# Boston University

# Comprehensive Examination for M.A.E.P. and M.A. in Economics

## May 18, 2006

## PART I: MICROECONOMICS (2 QUESTIONS)

I.1 [50 points] Rani's utility function is $U = X Y^2$,
where X and Y represent the quantities of the only two goods that Rani consumes. The prices of the two goods are $p_x = \$2$ and $p_y = \$4$. Rani's income is \$300 per month.

(a) Find the quantities of X and Y Rani would consume in order to maximize her utility.
(b) Suppose $p_y$ goes up to \$5. What will now be the utility maximizing quantities of consumption?
(c) Find the Compensating Variation of this price change.
(d) Find the Equivalent Variation of this price change.

I.2 [50 points] Widget Corp. is the only producer of widgets in Smalliland. Its cost function for the production of widgets is $C(Q) = Q^2/6$, where Q is the total number of widgets produced.

Widget Corp. can sell widgets domestically (where it is the only seller) or internationally (where it is a perfect competitor). Trade restrictions prevent the import of widgets and the export of widgets by anyone other than Widget Corp. The domestic demand curve for widgets is

$$q_h = 50 - p_h$$

where $q_h$ is the number of widgets transacted in the home market and $p_h$ is the price at home. The price at which Widget Corp can export widgets is \$10.

(a) Find how many widgets Widget Corp. would produce, how many it would sell at home and how many abroad, and at what prices, in order to maximize profits.
(b) Suppose the Government of Smalliland imposed an excise tax of \$5 per widget produced (whether to be sold at home or abroad). Answer the questions of part (a) under this scenario.
(c) Draw a single graph illustrating your solutions to (a) and (b).

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## PART II: MACROECONOMICS (2 QUESTIONS)

II.1 [50points] **Continuous time Solow model:** Let A(t), L(t), K(t),Y(t) denote the level of technology, labor, capital and output produced at time t.

Assume that technology and labor evolve according to:

$$dA/dt = gA(t)$$
$$dL/dt = nL(t),$$

where $g$, $n$ are the exponential rates of growth. Assume the production function is Cobb-Douglas:

$$Y(t)=K(t)^{\alpha} (A(t)L(t))^{1-\alpha},$$

and capital accumulation satisfies

$$dK/dt = sY(t) - \delta K(t),$$

where $\delta$ denotes the instantaneous rate of depreciation and $s$ denotes the savings rate.

(a) Derive the equation that describes the evolution of capital per effective unit of labor $k(t)= K(t)/(A(t)L(t))$

(b) Solve for the steady levels of k(t) and output per effective unit of labor $y(t)=Y(t)/(A(t)L(t))$.

(c) Plot savings versus break-even investment as a function of k(t). Show the effect of a decrease in the savings rate on k(t), the steady-state level of capital per effective unit of labor.

(d) Plot the time paths of k(t) and output per worker (Y(t)/L(t)) in response to a decrease in the savings rate, starting from the original steady-state balanced growth path.

(e) What effect does a reduction in the savings rate have on consumption per capita in the short-run and long-run. Does consumption per capita rise or fall? Explain.

II.2 [50points] Suppose that the economy is described by the following three equations:

- AS curve:

  $\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t$ , where $x_t$ is the output gap,

  $u_t = \rho u_{t-1} + \varepsilon_t$ is an AR(1) cost-push shock to inflation with $0<\rho<1$.

- IS curve:

  $x_t = -\varphi(i_t - E_t \pi_{t+1}) + E_t x_{t+1} + g_t$

  where $g_t = \rho_g g_{t-1} + v_t$ is an AR(1) shock to the IS curve with $0<\rho_g<1$.

- Monetary Policy: Assume that the monetary authority sets the nominal interest rate path $i_t$ to minimize the following loss function

  $$(1/2)E_t \left[\sum_{0\le s<\infty} \beta^s (\alpha x_{t+s}^2 + \pi_{t+s}^2)\right]$$

  subject to the IS and AS curves given above.

(a) Show that in a model without commitment, optimal monetary policy implies that inflation and the output gap satisfy the following optimality condition:

$$x_t = -(\kappa/\alpha)\pi_t$$

Interpret this condition.

(b) Using this optimality condition combined with the AS and IS curves, derive expressions for inflation and the output gap as functions of the underlying shocks to $u_t$ and $g_t$.

(c) Provide algebraic expressions for the standard deviation of output and inflation as a function of the standard deviations of the underlying shocks $\sigma_u = \sqrt{(var(u_t))}$ and $\sigma_g = \sqrt{(var(g_t))}$.

(d) In what sense does this model imply a tradeoff between inflation and output variability? Would there still be such a tradeoff if $\sigma_u = 0$? Explain.

