MAY 27, 2005

PART I: MICROECONOMICS (2 QUESTIONS)

I.1 [50 points] Jane has the utility function

$$U(F,C) = F^{1/3} C^{2/3},$$

where F and C are the quantities of food and clothing, respectively, that she consumes per month. The prices of F and C are each $1. Jane has a money income of $100 per month and, in addition, she receives an in-kind welfare allocation of 80 units of food, which cannot be resold.

(a) How much food and clothing will Jane consume per month, in order to maximize her utility? Draw a clearly-labeled diagram showing Jane's utility maximizing choice.
(b) If the welfare agency wished to change Jane's welfare allocation from an in-kind food gift to a money allocation, how much money would they need to pay her each month in order to make her just as well-off as she was under the in-kind allocation?

I.2 [50 points] There are two firms competing in the market fòr widgets. The demand curve for widgets is:

$$Q = 30 - 2P,$$

where Q and P are, respectively, the market quantity and price of widgets. There is no threat of entry. Firm 1 produces widgets at a constant average cost of $6 per widget, and firm 2 produces widgets at a constant average cost of $9 per widget.

(a) Suppose the firms are Cournot competitors. Find how many widgets each firm will produce, the market price of widgets, and the total profits of each firm in the Cournot equilibrium.
(b) Suppose the firms agreed to form a cartel. How many widgets would each firm produce under the cartel equilibrium? What would be the market price and the total profits of each firm under this solution?

How much would each firm be willing to pay to buy the other? What is the minimum each firm would require in order to be purchased?

MAY 27, 2005
PART II: MACROECONOMICS (2 QUESTIONS)

II.1 [50 POINTS] SOLOW MODEL AND PRODUCTIVITY SLOWDOWN

Suppose we have an economy described by the Solow growth model, with the following Cobb-Douglas production function

$$Y(t)=K(t)^{\alpha}\left[A(t)L(t)\right]^{1-\alpha},\ \ 1>\alpha>0$$

In particular, n is the population growth rate, g is the labor-augmenting productivity rate of growth, δ is the yearly capital depreciation rate, and s is the savings rate per unit of output $Y(t)$.

