Firms’ Heterogeneity and Incomplete Pass-Through

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Abstract

A large body of empirical work documents that prices of traded goods change by a smaller proportion than real exchange rates between the trading countries (incomplete pass-through). The wedge between exchange rates and relative prices also varies across countries (pricing-to-market). I present a model of trade and international price-setting with heterogeneous firms, where firms’ strategic behavior implies that: 1) firm-level pass-through is incomplete and a U-shaped function of firm market share; 2) exchange rate fluctuations affect both the prices of traded goods and the prices of goods sold domestically; and 3) firm-level pass-through varies across destination countries. Estimates from a panel data set of cars prices support the predictions of the model.

Keywords: Heterogeneous firms, incomplete pass-through, pricing to market.


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1 Introduction

A large body of empirical work documents the fact that prices of traded goods typically change by a smaller proportion than the real exchange rates between the trading countries (incomplete pass-through). The wedge between export prices and domestic prices appears also to depend on the countries involved in the trade relationship, with exporters charging different prices to different export markets (pricing-to-market). The basic fact of incomplete pass-through has received a lot of attention in the literature, and there is a recent and growing body of work on how the extent of pass-through and pricing-to-market varies across firms in a country. This paper contributes to this literature by proposing a novel channel driving incomplete pass-through at the firm level and by testing its implications empirically.

I present a simple two-country model of trade and international price-setting where firms are heterogeneous and the market for each good has the characteristics of an international oligopoly with imperfect information. National markets are segmented, and firms set their prices by taking into account the optimal responses of their foreign competitors, whose cost structure is unobservable. I show that for a wide range of parameterizations, firms’ strategic behavior generates residual demands with an elasticity that is increasing in the price charged, and hence incomplete pass-through of cost changes into prices and pricing-to-market. The optimal price adjustments following changes in marginal cost depend on a firm’s relative size in the destination market compared to the average (or expected) size of its competitors: as a result, pass-through is a U-shaped function of firm’s size. The intuition behind this result is as follows: the largest firms (who are also the most productive in the model) don’t fear external competition, and – with a probability approaching one - are the lowest price sellers, hence their pricing decisions are not characterized by any strategic consideration. Similarly, the smallest, least productive firms have tiny mark-ups, hence no room for absorbing cost increases through mark-up reductions, and pass most of their cost changes into changes in prices. Conversely, firms lying in the middle of the distribution take into consideration their competitors’ optimal responses, and – following a cost shock – increase prices only partially
and shrink their mark-ups to avoid losing market share in favor of their competitors.

The same reasoning implies similar firm-level variation in pricing-to-market behavior. Firms charge different prices and different mark-ups in different countries, depending on demand characteristics, the extent of competition, trade barriers, and so on. Firm heterogeneity implies that pass-through is incomplete also for transactions between identical countries, and is higher the lower the average productivity of the importing country. Firms fear less competition in countries with low average productivity, and can afford to pass their cost changes through changes in prices.

While the predictions on the model for pass-through on import prices can also be derived through other theoretical frameworks, strategic price setting in my model has a unique implication for the effect of fluctuations of the exchange rate on the prices of goods produced and sold domestically. More precisely, the model predicts that the prices of goods sold domestically increase following an exchange rate depreciation. The intuition behind this result is the following. An exchange rate depreciation makes foreign firms less competitive, so import prices increase. As a result, competition in the home market is less fierce and domestic firms can “afford” to charge higher mark-ups than before the depreciation, since the threat of being undercut by foreign competitors is less severe. This implication of the model is novel, and finds support in the empirical evidence I describe next.

I test the predictions of the model using a panel data set of car prices in five European markets.\footnote{I use the data collected by Penny Goldberg and Frank Verboven, described in Goldberg and Verboven (2001).} The estimates consistently confirm the model’s prediction linking the extent of pass-through and firm size and the result of pass-through on domestic prices. I exploit the multi-country structure of the data to assess the extent of pricing-to-market and to estimate the shape of the pass-through function by origin-destination country pairs. The results broadly support the predictions of the theory.