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## PART III: STATISTICS AND ECONOMETRICS (2 QUESTIONS)

III.1 [50 points] Let $X_1,\ldots,X_{10}$ be a random sample of size n=10 from a normal random variable $X$. The sample mean and second moment are $\overline{X} = n^{-1}\sum_{i=1}^{n} X_i$ =0.56 and $\overline{X^2} = n^{-1}\sum_{i=1}^{n} X_i^2$ =1.13.

a) Assume that Var($X$)=1. Construct a 95% confidence interval for the mean of $X$. Carefully explain your answer.

b) Now assume that Var($X$) is unknown. Construct a 95% confidence interval for the mean of $X$. Carefully explain the differences in your answer to parts a) and b). (Hint for a) and b): use the attached statistical tables.)

III.2 [50 points]

(a) Explain what the Chow test does. Provide the formula for the test statistic, its distribution and explain the theory behind it.

(b) Suppose I am interested in analyzing the relationship between two variables y and x. The null hypothesis is y is linearly related to x. The alternate hypothesis is that the relationship is piecewise linear with a kink at $x_0$ (see attached diagram). How will I go about testing the null against the alternative using a test of structural change framework? In answering this, please provide the unrestricted and restricted (linear) models, the test statistic, how you will compute it and finally the distribution of the test statistic. (Hint: Consider the variable $x - x_0$ as a regressor).

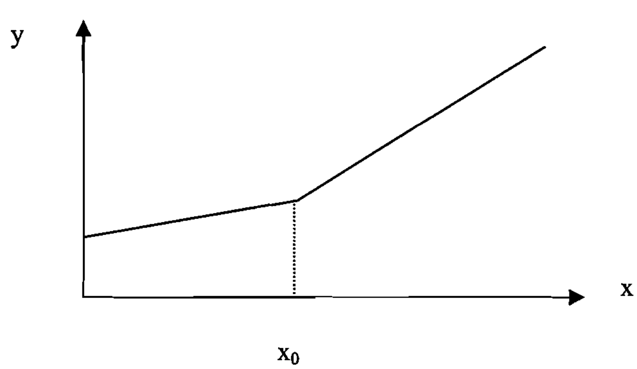


TABLE A1 *Standard Normal Distribution Function:* $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2}\, dy$

| $x$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| .1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| .2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| .3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| .4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| .5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| .6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| .7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| .8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| .9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

**TABLE A2** *Values of* $x^2_{\alpha,n}$

| $n$ | $\alpha = .995$ | $\alpha = .99$ | $\alpha = .975$ | $\alpha = .95$ | $\alpha = .05$ | $\alpha = .025$ | $\alpha = .01$ | $\alpha = .005$ |
|---|---|---|---|---|---|---|---|---|
| 1 | .0000393 | .000157 | .000982 | .00393 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | .0100 | .0201 | .0506 | .103 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | .0717 | .115 | .216 | .352 | 7.815 | 9.348 | 11.345 | 12.838 |
| 4 | .207 | .297 | .484 | .711 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | .412 | .554 | .831 | 1.145 | 11.070 | 12.832 | 13.086 | 16.750 |
| 6 | .676 | .872 | 1.237 | 1.635 | 12.592 | 14.449 | 16.812 | 18.548 |
| 7 | .989 | 1.239 | 1.690 | 2.167 | 14.067 | 16.013 | 18.475 | 20.278 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 15.507 | 17.535 | 20.090 | 21.955 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 16.919 | 19.023 | 21.666 | 23.589 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 18.307 | 20.483 | 23.209 | 25.188 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 19.675 | 21.920 | 24.725 | 26.757 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 21.026 | 23.337 | 26.217 | 28.300 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 22.362 | 24.736 | 27.688 | 29.819 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 4.601 | 5.229 | 6.262 | 7.261 | 24.996 | 27.488 | 30.578 | 32.801 |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 26.296 | 28.845 | 32.000 | 34.267 |
| 17 | 5.697 | 6.408 | 7.564 | 8.672 | 27.587 | 30.191 | 33.409 | 35.718 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 28.869 | 31.526 | 34.805 | 37.156 |
| 19 | 6.844 | 7.633 | 8.907 | 10.117 | 30.144 | 32.852 | 36.191 | 38.582 |
| 20 | 7.434 | 8.260 | 9.591 | 10.851 | 31.410 | 34.170 | 37.566 | 39.997 |
| 21 | 8.034 | 8.897 | 10.283 | 11.591 | 32.671 | 35.479 | 38.932 | 41.401 |
| 22 | 8.643 | 9.542 | 10.982 | 12.338 | 33.924 | 36.781 | 40.289 | 42.796 |
| 23 | 9.260 | 10.196 | 11.689 | 13.091 | 35.172 | 38.076 | 41.638 | 44.181 |
| 24 | 9.886 | 10.856 | 12.401 | 13.484 | 36.415 | 39.364 | 42.980 | 45.558 |
| 25 | 10.520 | 11.524 | 13.120 | 14.611 | 37.652 | 40.646 | 44.314 | 46.928 |
| 26 | 11.160 | 12.198 | 13.844 | 15.379 | 38.885 | 41.923 | 45.642 | 48.290 |
| 27 | 11.808 | 12.879 | 14.573 | 16.151 | 40.113 | 43.194 | 46.963 | 49.645 |
| 28 | 12.461 | 13.565 | 15.308 | 16.928 | 41.337 | 44.461 | 48.278 | 50.993 |
| 29 | 13.121 | 14.256 | 16.047 | 17.708 | 42.557 | 45.772 | 49.588 | 52.336 |
| 30 | 13.787 | 14.953 | 16.791 | 18.493 | 43.773 | 46.979 | 50.892 | 53.672 |

