Two Illustrations of the Quantity Theory of Money

By ROBERT E. LUCAS, JR.*

This paper presents empirical illustrations of two central implications of the quantity theory of money: that a given change in the rate of change in the quantity of money induces (i) an equal change in the rate of price inflation; and (ii) an equal change in nominal rates of interest. The illustrations were obtained by comparing moving averages of the three variables in question, using quarterly *U.S.* time-series for the period 1953–77. Readers may find the results of interest as additional confirmation of the quantity theory, as an example of one way in which the quantity-theoretic relationships can be uncovered via atheoretical methods from time-series which are subject to a variety of other forces, or as a measure of the extent to which the inflation and interest rate experience of the postwar period can be understood in terms of purely classical, monetary forces.

The theoretical background of the study is reviewed, very briefly as it is familiar material, in the next section. The data processing methods are described and rationalized in Section II. The illustrations resulting from the application of these methods are in Section III. Section IV contains some decompositions of postwar time-series and concluding comments.

I. Theoretical and Empirical Background

The two quantity-theoretic propositions stated in the introduction possess a combination of theoretical coherence and empirical verification shared by no other propositions in monetary economics. By "theoretical coherence," I mean that each of these laws appears as a characteristic of solutions to explicit theoretical models of idealized economies, models which give some guidance as to why one might expect them to obtain in reality, also as to conditions under which one might expect them to break down. For present purposes, Miguel Sidrauski's monetary version of the Solow-Swan one-sector model of economic growth (1967a, b) is perhaps the most useful, single theoretical illustration. In that model, both laws appear as explicit, necessary characteristics of the stationary solution of the differential equations which describe equilibrium in the system. To restate this in a way which is more suggestive empirically, they appear as characteristics of long-run average behavior in the model economy.

Both of these laws are, as is clear from the Sidrauski example, propositions about the consequences of a unit's change. Thus neither appears to depend crucially on particular features of the preferences and technology postulated by Sidrauski. It is not difficult to construct other examples to illustrate the insensitivity of these laws to variations in the structure of the economy. In particular, if stochastic elements are introduced, the laws are reinterpreted to apply to means of theoretical stationary distributions or, as before, to long-run average behavior.[1]

Sidrauski's example, together with variations appearing in the literature both before and since he wrote, also suggests some qualifications or limitations to these laws.

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[1]This interpretation of the quantity theory of money as a set of predictions about the long-run average behavior of a general equilibrium system is different from, though not inconsistent with, Milton Friedman. There, Friedman stresses the stability of the market demand function for money, a property which is neither necessary nor sufficient for the quantity theory to obtain in the sense used here.

First, Sidrauski's version of the neoclassical model does not exhibit the Mundell-Tobin effect of a monetary expansion: the possibility that an inflation, by reducing the real yield on money, will shift saving to real capital accumulation. If this effect is important, it would force us to modify the second law to predict interest rate increases by *less* than the increase in the monetary growth rate (due to the decline in the real return on capital, offsetting the inflation premium).[2] Theoretically, I think it is clear from related work (see, for example, David Levhari and Don Patinkin, Stanley Fischer, and Ronald Michener) that only a very coincidental combination of assumptions produces an absence of a Mundell-Tobin effect in Sidrauski's example and that, in general, one does not want to view this effect as ruled out on prior, logical grounds. This conclusion, of course, leaves us free to hope that the required modifications are minor enough to be neglected in some applications.

Second, and perhaps more fundamental, theory at this level gives no guidance as to the measurement of the quantity of money, or as to which (if any) of the available time-series on monetary aggregates corresponds to the variable theoretically termed "money." (Of course, it also gives no guidance as to the empirical definition of "the price level," but there is a good deal of other economic theory which does.) As recent theoretical work of John Bryant and Neil Wallace and Marco Martins has emphasized, this question of which monetary aggregate one would theoretically *expect* to move in proportion to prices is much more open than has traditionally been recognized. In the experiments reported below, money means $M1$, but the arbitrariness of this measurement choice should be emphasized at the outset, particularly as it is likely that very similar results would have been obtained under a variety of other choices.

