type i of the consumer but also on his overall portfolio holdings. We consider then the case where both the consumer's type and his portfolio holdings are observable by his trading partners. The loan contract offered by intermediaries is so an exclusive contract and the price depends both on the consumer's type and portfolio, $q^-_{i,\theta^i,\lambda^i_+,b^i,\lambda^i_-}$ as well as, obviously, on the return structure of the underlying equity. Hence the budget constraint faced by consumers at date 0 is now

$$c_0^i \ = w_0^i \ + [-k \ + q \ + p \ B \ ] \, \theta_0^i - q \ \theta^i \ - p \ b^i - q^+ \lambda^i_+ + q^-_{i,\theta^i,\lambda^i_+,b^i,\lambda^i_-} \lambda^i_- \tag{41}$$

An intermediary who is intermediating m units of the derivative by selling the short positions to consumers of type i, with portfolio $(\theta^i, \lambda^i_+, b^i, \lambda^i_-)$, faces a default rate on its loans equal to $\delta^i_s(\theta^i, \lambda^i_+, b^i, \lambda^i_-)$. As a consequence, the shortfall in its revenue at date 1 is:

$$\left[(f(k, \phi; s) - B) \ \ \delta^i_s(\theta^i, \lambda^i_+, b^i, \lambda^i_-)\right] m. \tag{42}$$

We consider still the case where only equity, an asset that is 'safe' as it is in positive net supply and backed by real claims, is used to hedge the consumers' default risk. To be able to fully meet the shortfall in (42) due to consumers' default, the intermediary must hold at least

$$\max_s \delta^i_s(\theta^i, \lambda^i_+, b^i, \lambda^i_-) m$$

units of equity. The total date 0 revenue of the intermediary is then:

$$\max_m \left[q^+ - q^-_{i,\theta^i,\lambda^i_+,b^i,\lambda^i_-} - q \left(\max_s \delta^i_s(\theta^i, \lambda^i_+, b^i, \lambda^i_-) \right) \right] m \tag{43}$$

The intermediary's choice problem consists in the choice of the amount m to issue of each type $i, \theta^i, \lambda^i_+, b^i, \lambda^i_-$ of derivative so as to maximize its profits, that is its revenue at date 0. Note that the intermediation technology still exhibits constant returns to scale, hence a solution exists provided

$$q \geq \frac{q^+ - q^-_{i,\theta^i,\lambda^i_+,b^i,\lambda^i_-}}{\max_s \delta^i_s(\theta^i, \lambda^i_+, b^i, \lambda^i_-)};$$

and is characterized by $m(i, \theta^i, \lambda^i_+, b^i, \lambda^i_-) > 0$ only if $q = \frac{q^+ - q^-_{i,\theta^i,\lambda^i_+,b^i,\lambda^i_-}}{\max_s \delta^i_s(\theta^i, \lambda^i_+, b^i, \lambda^i_-)}$.

The main difference with respect to the reduced form model is then the fact that the market for derivative claims is differentiated according to consumers' types and portfolio choices. This has the following implications for the consumers' optimization problem and the market clearing conditions.

Consumer i chooses his portfolio $\theta^i, \lambda^i_+, b^i, \lambda^i_-$ so as to maximize

$$u^i_0(c^i_0) + \mathbb{E}\left\{ \begin{array}{c} u^i_1\left[w^i(s) + b^i + (f(k,\phi;s) - B)(\theta^i + \lambda^i_+ - \lambda^i_-(1 - \delta^i_s(\theta^i, \lambda^i_+, b^i, \lambda^i_-))\right] \\ -\xi^i\delta^i_s(\theta^i, \lambda^i_+, b^i, \lambda^i_-) \end{array} \right\}$$

subject to the budget constraint (41), given the asset prices q, q^+, p and $q^-_{i,\cdot}$ and the default map $\delta^i_s(\cdot)$ obtained as above. Let $\bar\theta^i, \bar\lambda^i_+, \bar b^i, \bar\lambda^i_-$ denote the consumer's optimal choice in equilibrium. The asset market clearing conditions are then

$$\sum_i m(i, \bar\theta^i, \bar\lambda^i_+, \bar b^i, \bar\lambda^i_-)\left[\max_s \delta^i_s(\bar\theta^i, \bar\lambda^i_+, \bar b^i, \bar\lambda^i_-)\right] + \sum_{i\in I}\bar\theta^i = 1$$

for equity, and

$$\begin{array}{rcl} \bar\lambda^i_- & = & m(i, \bar\theta^i, \bar\lambda^i_+, \bar b^i, \bar\lambda^i_-) \text{ for each } i \\ 0 & = & m(i, \theta^i, \lambda^i_+, b^i, \lambda^i_-) \text{ for each } i,\ (\theta^i, \lambda^i_+, b^i, \lambda^i_-) \neq (\bar\theta^i, \bar\lambda^i_+, \bar b^i, \bar\lambda^i_-) \\ \sum_i m(i, \bar\theta^i, \bar\lambda^i_+, \bar b^i, \bar\lambda^i_-) & = & \sum_i \bar\lambda^i_+ \end{array}$$

for the derivative security.