*Other Chi-Square Probabilities:*

$x^2_{.9,9} = 4.2 \quad P\{x^2_{16} < 14.3\} = .425 \quad P\{x^2_{11} < 17.1875\} = .8976$

**TABLE A3** *Values of $t_{\alpha,n}$*

| $n$ | $\alpha = .10$ | $\alpha = .05$ | $\alpha = .025$ | $\alpha = .01$ | $\alpha = .005$ |
|---|---|---|---|---|---|
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 1.533 | 2.132 | 2.776 | 3.474 | 4.604 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| $\infty$ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |

*Other t Probabilities:*
$P\{T_8 < 2.541\} = .9825$ $P\{T_8 < 2.7\} = .9864$ $P\{T_{11} < .7635\} = .77$ $P\{T_{11} < .934\} = .81$ $P\{T_{11} < 1.66\} = .94$ $P\{T_{12} < 2.8\} = .984$.

# MA Comp, May 2006, Micro Solutions.

1\. (a) We know for this Cobb-Douglas utility function the demand functions will be $X = \frac{I}{3P_X}$ and $Y = \frac{2I}{3P_Y}$

Substituting $P_X$, $P_Y$ and $I$ :

$$X = \frac{300}{3*2} = \underline{\underline{50}} \quad ; \quad Y = \frac{2*300}{3*4} = \underline{\underline{50}}$$

(b) Substituting the new $P_Y$ :

$$X = \frac{300}{3*2} = \underline{\underline{50}} \quad ; \quad Y = \frac{2*300}{3*5} = \underline{\underline{40}}$$

(c) $CV = E(P_1, U_0) - E(P_1, U_1)$

Let $I_1 = E(P_1, U_0)$

If faced with $I_1$, $P_1$, Rani would consume

$$X_1 = \frac{I_1}{6} \quad , \quad Y_1 = \frac{2I_1}{15}$$

and then utility would be

$$U_0' = \frac{I_1}{6}\left(\frac{2I_1}{15}\right)^2$$

But we need $U_1$ to equal $U_0 = (50)(50)^2 = (50)^3$

$$\therefore \quad \frac{4}{6(15)^2} I_1^3 = (50)^3$$

$$I_1 = 50 * \sqrt[3]{\frac{3(15)^2}{2}} = 348.12$$

So $CV = 348.12 - 300 = \underline{\underline{\$ 48.12}}$

(d) $EV = E(P_0, U_1) - E(P_0, U_0)$

Let $I_2 = E(P_0, U_1)$ so that $X_2 = \frac{I_2}{6}$ and $Y_2 = \frac{2I_2}{12}$

Solving for $I_2$ :

$$\frac{I_2}{6}\left(\frac{2I_2}{12}\right)^2 = (50)(40)^2$$

$$\left(\frac{I_2}{6}\right)^3 = 80 * 10^3 \longrightarrow I_2 = 60 * \sqrt[3]{80} = 258.53$$

So $EV = 258.53 - 300 = -\$\underline{\underline{41.47}}$

2\. (a) Let $q_x$ represent the quantity of widgets exported. Then Widget Corp's profits would be

$$\pi = q_h(50 - q_h) + 10q_x - \frac{1}{6}(q_h + q_x)^2$$

To max:

$$\frac{\partial \pi}{\partial q_h} = 50 - 2q_h - \frac{1}{3}(q_h + q_x) = 0 \qquad (1)$$

$$\frac{\partial \pi}{\partial q_x} = 10 - \frac{1}{3}(q_h + q_x) = 0 \qquad (2)$$

Substituting (2) in (1):

$$50 - 2q_h - 10 = 0$$

$$2q_h = 40 \longrightarrow \underline{q_h = 20}$$

Then $\underline{p_h = 30}$

From (2): $\frac{1}{3}(q_h + q_x) = 10$

$$\underline{q_x = 10}.$$

Of course $p_x = 10$.