Incomplete pass-through in the data may arise from two main margins: mark-up variability, whereby firms absorb part of the cost increases through reductions in mark-ups, and distribution
margins, due to the fact that consumer prices are composed by a certain amount of non-tradeable distribution services, whose prices are not affected by exchange rate fluctuations. Several papers have shown the empirical importance of distribution margins: see Burstein et al. (2003), Burstein et al. (2005), and Campa and Goldberg (2006) among others. Nakamura and Zerom (2010) use micro-level data on the coffee industry to decompose price adjustments into the different components. The model presented in this paper abstracts from the distribution margin and concentrates on mark-up variability, driven by the non-linear optimal pricing strategies of firms.

Theoretical research on incomplete pass-through has achieved the result of mark-up variability through two main channels: exogenous price stickiness,\textsuperscript{2} or imperfect competition with non-constant elasticity of demand. In this paper prices are fully flexible, hence the results on incomplete pass-through should be interpreted as long-run results, and not as the product of short term frictions. Imperfect competition models with variable elasticity of demand generate incomplete pass-through because changes in prices determine changes in the elasticity of demand: firms may find optimal to adjust prices only partially in order not to loose market share in favor of their competitors. Variable elasticity of demand may be achieved by appropriate choices of preferences (like in Melitz and Ottaviano 2008 and in Gust et al. 2010) or by specific assumptions on the nature of imperfect competition, as illustrated in Dornbusch (1987) and more recently implemented by Atkeson and Burstein (2008). This paper contributes to this last strand of the literature. On the demand side, I assume that consumers have CES preferences over a given set of goods, and that they can acquire each good either by a domestic or by a foreign producer. The existence of an outside option (in this case, switching to another producer from a different country) generates a residual demand with non-constant elasticity. On the supply side, I assume that firms cannot observe their competitors’ cost structure, and set optimal prices based on their expectations about the prices charged by their competitors. Given that costs are unobservable, optimal prices depend on the probability that buyers switch to another supplier.

Models featuring incomplete pass-through at the firm level differ in terms of their implications

\textsuperscript{2}Gopinath and Rigobon (2008) document the low frequency and small size of price adjustments.
for how the extent of pass-through varies across firms. We can broadly separate the literature in
two groups. On one side, models with additive distribution costs (like Berman et al. 2012 and
Chatterjee et al. 2013) imply that prices are a linear affine function of marginal costs, mark-ups
are higher and pass-through is lower for larger and more productive firms.\(^3\) In these models, pass-
through incompleteness arises from the interaction of fixed costs and firm size, and there is no
role for firms’ strategic decisions. On the other side, there is a large class of models featuring some
form of imperfect competition coupled with non-linear demand systems and strategic interactions
in price setting. Dornbusch (1987) reviews this class of models and concludes that the exact shape
of the pass-through function depends on the precise choice of market structure that one considers.\(^4\)
Particularly, models with Bertrand competition and incomplete information (like Fisher 1989 and
this paper) and models with Cournot competition, product differentiation, and a nested CES
system (like Atkeson and Burstein 2008, Amiti et al. 2013, and Auer and Schoenle 2013) imply that
prices are concave functions of marginal costs, mark-ups are higher for larger and more productive
firms, and pass-through at the firm-level is a U-shaped function of firm size.\(^5\) In these models,
pass-through incompleteness arises from strategic price adjustments whose extent depends on the
amount of competition a firm faces in the export market (size and market share are proxies for the
amount of competition a firm is subject to). Hence the different implications of these two classes
of models for the relationship between pass-through and firm size are driven by different ways of
modeling competition across firms.

The model in this paper is closest in spirit to Feenstra et al. (1996), Fisher (1989), and Alessan-
dria (2004). In my model, like in Feenstra et al. (1996), following an exchange rate shock, a firm
with a large market share in the destination market faces little competition from local firms that

\(^3\)Models with imperfect competition and linear demand systems, like the one in Melitz and Ottaviano (2008),
share the same predictions.