1) Write the production function in its *intensive form*

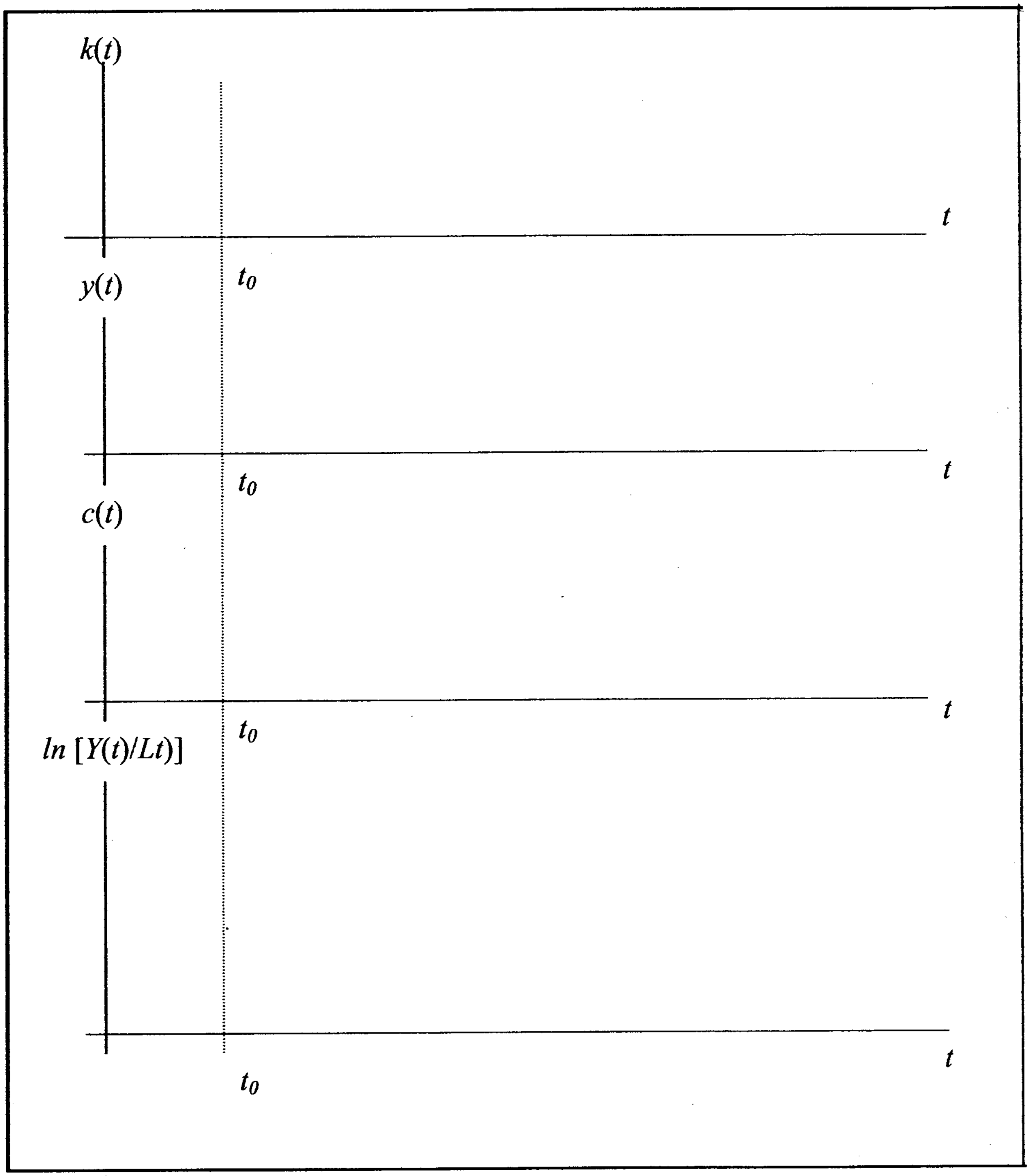
2) Construct the fundamental equation that sets *break-even investment* equal to *actual investment*. Solve formally for k^*, y^*, and c^* [where $c(t)$ is consumption per unit of effective labor].

3) Plot *break-even investment* and *actual investment* in a diagram that capital per unit of effective labor $k(t)$ on the x-axis. Show k^*, y^*, and c^* in the diagram.

4) Now suppose that g suddenly and permanently falls. In the diagram for $k(t)$ show the impact of the productivity slowdown on k^*, y^*, and c^*.

5) Now construct a set of diagrams to show the paths over time (before and after the slowdown) of $k(t)$, $y(t)$, $c(t)$, $ln[Y(t)/L(t)]$ [i.e. the LOG of labor productivity]. In the same diagrams, show also the paths for the case in which the slowdown in productivity growth did not occur.

The effect of a productivity slowdown



II.2 [50 POINTS] THE DISCRETIONARY MONETARY POLICY IN A NEW-KEYNESIAN SETUP

Assume a *discretionary* and forward-looking policymaker that sets the short-term nominal interest rate (the policy instrument) to maximize a standard quadratic objective function:

$$(-1/2)\, E_t\{\sum [\alpha (x_{t+j})^2 + (\dot{\pi}_{t+j})^2]\}$$, with summation taken over the range $j \in [0,\infty]$

s.t: $\dot{\pi}_t = \lambda x_t + u_t$ and $x_t = E_t(x_{t+1}) - \varphi [i_t - E_t(\dot{\pi}_{t+1})] + g_t$

In the firs expression, $\alpha \geq 0$ is the relative weight on output deviation x_t.

Note: this Phillips curve IS NOT expectation-augmented.

1) 1) What are the policy targets for inflation $\dot{\pi}_t$ and output y_t ?