In summary, then, we have specific theoretical examples exhibiting both quantity-

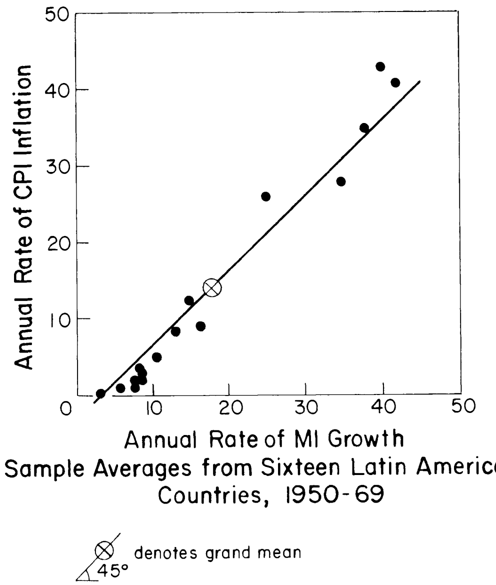


FIGURE 1

theoretic laws in clear, exact form, and others which suggest possibly important qualifications. This is all we can ever hope for from our theory: some strong clues as to what to look for in the data; some warnings as to potential sources of error in these predictions. This is the theoretical coherence of the neoclassical laws.

Since the two quantity-theoretic laws are obtained as characteristics of steady states, or limiting distributions, of theoretical models, the ideal experiment for testing them would be a comparison of long-term average behavior across economies with different monetary policies but similar in other respects. Many such tests of the first law are available;[3] a particularly clean example is shown in Figure 1. These data are taken from Robert Vogel's study of inflation in sixteen Latin American economies, using annual data for the period 1950–69.[4] Vogel does not report the interest rate data which would have permitted a comparable test of the Fisherian interest-inflation relationship. In general, such evidence is difficult to ob-

[2]Since interest payments are taxable, the maintenance of a given real yield on bonds would require interest rates to rise by more than the inflation rate. This effect will offset, and perhaps even reverse, the Mundell-Tobin effect.

[3]See in particular Anna Schwartz.

[4]The countries included in Vogel's study are Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Ecuador, El Salvador, Guatemala, Honduras, Mexico, Nicaragua, Paraguay, Peru, Uruguay, and Venezuela.

tain, no doubt due to the fact that in inflationary economies published interest rates are rarely left free to reach their equilibrium levels.

The line in Figure 1 is drawn through the grand mean of the $16 \times 20 = 320$ annual money growth rate inflation pairs in Vogel's sample. This is the one "free parameter" permitted by the theory. Its slope is 45°, as specified theoretically: it is not fit to the data. It is hard to imagine a nonvacuous economic prediction obtaining stronger confirmation than that shown in Figure 1. This is the kind of "empirical verification" of the quantity theory on which economists who assign it a central theoretical role base most of their confidence.

In the absence of the kind of decisive natural experiment used by Vogel, one could in principle test the neoclassical laws by deriving their implications for the parameters of a structural econometric model. This course, while attractive in theory (since it broadens considerably the class of data which might shed light on the laws), is in practice a difficult one, since it involves nesting the two hypotheses in question within a complex maintained hypothesis, which must be accepted as valid in order to carry out the test. The virtue of relatively atheoretical tests, such as carried out by Vogel, is that they correspond to our theoretically based intuition that the quantity theoretic laws are consistent with a wide variety of possible structures. If so, it would be desirable to test them independently and then, if confirmed, to *impose* them in constructing particular structural models, rather than to proceed in the reverse direction. It would be of value, then, to have measurement techniques which are atheoretical in the sense of Vogel's but which can be applied to continuous time-series for a single economy. The use of one such technique is illustrated below.

II. Data and Data Processing Methods

The time-series used in this study are the money supply ($M1_t$), the consumer price level (P_t) and the ninety-day Treasury bill rate (r_t). The value of $M1_t$ for quarter t is demand deposits plus currency outside banks, for the first month of the quarter, seasonally adjusted, taken from successive issues of the *Federal Reserve Bulletin*. The *CPI* is similarly timed, not seasonally adjusted, from the Consumer Price Index. The bill rate is that used and described by Eugene Fama.

I shall work with the following transformed variables:

$$X_{0t} = ln(M1_{t+1}) - ln(M1_t)$$

$$X_{1t} = ln(P_{t+1}) - ln(P_t)$$

$$X_{2t} = r_t$$

Scatter diagrams of X_{1t} and X_{2t} against X_{0t} are given in Figures 2 and 3, in the next section. These figures seem to capture fairly well what people mean when they say that the quantity theory of money is not a "short-run" relationship.