Total production will be $q_x + q_h = 10 + 20 = \underline{\underline{30}}$

(b) This pushes the costs up by $5Q$.

$$\pi = q_h(50 - q_h) + 10q_x - \frac{1}{6}(q_h + q_x)^2 - 5(q_h + q_x)$$

$$\frac{\partial \pi}{\partial q_h} = 50 - 2q_h - \frac{1}{3}(q_h + q_x) - 5 = 0 \qquad (3)$$

$$\frac{\partial \pi}{\partial q_x} = 10 - \frac{1}{3}(q_h + q_x) - 5 = 0 \qquad (4)$$

Substituting (4) in (3):

$$50 - 2q_h - 10 = 0 \longrightarrow q_h = 20$$

Then substituting in (4):

$$5 - \frac{1}{3}(20 + q_x) = 0 \longrightarrow q_x = -5 \;!!$$

Obviously $q_x$ cannot be negative. Therefore this solution must involve zero exports and we can simply solve by setting $q_h = Q$:

$$\pi = q_h(50 - q_h) - \frac{1}{6} q_h^2 - 5 q_h$$

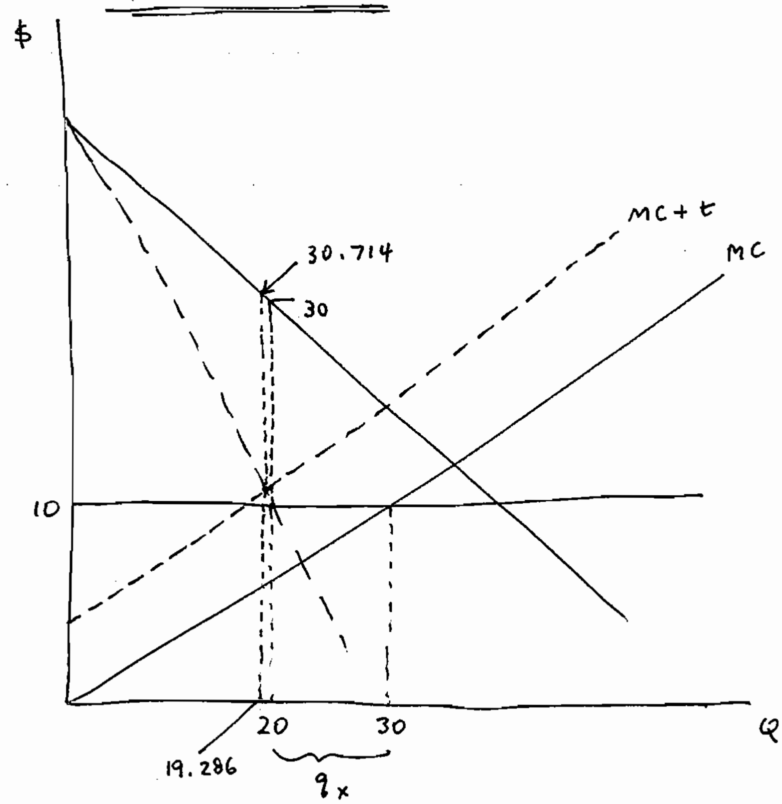
$$\frac{d\pi}{dq_h} = 50 - 2q_h - \frac{1}{3} q_h - 5 = 0$$

$$\frac{7}{3} q_h = 45$$

$$q_h = \frac{3}{7} * 45 = \underline{\underline{19.286}}$$

$$\underline{\underline{P_h = \$\ 30.714}}$$

(c)



Macro Comp Answers..
MAY 2006

1) a) $k(t) = K(t)/(A(t)L(t))$

$$y(t) = Y(t)/(A(t)L(t)) = k(t)^{\alpha}$$

$$\frac{dk(t)}{dt} = \frac{1}{A(t)L(t)}\frac{dK(t)}{dt} - \left(\frac{K(t)}{A(t)L(t)}\right)\frac{dA(t)}{A(t)} - \frac{K(t)}{A(t)L(t)}\frac{dL(t)}{L(t)}$$