2) Write the Lagrangian of the problem [Hint: under discretion, disregard all elements that are independent on the monetary policy]. First compute FOC relative to output gap and inflation. Then combine these FOC (get rid of the multiplier) and obtain a relationship between inflation and output gap. What is the economic interpretation for this relationship?

3) Now plug this relationship back into the Phillips curve. First solve for $\dot{\pi}_t$ and then for x_t [set $\omega \equiv 1/(\alpha + \lambda^2)$)].

4) In their paper CGG argue: "*The optimal policy incorporates inflation targeting in the sense that it requires to aim for convergence of inflation to its target over time*". Is this true in this case?

5) Now write the solution for $\dot{\pi}_{t+1}$ and take expectations $E_t(\dot{\pi}_{t+1})$. Write x_t and $E_t(x_{t+1})$, both in terms of $E_t(\dot{\pi}_{t+1})$.

6) Now plug these expressions for x_t and $E_t(x_{t+1})$ into the New-Keynesian IS curve. Write the nominal interest rate as a function of expected inflation $E_t(\dot{\pi}_{t+1})$ and g_t. List <u>at least three</u> properties of this feedback rule for i_t.

MAY 27, 2005

PART III: STATISTICS AND ECONOMETRICS (2 QUESTIONS)

III.1 [50 points] Let $X_1,\dots,X_{10}$ be a random sample of size n=10 from a normal distribution. The sample mean and second moment are $\overline{X} = n^{-1}\sum_{i=1}^{n} X_i$ =0.56 and $n^{-1}\sum_{i=1}^{n} X_i^2$ =1.93 respectively.

(a) Assume that Var(X_1)=1. Construct a test statistic for the null hypothesis $H_0 : \mu=0$ against the two sided alternative $H_1 : \mu \neq 0$. Is your test statistic significant at the 10% level? What is the critical value you use to carry out the test? Carefully justify your answer.

(b) Now assume that Var(X_1) is unknown. Construct a test with a significance level of 5% for the Null hypothesis $H_0 : \mu=0$ against the one sided alternative $H_1 : \mu>0$. What is the test statistic you use to carry out the test? Carefully justify your choice of the test statistic and the critical value. (Hint for a) and b): use the attached statistical tables).

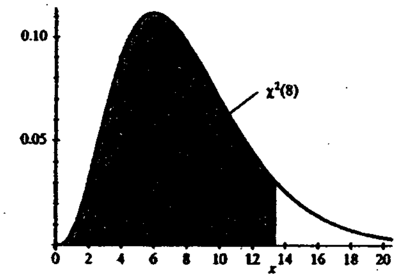
III.2 [50 points]

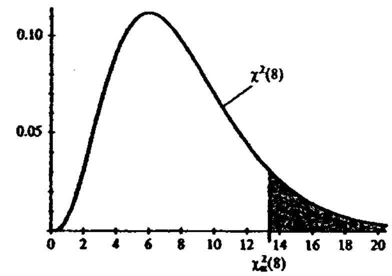
(a) Explain why consistency neither implies nor is implied by unbiasedness. [10 pts]

(b) Show that in the classical linear model, assuming exogeneity, normality and homoskedasticity of the error term and independence of observations, the variance of the error term can be unbiasedly estimated by the traditional estimator RSS / (n- k), where RSS is the residual sum of squares, n is the number of observations and k the number of variables in the model. (You may assume results on distribution of quadratic forms). [20 pts]

(c) Under those above assumptions, show that the above estimator for the variance term is consistent as well. [20 pts]

