MA Comprehensive Exam
Microeconomics

1. Consider a pure exchange economy with two commodities (x, y) and total resources $(e_x, e_y) = (5, 10)$. Consumers A and B have identical preferences represented by the following utility function:

$$u_i(x_i, y_i) = x_i y_i$$

for $i = \{A, B\}$. Suppose initial endowments are $(e_x^A, e_y^A) = (2, 1)$ for A and $(e_x^B, e_y^B) = (3, 9)$ for B.

   (a) Draw an Edgeworth box and indicate the endowment point. Define the set of Pareto efficient allocations and draw them in the diagram.

   (b) Compute the Walrasian (or perfectly competitive) equilibrium and the associated allocation. Is the equilibrium efficient?

   Solution: *a) The set of Pareto efficient allocations (or contract curve) is the locus of points in the box such that the indifference curves of the two agents are tangent. The equilibrium must have that* $MRS^i_{x,y} = \frac{y_i}{x_i}$ *and that resources are exhausted. This gives the set of PE allocations as*:

$$\left\{x^A, y^A, x^B, y^B : 0 < x^A < 5, y^A = 2x^A, x^B = 5 - x^A, y^B = 2x^B\right\}$$

   b) Maximizing utility gives the offer curves:

$$\begin{aligned} OC^A(p) &= \left(1 + \frac{p_y}{2p_x}, \frac{p_x}{p_y} + \frac{1}{2}\right) \\ OC^B(p) &= \left(\frac{3}{2} + \frac{9p_y}{2p_x}, \frac{3p_x}{2p_y} + \frac{9}{2}\right) \end{aligned}$$

   Adding the offer curves for good x *and equating it to total endowment gives the equilibrium condition:*

$$1 + \frac{p_y}{2p_x} + \frac{3}{2} + \frac{9p_y}{2p_x} = 5$$

   or:

$$\frac{p_x}{p_y} = 2$$

   The equilibrium allocations are:

$$\begin{aligned} OC^A(2) &= \left(\frac{5}{4}, \frac{5}{2}\right) \\ OC^B(2) &= \left(\frac{15}{4}, \frac{15}{2}\right) \end{aligned}$$

   All Walrasian equilibria are Pareto efficient (First Welfare Theorem).

2. Suppose a monopolist can sell electricity y in two markets, 1 and 2. The demand curves in the two markets are:

$$\begin{aligned} p_1(y_1) &= 3 - \frac{y_1}{2} \\ p_2(y_2) &= 2 - \frac{y_2}{2}. \end{aligned}$$

The monopolist's cost function is:

$$c(y_1, y_2) = \frac{7}{3} + y_1 + y_2.$$

(a) What are the profit-maximizing prices in each market and what are the corresponding demand elasticities at the profit-maximizing price?

(b) What would happen to output if the monopolist is prohibited from charging different prices in the two markets?

(c) Argue whether the monopolist should have the right to charge different prices in the two markets.

Solution a) Setting $MR_i = MC_i$ gives $(y_1, y_2) = (2, 1)$, $(p_1, p_2) = (2, \frac{3}{2})$. Total revenue is $\frac{11}{2}$ and total cost $\frac{16}{3}$, so $\pi = \frac{1}{6}$. The elasticities are $(\varepsilon_1, \varepsilon_2) = (2, 3)$; b) The aggregate demand function in the relevant nbh is $p(y) = \frac{1}{4}(10 - y)$. Setting $MR = MC$ gives $p = \frac{7}{4}$, $y = 3$ and total revenue $\frac{21}{4}$. Costs are $\frac{16}{3} > \frac{21}{4}$, so $\pi < 0$. W/o price discrimination, nothing would be produced so pd is a condition for the firm to exist; c) Yes, production generates consumer surplus.