$$= s\,y(t) - \delta k(t) - g\,k(t) - n\,k(t)$$

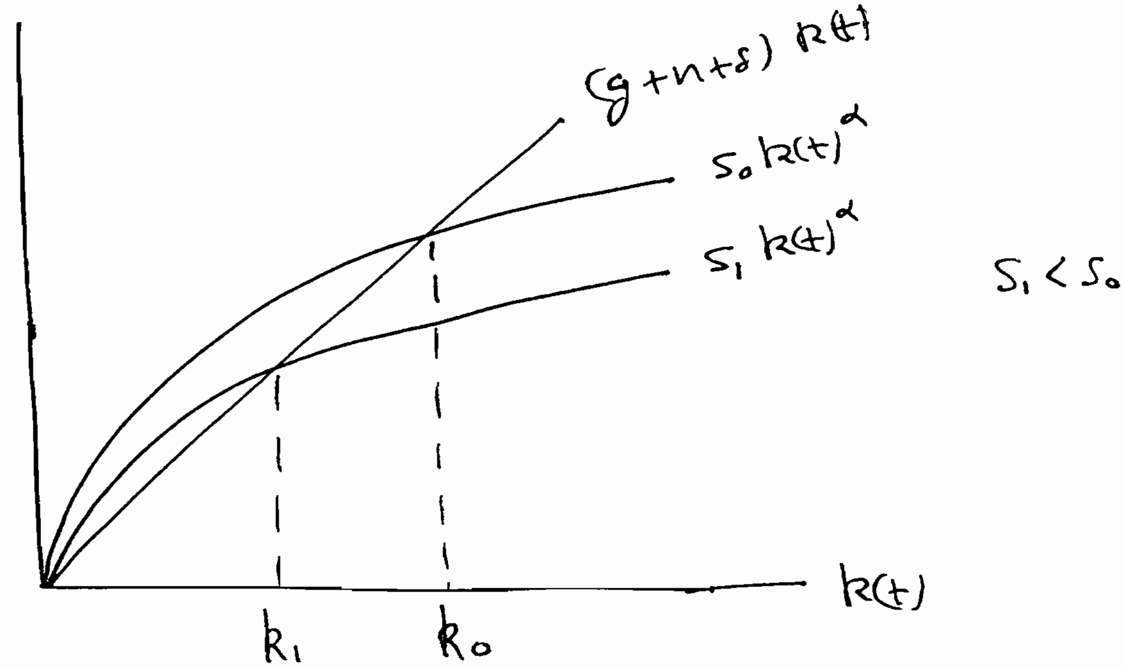
$$\Rightarrow \boxed{\frac{dk(t)}{dt} = s\,k(t)^{\alpha} - (g+n+\delta)k(t)}$$

b) In steady-state $\frac{dk(t)}{dt} = 0$

$$s\,k(t)^{\alpha} = (g+n+\delta)k(t)$$

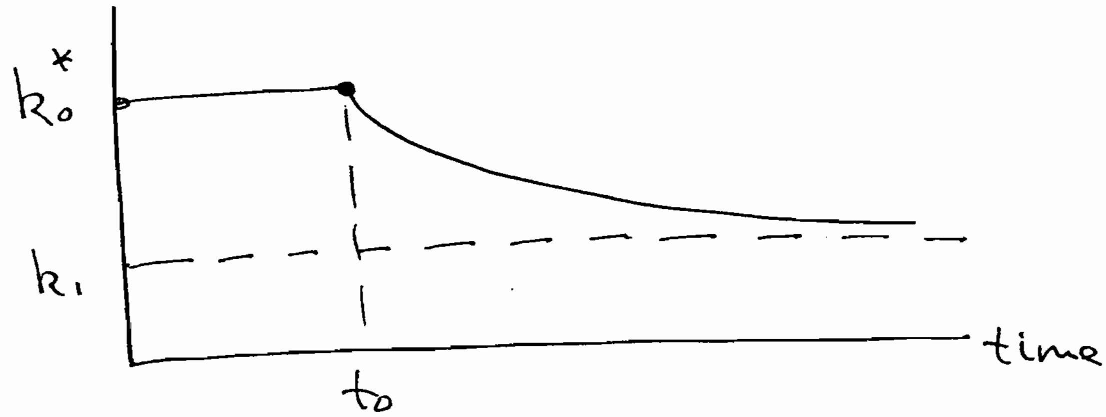
$$\Rightarrow \boxed{k(t) = \left(\frac{s}{g+n+\delta}\right)^{\frac{1}{1-\alpha}}}$$

c)




line $sk^\alpha$ shifts down. The steady-state value of $k$ occurs at the intersection of the break even line $(g+n+\delta)k(t)$ and the savings line $s\,k(t)^\alpha$. With a lower savings rate, the economy can no longer maintain the savings required to keep $k = k_0$. As a result, savings is less than break-even investment at $k = k_0$. The capital stock falls until savings now equals break even investment again. This occurs at $k = k_1$.