Table IV The Chi-Square Distribution



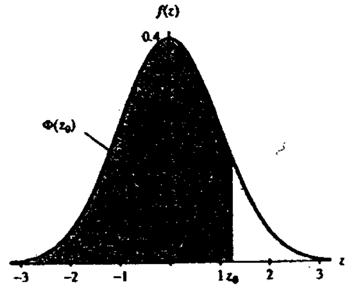


$$P(X \le x) = \int_0^x \frac{1}{\Gamma(r/2)2^{r/2}} w^{r/2-1} e^{-w/2} dw$$

				$P(X \le x)$				
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
r	$\chi^2_{0.99}(r)$	$\chi^2_{0.975}(r)$	$\chi^2_{0.95}(r)$	$\chi^2_{0.90}(r)$	$\chi^2_{0.10}(r)$	$\chi^2_{0.05}(r)$	$\chi^2_{0.025}(r)$	$\chi^2_{0.01}(r)$
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48
8	1.646	2.180	2.733	3.490	13.36	15.51	17.54	20.09
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21
11	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72
12	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22
13	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69
14	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14
15	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58
16	5.812	6.908	7.962	9.312	23.54	26.30	28.84	32.00
17	6.408	7.564	8.672	10.08	24.77	27.59	30.19	33.41
18	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.80
19	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19
20	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57
21	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93
22	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64
24	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31
26	12.20	13.84	15.38	17.29	35.56	38.88	41.92	45.64
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96
28	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89
40	22.16	24.43	26.51	29.05	51.80	55.76	59.34	63.69
50	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15
60	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38
70	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4
80	53.34	57.15	60.39	64.28	96.58	101.9	106.6	112.3

This table is abridged and adapted from Table III in *Biometrika Tables for Statisticians*, edited by E.S. Pearson and H.O. Hartley. It is published here with the kind permission of the *Biometrika* Trustees.

Table Va The Normal Distribution



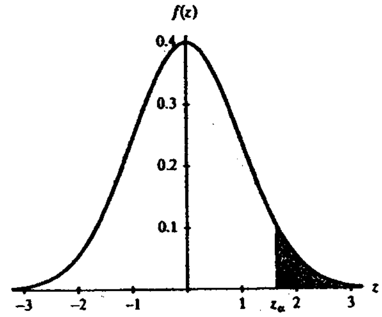
$$P(Z \le z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

$$\Phi(-z) = 1 - \Phi(z)$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9545	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

α	0.400	0.300	0.200	0.100	0.050	0.025	0.020	0.010	0.005	0.001
z_α	0.253	0.524	0.842	1.282	1.645	1.960	2.054	2.326	2.576	3.090
$z_{\alpha/2}$	0.842	1.036	1.282	1.645	1.960	2.240	2.326	2.576	2.807	3.291

Table Vb The Normal Distribution

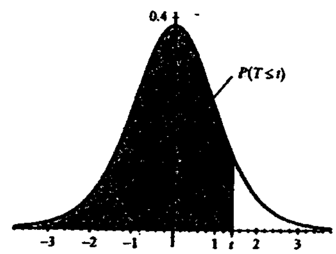


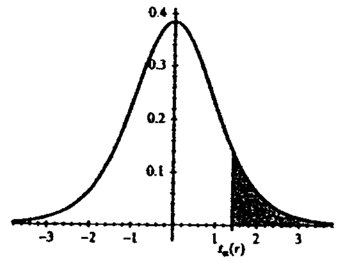
$P(Z > z_\alpha) = \alpha$

$P(Z > z) = 1 - \Phi(z) = \Phi(-z)$

z_α	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002

Table VI The t Distribution





$$P(T \le t) = \int_{-\infty}^{t} \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r}\,\Gamma(r/2)(1+w^2/r)^{(r+1)/2}}\,dw$$

$$P(T \le -t) = 1 - P(T \le t)$$

	$P(T \le t)$						
	0.60	0.75	0.90	0.95	0.975	0.99	0.995
r	$t_{0.40}(r)$	$t_{0.25}(r)$	$t_{0.10}(r)$	$t_{0.05}(r)$	$t_{0.025}(r)$	$t_{0.01}(r)$	$t_{0.005}(r)$
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012
14	0.258	0.692	1.345	1.761	2.145	2.624	2.997
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576

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