The general idea of what follows will be to examine scatter diagrams of $X_{it}(\beta)$, $i=1$, 2, against $X_{0t}(\beta)$ where for $i=0$, 1, 2, $X_{it}(\beta)$ is the two-sided exponentially weighted moving average given by[5]

$$X_{it}(\beta) = \alpha \sum_{k=-\infty}^{\infty} \beta^{|k|} X_{i,t+k} \tag{1}$$

where $\alpha = \dfrac{1-\beta}{1+\beta}$, $0 \leqslant \beta < 1$

The effect of the filter (1) is to smooth the original series; indeed, as β approaches unity, the filtered observations $X_{it}(\beta)$ ap-

[5]Here and below I write as though the entire doubly infinite record were available for each variable. In the calculations, the algorithm described by Thomas Cooley, Barr Rosenberg, and Kent Wall was used. This algorithm permits the assignment of a diffuse prior on X_{it} values outside the sample period which appear in the doubly infinite sum (1). With beliefs about points prior to 1953 and after 1977 so described, it calculates posterior means of the slowly moving "signal," called s_t below. Except for points near the beginning and the end of the sample period, virtually identical results were obtained simply by replacing missing observations in (1) by zeros. So as not to present results which are unduly dependent on the way out-of-sample X_{it} values are treated, numbers for 1953-54 and 1976-77 are not plotted.

proach the sample average values of the original series. In the latter case, plots of $X_{1t}(\beta)$ and $X_{2t}(\beta)$ against $X_{0t}(\beta)$ will degenerate to a point, vacuously lying on a line with slope 45°. Our interest will be in seeing whether the points $(X_{0t}(\beta), X_{it}(\beta))$, $i=1, 2$, fall on a 45° line for β-values less than unity, providing a time-series confirmation of the cross-country results obtained by Vogel and others. Viewed as a measurement procedure, the test of this method will be the quality of the pictures it yields. It may be useful first, however, to look in more detail into what the filter (1) does to a time-series, and what statistical and economic rationales may underlie its use.

The Fourier transform of the filter given in (1), with α free, is for $0 \leqslant \omega \leqslant \pi$,

$$(2) \quad f(\omega; \alpha, \beta) = \alpha \sum_{k=-\infty}^{\infty} \beta^{|k|} e^{-i\omega k}$$

$$= \frac{\alpha(1-\beta^2)}{1+\beta^2-2\beta cos(\omega)}$$

One verifies that $f(0)=1$ if $\alpha=(1-\beta)/(1+\beta)$, that $f(\pi)>0$ and that $f'(\omega)<0$ for all $0 \leqslant \omega \leqslant \pi$. Also $f''(0)<0$ and $f''(\pi)>0$; $f''(\omega)$ changes sign once, at the unique ω value at which $X=cos(\omega)$ is a positive root of

$$X^2 + \frac{1+\beta^2}{2\beta} X - 2 = 0$$

For high β-values (for example, near 0.9) this root occurs very near $\omega=0$. Since the spectral density of the filtered series $X_{it}(\beta)$ is just the spectral density of X_{it} multiplied by $f(\omega;\ \alpha_i\beta)$, one sees that the filter (1) retains power at very low frequencies, while sharply reducing power at higher frequencies.

Filters of the form (1) are solutions to a well-known signal-extraction problem, the form of which may also be instructive.[6] Let $\{v_t, w_t\}$ be a white noise process with mean (0,0) and covariance matrix $\sigma^2 \begin{bmatrix} \theta & 0 \\ 0 & 1 \end{bmatrix}$.

Define the processes u_t and s_t by

$$u_t = s_t + v_t$$

$$s_t = \rho s_{t-1} + w_t$$

$$0 < \rho < 1$$

Imagine that this structure, including the values of the parameters θ, σ^2, and ρ, is known and that one has observations on the u_t, $t=-\infty,\ldots,\infty$. It is desired to obtain minimum variance unbiased estimators $\hat{s}_t$ of the sequence of signals s_t. Projecting s_t on u_t, $t=-\infty,\ldots,\infty$

$$s_t = \hat{s}_t + \eta_t$$

$$= \sum_{k=-\infty}^{\infty} \gamma_k u_{t+k} + \eta_t$$

where $E(u_t \eta_s)=0$, all s, t. The coefficients γ_k must satisfy the normal equations:

$$E(u_{t+j} s_t) = \sum_{k=-\infty}^{\infty} \gamma_k E(u_{t+j} u_{t+k})$$

$$j = -\infty, \ldots, \infty$$

Taking the Fourier transform of both sides:[7]

$$f_{us}(\omega) = f_{uu}(\omega) f_\gamma(\omega),\ 0 \leqslant \omega \leqslant \pi$$

or, exploiting the particular structure of the process assumed here,

$$f_{ss}(\omega) = [f_{ss}(\omega) + f_{vv}(\omega)] f_\gamma(\omega)$$

Solving for $f_\gamma(\omega)$ gives

$$(3) \quad f_\gamma(\omega) = \frac{1}{1+\theta[1+\rho^2-2\rho cos(\omega)]}$$

since $f_{vv}(\omega)=\theta\sigma^2$ and $f_{ss}(\omega)=[1+\rho^2-2\rho\ cos(\omega)]^{-1}\sigma^2$.