\(^4\)Yang (1997) illustrates the implications of a special case of the models described in Dornbusch (1987) for the
relationship linking pass-through and market shares.

\(^5\)Even if their model implies a U-shape, Amiti et al. (2013) argue that the relevant comparative statics exercise
to assess the shape of the empirical pass-through function is performed by defining the mark-up elasticity by keeping
the price index constant. With this modification, their model delivers a pass-through function that is decreasing in
firm size. In Feenstra et al. (1996), Bertrand competition and CES preferences over a discrete number of products
imply a U-shaped relationship between pass-through and the exporting country share in the destination market.
have not experienced a similar change in unit costs, and can pass through more fully the exchange rate change. However, while in Feenstra et al. (1996) market share is a country-level characteristic, firm heterogeneity in my model allows to link market share and the extent of pass-through at the firm-level.

My model shares with the one in Fisher (1989) the concept of equilibrium considered: a Bayesian Nash equilibrium where there is strategic interdependence of the pricing rules set by firms. Bertrand competition under uncertainty implies that each firm chooses the optimal price based on the expectation it has of the prices charged by its competitors. My paper adds to the analysis in Fisher (1989) firms’ heterogeneity and the fact that the choke price is endogenous and firm-specific.

The idea of incomplete price adjustments motivated by the possibility of consumers to switch to other producers is also present in Alessandria (2004). In his paper, agents have CES preferences and incomplete pass-through is driven by the possibility that consumers stop buying if the price charged is too high. Switching to another supplier involves costly search, hence optimal prices are set by keeping into account the consumers’ threat of switching supplier. The result is a reservation price rule similar to the one assumed by Feenstra et al. (1996), with optimal mark-ups depending on search and transport costs and on the number of firms competing in the same market. The model generates a pass-through function that is U-shaped in the firm’s market share. The mechanism is very similar to this paper for the presence of a threat of switching to another supplier. In my model the threat is instantaneous and has implications on prices via the non-observability of marginal costs. In Alessandria (2004), the threat takes effect over time due to the presence of search frictions.

Finally, this paper is closely related to a number of empirical contributions that have been testing the predictions of the various models regarding the shape of the empirical pass-through function. Berman et al. (2012), Chatterjee et al. (2013), and Amiti et al. (2013) all find support for the prediction that the extent of pass-through is inversely related to the size of the firm, using large across-industries firm-level datasets from France, Brazil, and Belgium, respectively.
As Feenstra et al. (1996), the empirical analysis in my paper provides evidence in support of the fact that, at least for certain industries, the relationship between pass-through and size (or market share) is U-shaped.\textsuperscript{6} Like Feenstra et al. (1996), my empirical analysis concentrates on the cars market, but exploits more the detailed micro-structure of the data, following Goldberg and Verboven (2005). Moreover, I test and find support for a novel prediction of my model, about the presence of exchange rate pass-through on domestic prices, illustrating that strategic complementarities in pricing are important empirically.

The car industry is a good laboratory to test the predictions of a model whose results derive from the strategic decisions taken by firms with market power and imperfect information about their competitors. The small number of competitors in the car industry makes strategic complementarity in pricing plausible. Moreover, the data contains information on product characteristics that can be used to identify the fact that price changes do not reflect quality changes.

The rest of the paper is organized as follows. In Section 2, I present a two-country model of trade with heterogeneous firms and strategic price-setting. I characterize the optimal pricing strategy, derive the conditions that assure incomplete pass-through and pricing-to-market, and discuss the dependence of pass-through on firm size. Section 3 contains numerical examples illustrating the properties of the pass-through function and its dependence on the aggregate parameters of the model. In Section 4 I test the predictions of the model using product-level data from the European car industry. Section 5 concludes.

\section{A Simple Model of Trade with Strategic Price Setting}

In this section I introduce a simple model of trade where firms’ heterogeneity and imperfect competition generate incomplete pass-through of changes of marginal costs into prices.