D/

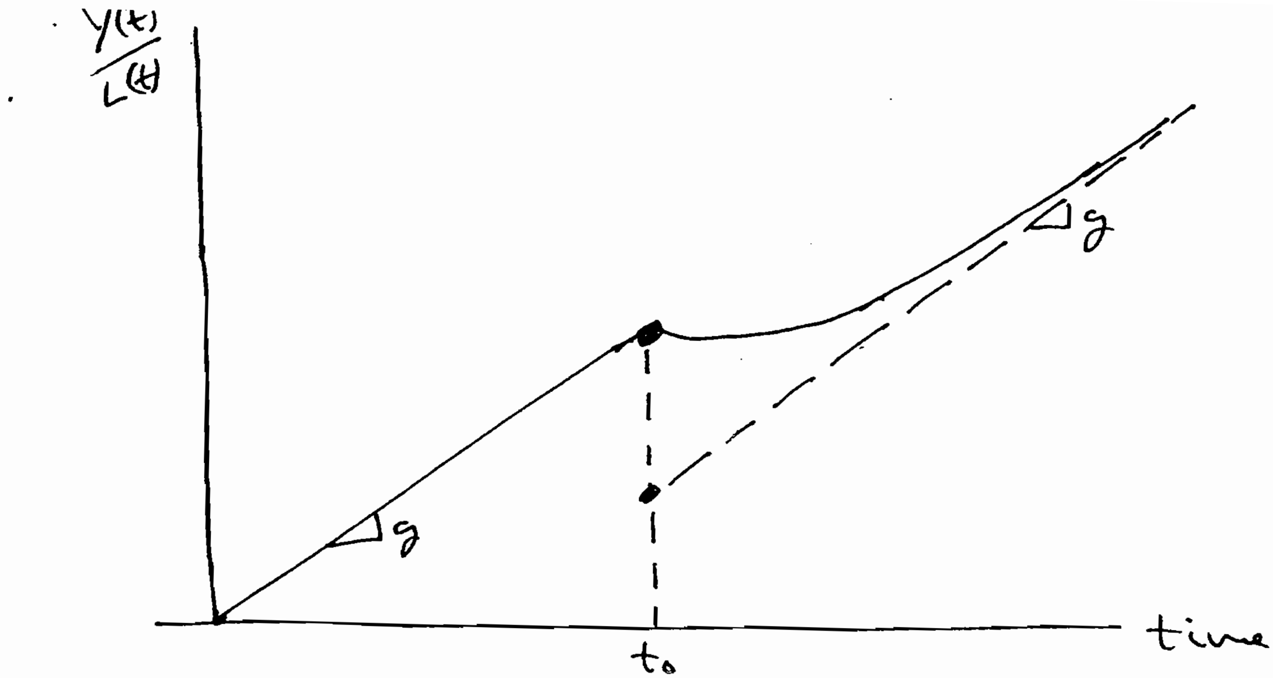



At time $t_0$ the economy is in its original steady-state with $k(t) = k_0$.

fall in $k(t)$. At the lower savings rate the economy no longer accumulates enough capital to maintain the capital stock at $k = k_0$.

The reduction in the capital stock is gradual with $k(t)$ eventually converging to the new steady state $k_1$.

Along the path, the initial reduction is rapid ( $k$ falls more in percent terms the farther away it is from the new steady-state). Eventually the growth rate of $k(t)$ returns to zero.




initially. Output per capita [illegible] at the constant rate $g$. At time $t_0$ the savings rate falls. This implies a reduction in steady-state $k(t)$ and therefore $y(t)$. The new long run path is the dashed line which also grows at rate $g$ but is shifted down by the amount $(k_1^\alpha - k_0^\alpha)A(t)$.

- In the shortrun the reduction in savings will lead to less capital accumulation and therefore a lower growth rate of output. Over time, the economy approaches the new balanced growth path and output per capita again grows at rate $g$.

e) Consumption: $C(t) = (1-s)\,Y(t)$

In effective units of labor:

$$c(t) = \frac{C(t)}{A(t)L(t)} = (1-s)\,y(t)$$

At the steady-state $y(t) = \left(\frac{s}{(g+n+\delta)}\right)^{\frac{\alpha}{1-\alpha}}$

so

$$c(t)^{ss} = (1-s)\left(\frac{s}{g+n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\frac{d\,c(t)^{ss}}{ds} = -\,y(t) + (1-s)\frac{d\,y(t)}{ds}$$

$$= \left[-1 + \frac{1-s}{s}\,\frac{\alpha}{1-\alpha}\right] y(t)$$

If $\alpha = s$, savings rate is at the golden rule and $\frac{d\,c(t)}{ds} = 0$.

If $s < \alpha$ $\frac{d\,c(t)}{ds} > 0$, a reduction in the savings rate will cause per capita consumption to fall. I.e. if savings rate is below golden rule rate, then reductions in savings rate cause consumption to fall whereas, if $s > \alpha$, savings rate is above golden rule rate consumption will rise as savings rate falls.

2) a) Without commitment, monetary authority solves

$$\min_{\pi_t, x_t} \quad \frac{1}{2}(\alpha x_t^2 + \pi_t^2) + F_t$$

subject to $\quad \pi_t = \kappa x_t + f_t$

taking $\quad F_t = \frac{1}{2}\sum_{s=1}^{\infty} E_t \, \beta^s (\alpha x_{t+s}^2 + \pi_{t+s}^2)$

$$f_t = \beta E_t \pi_{t+1} + u_t$$

as given.

F.O.C.:

$$\alpha x_t + \pi_t \cdot \frac{\partial \pi_t}{\partial x_t} = 0$$

$$\alpha x_t + \pi_t(+\kappa) = 0$$

$$x_t = -\frac{\kappa}{\alpha}\pi_t.$$

This is a lean against the wind policy. When inflation is above target, the monetary authority raises the nominal interest rate by a sufficient amount to cause a reduction in the output gap.