[6]See Peter Whittle (ch. 5) for a discussion of this and other examples.

[7]For two time-series $\{x_t\}$ and $\{y_t\}$, the notation $f_{xy}(\omega)$ means

$$f_{xy}(\omega) = \sum_{k=-\infty}^{\infty} e^{-i\omega k} Cov(x_{t+k}, y_t)$$

For the functions $f(\omega;\ \alpha,\beta)$ in (2) and $f_Y(\omega)$ in (3) to be the transforms of the same filter, it is necessary that the right-hand sides of these equations be identically equal in ω on $[0,\pi]$. This requires that β in (2) be that root of

$$0=1-\beta\left[\frac{1+\theta(1+\rho^2)}{\theta\rho}\right]+\beta^2$$

which lies in (0,1). Given β, α must satisfy

$$\alpha=\frac{\beta}{\theta\rho(1-\beta^2)}$$

The particular filter given in (1) is a one-parameter family in which α and β are constrained by $\alpha=(1-\beta)(1+\beta)^{-1}$. This case is seen to correspond to the limiting situation where $\rho=1$ and β solves[8]

$$0=1-\left[\frac{1+2\theta}{\theta}\right]\beta+\beta^2 \tag{4}$$

Hence if the variance of the "noise" v_t is small relative to the variance of w_t ($\theta\cong 0$), the root β of (4) in (0,1) will be near 0. This means that the current observation u_t is a good estimate of the true signal s_t. In our economic application, where s_t is taken to be that part of a time-series which is dominated by quantity-theoretic forces, this would correspond to a situation in which other "real" forces play a negligible role. At the other extreme, when the noise variance is high (θ large), β will be near one, and the best estimate of the true signal at t will be a very long moving average of the observed u_t.[9]

This purely statistical rationale for experimenting with the filter (1) has no basis in economic theory, and a little reflection suggests that none will be forthcoming: a *good* economic theory accounting for both quantity-theoretic and other forces on interest rate and price series would surely suggest the use of a "sharper" filter than (1). Nevertheless, the following scenario may be helpful. Imagine an economy in which the rate of monetary growth is a constant, known to agents, plus noise. The known, constant component is incorporated exactly into inflation and interest rates, with a negligible Mundell-Tobin effect. The monetary noise induces noise in interest and prices. In this example, the signal s_t represents the "constant" known, common component in monetary growth, price inflation, and interest rates. The noise v_t will be different for the different series.

Next, imagine that s_t, while constant for long stretches of time, infrequently changes to a new value from time to time. That is, model s_t by

$$s_t=\begin{cases} s_{t-1} & \text{with} \quad \textit{prob}\ \lambda \\ \tilde{s}_t & \text{with} \quad \textit{prob}\ 1-\lambda \end{cases}$$

where $\tilde{s}_t$ is serially independent with mean 0, and variance τ^2, and where $1-\lambda$ is "small." This process has the same covariance structure as the "signal" used in the statistical example above, with $\rho=\lambda$ and $\sigma^2=(1-\lambda)\tau^2$.

For an econometrician to treat this economy as posing a signal processing problem of the above type, one assumes that the "structural changes" in s_t are perfectly understood by agents as they occur, but cannot be observed by the econometrician. Hence the use of a *two-sided* moving average filter.[10]

The hope in applying this filter is not, of course, that an economic model of this type holds *exactly*. It is rather the general idea that the actual series may be generated by a *very* slowly changing structure of monetary policy, with business cycle activity occurring

[8]This is the quadratic John Muth arrived at, for the same reasons, in his study of the permanent income hypothesis.

[9]In the application below, the noise component is not serially uncorrelated as assumed in the example just discussed. For a more general discussion of the rationales for the use of a filter such as that described by (1), see Christopher Sims.

[10]This is not, of course, a compelling reason for using a two-sided filter. It is simply the condition under which it would be optimal to do so. In general, agents *know* only the past (arguing for a one-sided backward filter) but they *care* only about the future, and probably process much more information in forecasting that part of the future relevant to their own decisions than we econometricians can observe (arguing for a one-sided forward filter).