I consider a world of two countries, Home ($h$) and Foreign ($f$). Each country is populated by a

\textsuperscript{6}Auer and Schoenle (2013) also provide empirical evidence in support of a U-shaped firm-level pass-through function.
large number of identical consumers that have CES preferences over a continuum of differentiated goods:

\[ U_j = Q_j = \left[ \int (q^i_j)^{-1/\eta} \, di \right] ^{\eta/(\eta-1)} \]

where \( q^i_j \) is the quantity consumed of good \( i \) in country \( j \) \((j = h, f)\), \( Q_j \) denotes aggregate consumption in country \( j \), and \( \eta > 1 \) denotes the elasticity of substitution across goods. Each good can be acquired from a domestic or from a foreign producer, and consumers buy it from the producer that charges the lowest price.

Each country is populated by a continuum of heterogeneous producers. Each producer is specialized in the production of a single good \( i \). A producer of good \( i \) in country \( j \) has a constant return to scale technology described by \( q^i_j = l^i_j / w_j z^i_j \), where \( l^i_j \) is the quantity of labor hired to produce \( q^i_j \), \( w_j \) is the labor cost in country \( j \), and \( 1 / z^i_j \) denotes the firm’s productivity (\( z^i_j \) is the number of units of labor a firm in country \( j \) needs to produce one unit of good \( i \)). Producers in each country are heterogeneous in their costs: let \( G_j(\bar{z}_j) \) denote the mass of producers from country \( j \) that have unit cost \( z_j \leq \bar{z}_j \). In this economy, the only features differentiating goods are the cost parameters \( z^i_j \), for \( j = h, f \). Consequently, I drop the index \( i \) and denote each good by the couple of unit labor requirements of its suppliers in the two countries.\(^7\) Let \( z = (z_h, z_f) \in \mathbb{R}^2_+ \) and \( q_j(z) \) the quantity purchased in country \( j \) of a good for which the domestic supplier has unit cost \( z_h \) and the foreign supplier has unit cost \( z_f \). Finally, let \( p_{jk}(z_k) \) denote the price charged by a producer from country \( k \) with cost draw \( z_k \) for its sales in country \( j \) \((j, k = h, f)\).

The timing is the following: (i) producers in both countries observe their own productivity and the aggregate parameters of the economy; (ii) based on his own productivity and on the expectations on the prices charged by his competitors, each producer declares a selling price; (iii) for each good, consumers decide whether to buy it domestically or abroad, depending on which seller charges the lowest price; (iv) producers whose realized demand is positive produce, sell and make profits.

\(^7\)This is a common convention in the Ricardian trade literature. See Eaton and Kortum (2002), Alvarez and Lucas (2007).
The demand side of the economy is standard. A consumer in country $j$ chooses the optimal quantity of each good $q_j(z)$, and whether to buy it domestically or abroad, to minimize total expenditure. The consumer’s problem is:

$$\min_{q_j(z)} \int_{\mathbb{R}^2_+} \min\{p_{jh}(z_h), p_{jf}(z_f)\} q_j(z) g(z) dz$$

s.t. $$\left[ \int_{\mathbb{R}^2_+} q_j(z)^{1-1/\eta} g(z) dz \right]^{\eta/(\eta-1)} \geq Q_j$$

where $g(z) = g_h(z_h) \cdot g_f(z_f)$ is the joint density of the cost distributions in the two countries, which are assumed to be independent. Problem (1) has solution:

$$q_j(z) \equiv q_{jh}(z) = \left( \frac{p_{jh}(z_h)}{P_j} \right)^{-\eta} Q_j$$ if $p_{jh}(z_h) \leq p_{jf}(z_f)$  \hspace{1cm} (2)

$$q_j(z) \equiv q_{jf}(z) = \left( \frac{p_{jf}(z_f)}{P_j} \right)^{-\eta} Q_j$$ if $p_{jh}(z_h) \geq p_{jf}(z_f)$  \hspace{1cm} (3)