2 b)

$$\pi_t = K\left(-\frac{K}{\alpha}\right)\pi_t + \beta E_t \pi_{t+1} + u_t$$

$$\left(1+\frac{K^2}{\alpha}\right)\pi_t = \beta E_t \pi_{t+1} + u_t$$

$$\pi_t = \left(\frac{\alpha}{\alpha+K^2}\right)\left(u_t + \beta E_t \pi_{t+1}\right)$$

$$= \left(\frac{\alpha}{\alpha+K^2}\right)\sum_{s=0}^{\infty} \beta^s \left(\frac{\alpha}{\alpha+K^2}\right)^s E_t(u_{t+s})$$

$$\pi_t = \left(\frac{\alpha}{\alpha+K^2}\right)\left(\sum_{s=0}^{\infty}\left(\frac{\beta e \alpha}{\alpha+K^2}\right)^s\right)u_t$$

$$= \left(\frac{\alpha}{\alpha+K^2}\right)\frac{1}{1-\frac{\beta e \alpha}{\alpha+K^2}}\, u_t$$

$$\boxed{\begin{aligned} \pi_t &= \alpha q u_t \\ x_t &= -K q u_t \end{aligned} \qquad q = \frac{1}{K^2 + \alpha(1-\beta e)}}$$

2) c) $\sigma_\pi = \sqrt{\text{var}\, \pi_t} = \alpha q \sigma_u$

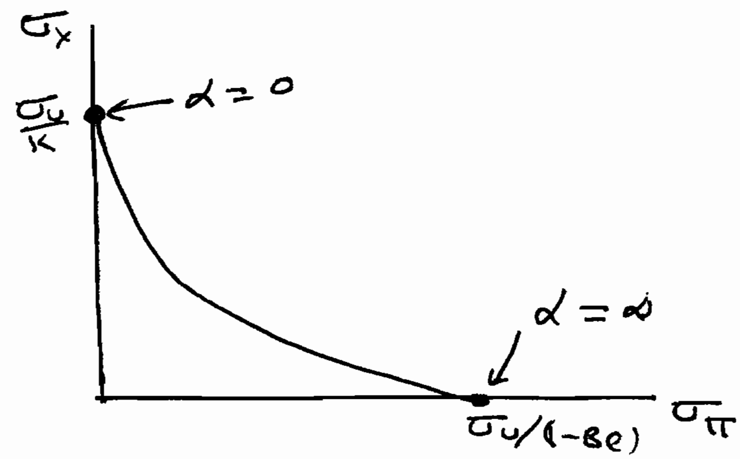
$$\sigma_x = \sqrt{\text{var}\, x_t} = \kappa q \sigma_u$$

$$\alpha q = \frac{\alpha}{\kappa^2 + \alpha(1-\beta e)}, \quad \kappa q = \frac{\kappa}{\kappa^2 + \alpha(1-\beta e)}$$

As $\alpha \to 0$ $\quad \sigma_\pi \to 0, \quad \sigma_x \to \frac{\sigma_u}{\kappa}$

As $\alpha \to \infty$ $\quad \sigma_\pi \to \frac{\sigma_u}{1-\beta e}, \quad \sigma_x \to 0$

D)




- This model implies a tradeoff between output and inflation variability in response to cost push shocks $\sigma_u$. If the monetary authority puts zero weight on output ($\alpha = 0$), it can achieve zero variability in inflation $\sigma_u = 0$. For positive $\alpha$, ~~t~~ the more weight the monetary authority puts on the output gap, (higher $\alpha$) the lower will be output variability ($\sigma_x$) but this comes at the cost of higher variability in inflation ($\sigma_\pi$).

**EC507 Question**
**MA Comprehensive Exam**
**May 2006**

1. Let $X_1,...,X_{10}$ be a random sample of size n=10 from a normal random variable $X$. The sample mean and second moment are $\overline{X} = n^{-1}\sum_{i=1}^{n} X_i$ =0.56 and $\overline{X^2} = n^{-1}\sum_{i=1}^{n} X_i^2$ =1.13.

   a) Assume that $Var[X]=1$. Construct a 95% confidence interval for the mean of $X$. Carefully explain your answer.

   Answer: Let μ denote the mean of $X$. First note that $E[\overline{X}] = \mu$ and

   $Var[\overline{X}] = Var[X]/n$ by the properties of $\overline{X}$ in random samples. Moreover, since the underlying population is normal, $\overline{X} \sim N(\mu, Var[\overline{X}])$. Finally, since $Var[\overline{X}]$ is known, we can construct a 95% confidence interval for μ as $\overline{X} \pm z_{0.975}\sqrt{Var[\overline{X}]}$, where $z_{0.975} = 1.96$ is the 97.5% (100 – 5/2) quantile of the standard normal distribution (obtained from Table Va). Using the information provided in the problem

$$\overline{X} \pm z_{0.975}\sqrt{Var[\overline{X}]} = 0.56 \pm 1.96\sqrt{1/10} = [-0.06, 1.18]$$

   b) Now assume that Var($X$) is unknown. Construct a 95% confidence interval for the mean of $X$. Carefully explain the differences in your answer to parts a) and b). (Hint for a) and b): use the attached statistical tables).