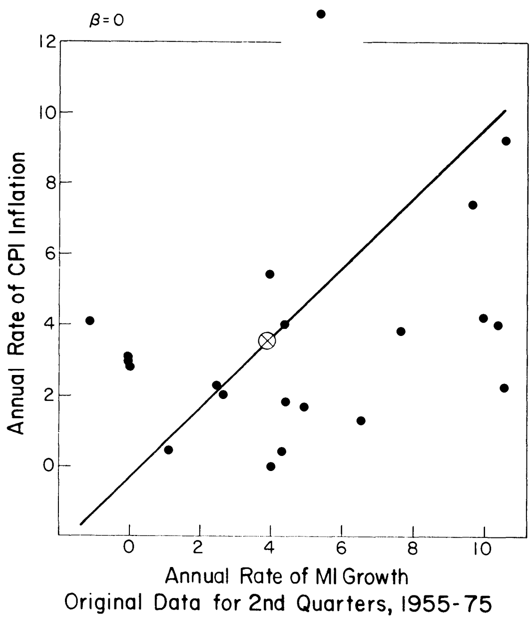


FIGURE 2

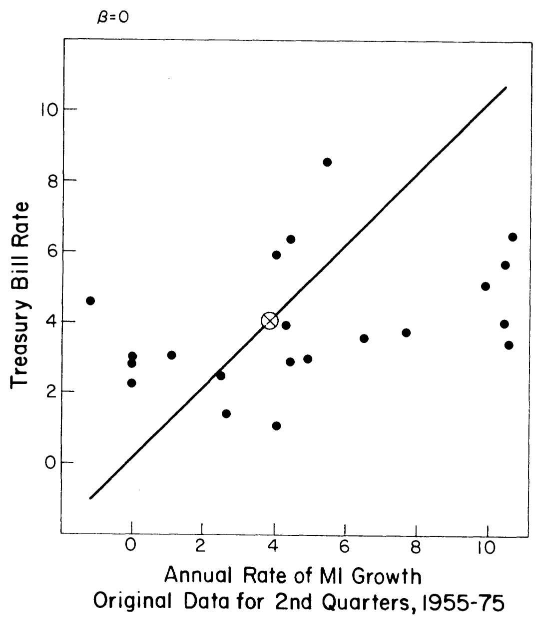


FIGURE 3

at higher frequencies superimposed. One can construct examples of economies fitting this description rather well, and one can construct other examples deviating very sharply from this description.

III. Illustrations

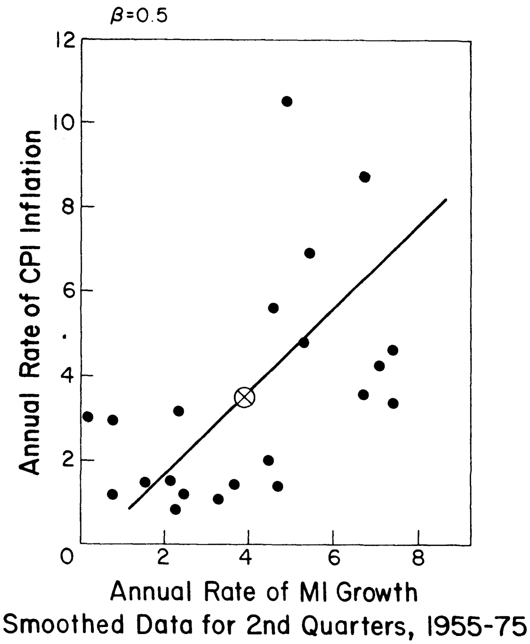
Figures 2 and 3 present scatter diagrams of inflation and interest rates, respectively, against rates of $M1$ growth. As remarked earlier, no relationship is evident.

Figures 4 and 5 are plots of moving averages with weights (on all series) equal to 0.5. That is, Figure 4 plots $X_{1t}(0.5)$ against $X_{0t}(0.5)$ and Figure 5 plots $X_{2t}(0.5)$ against $X_{0t}(0.5)$. Figures 6 and 7 utilize $\beta=0.8$; 8 and 9, $\beta=0.9$; and 10 and 11, $\beta=0.95$. All figures are drawn to the same scale. To avoid clutter, only points for the second quarter of each year are plotted. For high β-values, it is clear that this choice, while arbitrary, is of no consequence. Points for the first two years (1953–54) and last two (1967–77) are not plotted, though they were used in calculating the 1955–75 observations.