where $q_{jh}(z) \ (q_{jf}(z))$ is the quantity of good $z$ that a consumer in country $j$ purchases from a producer in country $h \ (f)$. $P_j$ is the consumer price index in country $j$:

$$P_j = \left[ P_{jh}^{1-\eta} + P_{jf}^{1-\eta} \right]^{1/(1-\eta)}$$

and:

$$P_{jh} = \left[ \int_0^\infty \int_0^{P_{jh}(z_f)} p_{jh}(z_h)^{1-\eta} g(z) dz \right]^{1/(1-\eta)}$$

$$P_{jf} = \left[ \int_0^\infty \int_0^{P_{jf}(z_f)} p_{jf}(z_f)^{1-\eta} g(z) dz \right]^{1/(1-\eta)}$$

It remains to determine the prices $p_{jk}(z_j)$, for $j, k = h, f$. Markets are segmented. The set of goods consumed in each country is fixed, and in each country there is exactly one producer of each good.\footnote{I assume that there is only one producer of each good in each country. This assumption can be relaxed by inter-}
both countries, and may charge different prices to buyers in different countries. By assuming that no resale is possible, I study the pricing problem country by country.

In choosing the optimal price to charge in country $j$, a producer from country $k$ must consider both direct competition from the producer of the same good in country $j$ and indirect competition from the producers of other, imperfectly substitutable goods in both countries. Consumers buy from the producer charging the lowest price, so expected profits are given by profits in case of sale times the probability that the price charged is below the price charged for the same good by a producer in the other country. In formulating this problem, I assume that each producer in each country knows the aggregate parameters of the cost distributions, but cannot observe the individual unit cost of the foreign producer that produces his same good. The price setting mechanism has the properties of a potentially asymmetric first-price sealed-bid auction. The assumption of incomplete information seems natural in the international context, where it may be too costly to monitor a foreign competitor’s cost structure. Consequently, each producer sets the price as a function of his own marginal cost in a way that, given that all the other producers set their price in the same way, no individual producer could do better by choosing the price differently. The resulting equilibrium is a Bayesian Nash equilibrium, where each producer chooses its optimal price based on his guess (correct in equilibrium) of the pricing rule followed by the producer of the same good in the other country.

interpreting each producer’s production function as the aggregate production function of a set of “lower-level” producers of the same good in a country. By assuming that the technology for producing a good is country-specific (i.e., that $z_j$ is constant across all producers of the same good in country $j$) and that each lower-level producer has a decreasing returns to scale production function and pays a fixed cost to enter the market (along the lines of Rossi-Hansberg and Wright 2007), it can be shown that the aggregation of lower-level producers generates a constant returns to scale technology that is isomorphic to the linear technology of each producer in the model. This is achieved by appropriately redefining the technology draw $z$ as a function of the fixed entry cost and of the parameter ruling decreasing returns for the lower-level producers.

In their survey of the auctions literature, McAfee and McMillan (1987) report that “sealed-bid tenders are [...] used by firms procuring inputs from other firms”. Asymmetric auctions seem a natural tool to study pricing in international markets, “when both domestic and foreign firms submit bids and, for reasons of comparative advantage, there are systematic cost differences between domestic and foreign firms”. Gareto (2013) uses a similar price setting mechanism to model optimal pricing of intermediate goods when the buyers have the possibility of integrating production. Dvir (2010) studies the final good producers optimal procurement problem in a setting with the same informational assumptions.

The equilibrium concept is the same as in Fisher (1989). I add to his framework firm heterogeneity and endogenous choke prices.
A producer from country $k$ with unit cost $z_k$ chooses the price to charge in country $j$ to maximize:

$$\max_{p_{jk}(z_k)} \left[ p_{jk}(z_k) - c_{jk}z_k \right] \left( \frac{p_{jk}(z_k)}{P_j} \right)^{-\eta} \cdot Q_j \left[ 1 - F_{j,\sim k}(p_{jk}(z_k)) \right].$$

(7)

The term $c_{jk}z_k$ denotes the marginal cost of a producer with unit labor requirement $z_k$, where:

$$c_{jk} = \begin{cases} w_k & \text{if } k = j \\ \eta w_k & \text{if } k \neq j \end{cases},$$