The consistency condition M^{SS})) on the firms' equity conjectures must also be properly modified to reflect the different specification of the value of intermediation in the present context:

$$M\text{''})\ \ q(k,\phi,B) = \max\left\{ \begin{array}{c} \max_i \mathbb{E}\left[\overline{MRS}^i(s)\,(f(k,\phi;s) - B)\right], \\ \max_{i,\theta^i,\lambda^i_+,b^i,\lambda^i_-} \frac{\max_i \mathbb{E}\left[\overline{MRS}^i(s)(f(k,\phi;s)-B)\right] - q^-(i,\theta^i,\lambda^i_+,b^i,\lambda^i_-;k,\phi,\bar U^i)}{\max_s \delta^i_s(\theta^i,\lambda^i_+,b^i,\lambda^i_-;k,\phi)} \end{array} \right\}, \forall k,\phi,B$$

where $q^-(i,\theta^i,\lambda^i_+,b^i,\lambda^i_-;k,\phi,B,\bar U^i)$ is constructed as follows. For any k,ϕ,B and $i,\theta^i,\lambda^i_+,b^i,\lambda^i_-$, set $q^-(i,\theta^i,\lambda^i_+,b^i,\lambda^i_-;k,\phi,B,\bar U^i)$ as the value of q^- that satisfies the following equation:

$$\begin{array}{c} \bar U^i \ = \ u^i_0(w^i_0 \ + \left[-\bar k \ + \bar q \ + \bar p \ \bar B\ \right]\theta^i_0 - \bar q\ \theta^i \ - \bar p\ b^i - \bar q^+\lambda^i_+ + q^-\lambda^i_-) + \\ \mathbb{E}\left\{ \begin{array}{c} u^i_1\left[w^i(s) + b^i + (f(\bar k,\bar\phi;s) - \bar B)(\theta^i + \lambda^i_+) - \lambda^i_-\left[f(k,\phi;s) - B\right]\left(1 - \delta^i_s(\theta^i,\lambda^i_+,b^i,\lambda^i_-;k,\phi,B)\right)\right] \\ -\xi^i\delta^i_s(\theta^i,\lambda^i_+,b^i,\lambda^i_-;k,\phi,B)\left[\lambda^i_-\left(f(k,\phi;s) - B\right)\right] \end{array} \right\} = \bar U^i \end{array}$$

where $\bar U^i$ denotes the utility level of type i consumers at the equilibrium choices $\bar\theta^i, \bar\lambda^i_+, \bar b^i, \bar\lambda^i_-$ and the map $\delta^i_s(\theta^i,\lambda^i_+,b^i,\lambda^i_-;k,\phi,B)$ is similarly obtained by maximizing the expected utility term on the right hand side of the above expression with respect to δ^i_s. That is,

$q^{-}(i, \theta^i, \lambda^i_+, b^i, \lambda^i_-; k, \phi, B, \bar{U}^i)$ identifies the maximal willingness to pay in equilibrium of consumer i for a short position equal to λ^i_- in the firm with plan k, ϕ, B when the rest of his portfolio is given by $\theta^i, \lambda^i_+, b^i$.[49] At these prices consumers are indifferent between choosing the equilibrium portfolio $\bar{\theta}^i, \bar{\lambda}^i_+, \bar{b}^i, \bar{\lambda}^i_-$ and any other portfolio with a short position λ^i_- in the equity of a firm with plan k, ϕ, B.

An important difference with respect to the previous analysis is the fact that here the price of short positions is no longer defined at the margin. This is due to the exclusive nature of loan contracts corresponding to short positions. Also, at the same prices intermediaries are indifferent between issuing the derivatives traded in equilibrium and any other derivative on equity of firms with plan k, ϕ, B such that $q = \frac{\max_i \mathbb{E}\left[\overline{MRS}^i(s)(f(k,\phi;s)-B)\right] - q^{-}(i,\theta^i,\lambda^i_+,b^i,\lambda^i_-;k,\phi,B,\bar{U}^i)}{\max_s \delta^i_s(\theta^i,\lambda^i_+,b^i,\lambda^i_-;k,\phi,B)}$.

The unanimity and constrained optimality properties still hold in this environment. The argument again is very similar and relies on the the fact that, given the above specification of the intermediation technology and the price conjectures, the model is equivalent to a setup where the markets for all types of equity and all types of corresponding derivatives are available for trade. The notion of completeness here also requires the exclusivity of the loan contracts associated to short positions, so that the market for all types of derivatives can also be differentiated according to the type of a consumer and the level of his trades.

[49] In the specification of $q^{-}(i, \theta^i, \lambda^i_+, b^i, \lambda^i_-; k, \phi, B, \bar{U}^i)$ we have implicitly assumed that all the long positions of a consumer are in the assets corresponding to the firms' equilibrium choices. This is with no loss of generality and to avoid excessive notational complexities.