Given the preparatory discussion in Section II, little need be said about these figures. It is evident that a filter with $\beta=0.5$ does not quite extract the quantity-theoretic signal. A β-value of 0.9 reveals a clear 45° line, as predicted by the quantity theory and produces a picture about as clear as Vogel's cross-country estimates (Figure 1); $\beta=0.95$ is clearer still. If a Mundell-Tobin effect were present, and if it dominated tax effects, this would show up in the odd-numbered figures as a line with slope less than 45°. Perhaps this may be seen, for example, in Figure 9. Since deviations of the moving averages from the 45° line are sure to exhibit patterns, the temptation to read Figure 9 (or 11) this way should probably be resisted.

It should be added that subjecting any two series to moving-average filtering of the type used here will cause a "pattern" of some kind to emerge. To illustrate, Figure 13 plots a two-sided moving average of the unemployment rate,[11] with $\beta=0.9$, against

[11]Last month of quarter, not seasonally adjusted, from *Employment and Earnings*.



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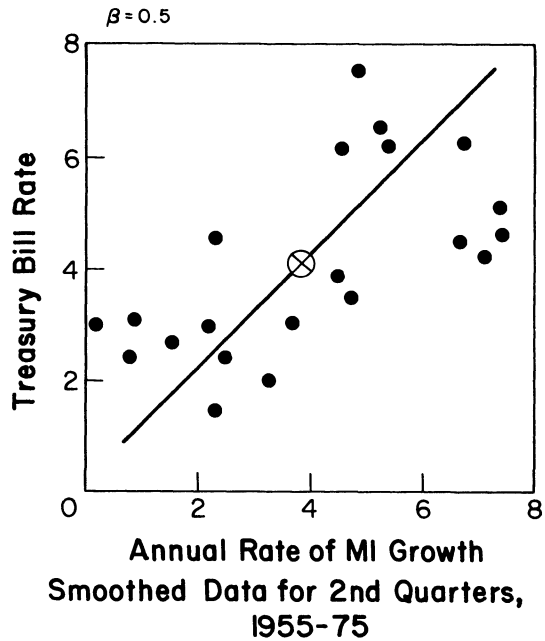

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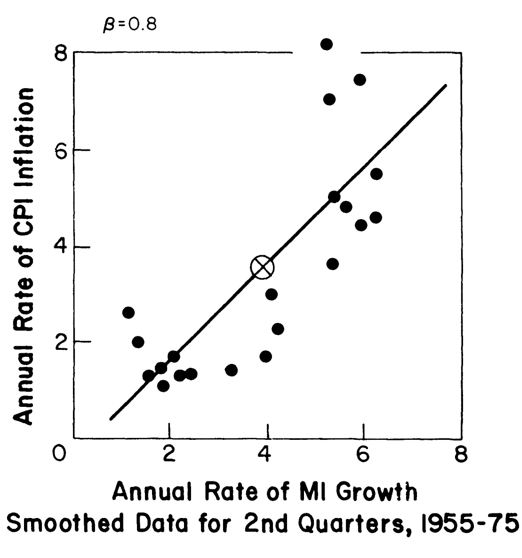

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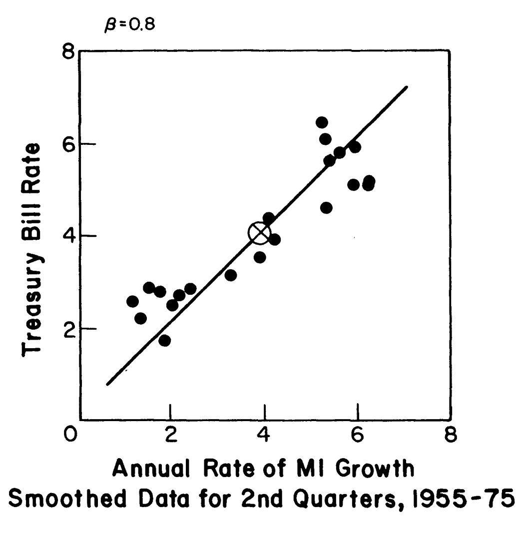

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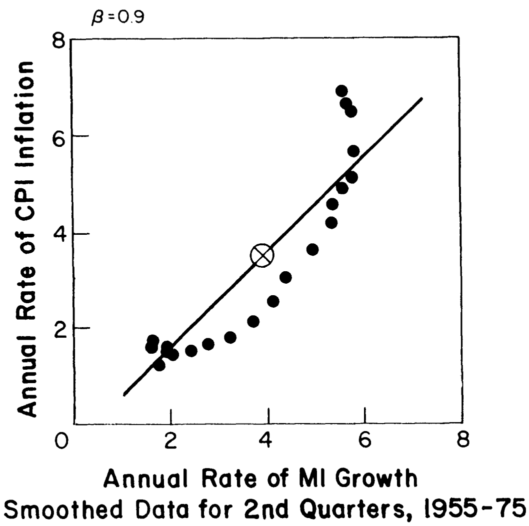