$t$ is the iceberg cost of trade between the two countries, and $e$ is the real exchange rate, expressed in units of domestic consumption per units of foreign consumption. $F_{j,\sim k}(\cdot)$ is the c.d.f. of the prices charged in country $j$ by suppliers not from country $k$. The term $[1 - F_{j,\sim k}(p_{jk}(z_k))] = \text{prob}\{p_{jk}(z_k) \leq p_{j,\sim k}(z_{\sim k})\}$ is the probability that consumers in country $j$ buy good $z$ from suppliers from country $k$.

The first order condition of problem (7) can be written as:

$$p_{jk}(z_k) = \left[ 1 - \frac{1}{\eta + H_{j,\sim k}[p_{jk}(z_k)] \cdot p_{jk}(z_k)} \right]^{-1} c_{jk}z_k$$

(8)

where $H_{j,\sim k}[p_{jk}(z_k)]$ is the hazard rate:

$$H_{j,\sim k}[p_{jk}(z_k)] = \frac{f_{j,\sim k}[p_{jk}(z_k)]}{1 - F_{j,\sim k}[p_{jk}(z_k)]}$$

and $f_{j,\sim k}[p_{jk}(z_k)]$ is the density associated with $F_{j,\sim k}[p_{jk}(z_k)]$.

In equation (8), the term $\eta_{jk} \equiv \eta + H_{j,\sim k}[p_{jk}(z_k)] \cdot p_{jk}(z_k)$ is the elasticity of residual demand perceived by a producer in country $k$ selling in country $j$. The expression of the elasticity of demand summarizes the two forces that affect optimal price setting: a supplier must choose its optimal price by keeping into account both the possibility of substitution across different goods ($\eta$) and direct competition from the producer of exactly the same, perfectly substitutable good produced in the other country. This second force is summarized by the hazard rate $H_{j,\sim k}[p_{jk}(z_k)]$, which is the
probability that – after an infinitesimal increase in the price charged by a producer in country $k$ – a consumer in country $j$ switches to buying the same good from the producer in the other country, conditional on buying from country $k$ before the price increase. Notice that when the hazard rate is equal to zero, $\eta_{jk} = \eta$ and equation (8) reduces to the standard constant mark-up pricing rule induced by imperfect competition and CES preferences. A positive hazard rate, i.e. a positive probability that potential buyers switch to another producer of the same good, generates a non-constant elasticity, and is the driving force behind the mechanism illustrated in this paper.\footnote{The result of endogenous mark-ups holds for any functional specification of the cost distributions $G_j(\cdot)$ except for the Pareto, for which the elasticity of demand is constant and hence mark-ups are constant too. I consider this particular case of limited empirical relevance, since there is large evidence about the fact that the empirical productivity distribution – not the cost distribution – can be well approximated by a Pareto.}

Given functional forms for the cost distributions $G_h(\cdot), G_f(\cdot)$, the model is solved up to the scale of production $Q_h, Q_f$ and the equilibrium wages $w_h, w_f$. Full employment and market clearing conditions allow to close the model.

The next sections characterize the optimal changes in prices following shocks to marginal costs. I start by providing conditions under which a shock to firms’ costs is not reflected one-to-one into the price charged (incomplete pass-through). Then I move to illustrate the relationship between pass-through and firm size. The model has implications for the effects of exchange rate changes on both domestic prices and import prices, and for pricing-to-market.

### 2.1 Incomplete Pass-Through of Cost Changes into Prices

The optimal price adjustment following a change in marginal cost depends crucially on the elasticity of demand. As equation (8) shows, the fact that each supplier in the model must keep into account his competitors’ strategies induces a variable component into the elasticity of demand, and the dependence of this component on prices determines the extent of pass-through. More precisely, a supplier will find optimal to adjust its price less than proportionately after a change in marginal...
cost when the elasticity of demand is increasing in the price charged. When this is true, the percentage reduction in demand caused by an increase in price is larger than the percentage increase in demand caused by a drop in price of similar size, inducing firms to be reluctant to adjust their prices proportionately to their costs.