   Answer: Here note that the previous method is no longer operational because $Var[\overline{X}] = Var[X]/n$ is now an unknown constant. An unbiased estimator of $Var[X]$ can be constructed from

$$s_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 = \frac{n}{n-1}\left(\frac{1}{n}\sum_{i=1}^{n} X_i^2 - \overline{X}^2\right) = \frac{10}{9}\left(1.13 - .056^2\right) = 0.91$$

   Moreover, we showed in class that if $X$ is normal

$$\frac{\overline{X} - \mu}{\sqrt{s_n^2 / n}} \sim t_{(n-1)}$$

   We can therefore construct a 95% confidence interval for μ as $\overline{X} \pm t_{0.975,(9)}\sqrt{s_n^2/n}$, where $t_{0.975,(9)} = 2.262$ is the 97.5% (100 – 5/2) quantile of the $t$-distribution with 9 degrees of freedom (obtained from Table VI). In this case

$$\overline{X} \pm t_{0.975,(9)}\sqrt{s_n^2/n} = 0.56 \pm 2.262\sqrt{.91/10} = [-0.12, 1.24]$$

Answer to Econometrics question
May 2006 Comp
Boston University

First part:

The Chow test is a test of structural stability applied to the special case when the data series for one of the regimes is rather short compared to the number of parameters the stability of which we are testing.

Suppose we have two regimes; the first one contains $n_1$ data points and the second $n_2$ data points. Suppose that there are also k explanatory variables. We assume $n_1 > k > n_2$. Then the Chow statistic is

$\frac{RSS_w - RSS_1/n_2}{RSS_1/(n_1 - k)}$ which under the null hypothesis of identical coefficients across the two regimes is distributed as an F statistic with $n_2$ and $(n_1 - k)$ degrees of freedom.

Here

$RSS_w$ is the RSS for the whole regression.
$RSS_1$ is the RSS for the regression with the longer dataset.

To demonstrate the validity of the distributional claim on the test statistic, let us write the equations for the two regimes:

$Y_1 = X_1 \beta_1 + u_1$ (1)
$Y_2 = X_2 \beta_2 + u_2$ (2)

where $Y_1$, $Y_2$ are the left hand side variable vectors (of size $n_1$ and $n_2$ respectively), $X_1$, $X_2$ are the matrices of explanatory variables, $\beta_1$, $\beta_2$ are the coefficient vectors while $u_1$, $u_2$ are the disturbance vectors.

The null $H_0$ is $\beta_1 = \beta_2$. Now let us rewrite the second equation slightly differently:

$Y_2 = X_2 (\beta_2 - \beta_1) + X_2 \beta_1 + u_2$ (3)

Letting $X_2 (\beta_2 - \beta_1) = \gamma$, we have the following unrestricted model:

$Y_1 = X_1 \beta_1 + u_1$ (4)
$Y_2 = \gamma + X_2 \beta_1 + u_2$ (5)

And we are testing the null $\gamma = 0$.

Clearly $RSS_{restricted}$ is obtained by running the full regression ($RSS_w$) and $RSS_{unrestricted}$ is $RSS_1$ since estimated $\gamma$ coefficients can always be so chosen so as to make the error terms on the last $n_2$ equations disappear. Now notice that the number of restriction is the size of $\gamma$ which happens to be $n_2$, and the total number of data points in the model minus the total number of variables is $(n_1 + n_2 - (k + n_2))$ which simplifies to $n_1 - k$. This then justifies the Chow procedure.

Second Part:

Divide the dataset into two subsets: those whose x values are less than $x_0$ and those whose x values are greater than $x_0$ (if there are points the x value of which happen to be exactly $x_0$ assign them to either group). Now for each point create the variable $z = x - x_0$. We can now write the model as follows:

$$y = \beta_0 + \beta_1 z \quad \text{if } z < 0$$
$$y = \beta_0 + \beta_2 z \quad \text{if } z > 0$$

and what we are testing is $H_0$: $\beta_1 = \beta_2$.

This can be done in the standard way following these steps:

1. Run a linear regression for the full sample (with two explanatory variables: a constant and z) and collect the RSS. This is $RSS_{restricted}$.
2. Now run a regression of y on a constant, D*z and (1-D)*z where D is a dummy variable taking the value 1 if $z < 0$ and 0 otherwise. The RSS from this procedure will give us $RSS_{unrestricted}$.
3. Now $(RSS_{restricted} - RSS_{unrestricted}) / (RSS_{unrestricted} / (n-3))$ is F distributed with degrees of freedom 1 and $n - 3$ (where n is the full sample size). This is because the number of parameters in the full model is 3 and the number of restrictions is 1. Checking the sample test statistic against an F table (or using STATA) we can then decide whether the hypothesis is rejected or not at some prespecified significance level.