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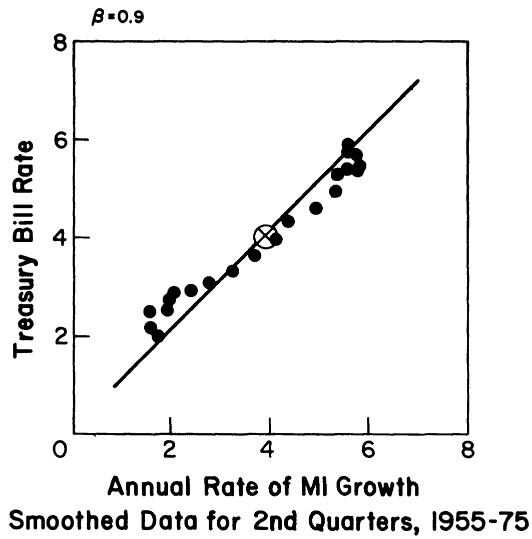

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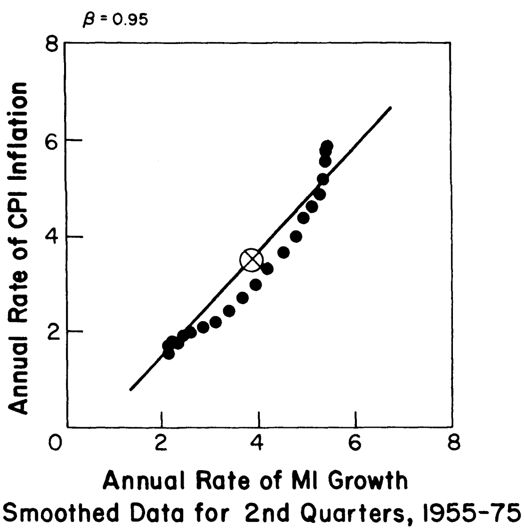

FIGURE 10

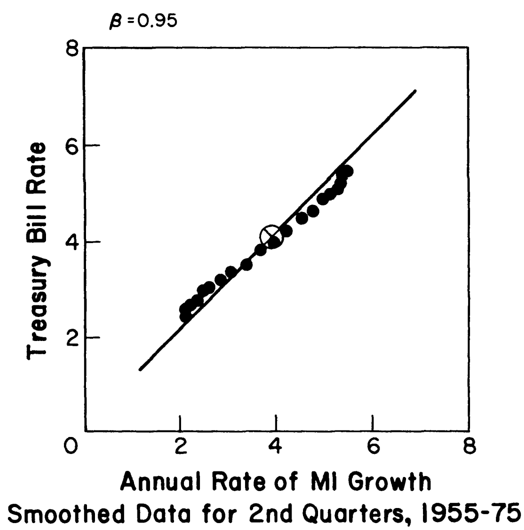

FIGURE 11

the smoothed monetary change $X_{0t}(0.9)$, while Figure 12 plots one raw variable against the other. Again, one sees order of a sort emerging from confusion but it is an order that makes no sense economically. The difference between this order and that displayed in Figures 8 and 9 is that the latter is an implication of a coherent economic theory.

Since the comparison of $X_{it}(.9)$, $i=1, 2$, to $X_{0t}(.9)$ in Figures 8 and 9 utilizes only low-frequency components of the original series, these figures will illustrate the quantity theory well only if the time-series used convey information on low-frequency movements in X_{0t}. In the absence of such information, the method applied above will produce merely a "blob" at the sample means of

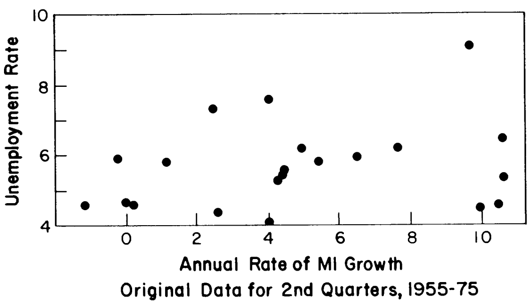

FIGURE 12

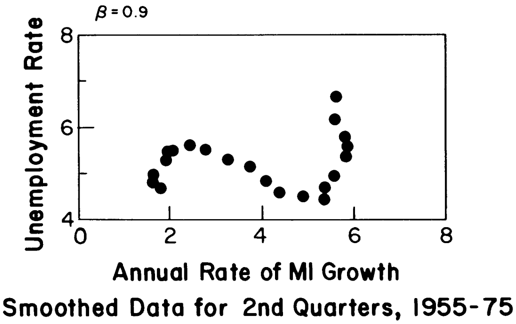

FIGURE 13

$X_{it}(\beta)$, $i=1, 2$, and $X_{0t}(\beta)$ as β approaches unity, even if the quantity theory is valid. This is the time-series equivalent of the observation that if the countries studied by Vogel had had similar rates of monetary growth over his sample period, his method would not have produced a clear 45° line. That is, these methods will yield clear results only if a good enough "experiment" has been run by "nature" over the sample period used.

IV. Concluding Remarks

The filtering techniques described and applied in Sections II and III represent what might be called a "minimal" use of the quantity theory of money, in the sense that they utilize only the widely agreed-upon "long-run" implications of that theory. To this was added the hunch that identifying long-run with "very low frequency" might isolate those movements in postwar inflation and interest rates which can be accounted for on purely quantity-theoretic grounds. Figures 8 and 9 (or 10 and 11) confirm both the hunch and the underlying theory.

Figures 14 and 15 plot actual postwar inflation and interest rates, respectively, against time (i.e., X_{1t} and X_{2t}). On each diagram is also plotted the corresponding series with the smoothed portion subtracted (that is, $X_{1t}-X_{1t}(0.9)$ and $X_{2t}-X_{2t}(0.9)$). Evidently, both the inflation and the high interest rates of the 1970's are well accounted for by the quantity theory or, to put the same point backwards, any nonmonetary explanation of these trends would lead to

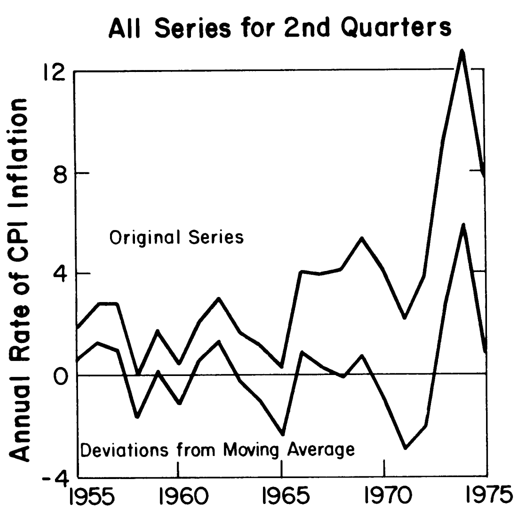

FIGURE 14

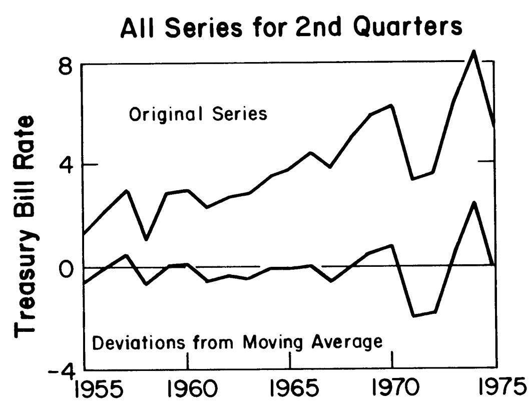

FIGURE 15

large, unexplained deviations from the relationships depicted so clearly in Figures 8–11.

The method applied in this paper involves decomposing movements in money and other nominal variables into two components, one of which I have called quantity theoretic and the other of which has been left unlabeled. This raises the question of the relationship of this decomposition to the clearly related decompositions of Thomas Sargent, and Robert Barro, among others, of monetary movements into "anticipated" and "unanticipated" components. Though it would be hard to spell out the details, my opinion would be that *all* of what I have called $X_{0t}(0.9)$ should be identified as anticipated in the Sargent-Barro sense, and *in addition*, that much of my $X_{0t}-X_{0t}(0.9)$ should also be thought of as anticipated. Indeed, this is what I mean by referring to the methods above as a *minimal* use of the quantity theory.

Putting the matter in this way should make it clear that no one decomposition method can dominate the other. By using weaker theory, one is more confident that his filter has not incorrectly labeled noise as signal; on the other side, there is no doubt that the methods used in this paper have not fully extracted from the series all that the quantity theory can account for.

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