In the model outlined in the previous section, the elasticity of demand that a producer from country \( k \) faces when selling in country \( j \) is:

\[
|\varepsilon_{jk}(z_k)| = \eta + \frac{f_{j,\sim k}[p_{jk}(z_k)]}{[1 - F_{j,\sim k}(p_{jk}(z_k))]}p_{jk}(z_k).
\] (9)

Whether the elasticity of demand is increasing in the price charged only depends on the shape of the competitors’ price distribution \( F_{j,\sim k}(\cdot) \), and hence on the cost distributions \( G_h(\cdot), G_f(\cdot) \). The following theorem states a necessary and sufficient condition for the elasticity of demand to be increasing in the price charged.

**Theorem 1.** The elasticity of demand \(|\varepsilon_{jk}|\) is increasing in the price charged if and only if the competitors’ cost distribution \( G_{\sim k}(\cdot) \) satisfies:

\[
g'(z) < -\frac{g(z)}{z^2} \left[ 1 + \frac{g(z)}{1 - G(z)} z \right] \forall z.
\] (10)

**Proof:** See Appendix A.

By applying Theorem 1, it is possible to derive the implications of any cost distribution for the shape of the elasticity of demand. The Pareto distribution is the cutoff between two sets of distributions that imply different results for the responsiveness of prices to changes in marginal costs. The relevant set for this exercise is the one composed by those distributions such that the

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\(^{12}\)It is easy to prove that if the elasticity of demand is increasing in the price charged, the model exhibits incomplete pass-through. We have incomplete pass-through when:

\[
\frac{\partial \log p(z)}{\partial \log z} = \left[ 1 - \frac{p(z)}{|c|^2} \right] \frac{\partial |c|}{\partial z} < 1,
\]

or – equivalently – if:

\[
\frac{p(z)}{|c|^2} \cdot \frac{\partial |c|}{\partial z} > 0,
\]

which is always true if the elasticity of demand is increasing in \( p(z) \).
slope of the density function is larger than in the Pareto case for each value of \( z \). For example, the exponential, Fréchet and Weibull distributions satisfy condition (10). Finally, this condition is closely related to the log-concavity of the survival function \([1 - G(\cdot)]\):

**Corollary 1.** If the survival function \([1 - G(z)]\) is log-concave \( \forall z \), then condition (10) holds and the elasticity of demand is increasing in the price charged.

**Proof:** See Appendix A.

Log-concavity is sufficient but not necessary to drive the result. Theorem 1 is a weaker requirement: the Weibull distribution, for example, exhibits a log-concave survival function only for certain values of its parameters, but satisfies Theorem 1 for the entire range of them.

### 2.2 Incomplete Pass-Through and Firm Size

Theorem 1 establishes a condition that disciplines the relationship between the firms’ productivity distribution and the extent of pass-through in the model. However, firms’ heterogeneity also implies that the elasticity of demand is firm-specific, and so is the extent of pass-through. In this section I characterize the dependence of pass-through on firm sales and market share in the destination market.

Optimal prices in this economy can be expressed as:

\[
p_{jk}(z_k) = \frac{|\varepsilon_{jk}(z_k)|}{|\varepsilon_{jk}(z_k)| - 1} c_{jk} z_k \quad \text{for } j, k = h, f
\]

where the elasticity of demand \(|\varepsilon_{jk}(z_k)|\) is given by:

\[
|\varepsilon_{jk}(z_k)| = \eta + \frac{f_{j, \sim k}[p_{jk}(z_k)]}{[1 - F_{j, \sim k}[p_{jk}(z_k)]]} p_{jk}(z_k).
\]
Pass-through is given by:

\[ PT_{jk}(z_k) = \frac{\partial \log(p_{jk}(z_k))}{\partial z_k} = 1 - \frac{p_{jk}(z_k)}{|\varepsilon_{jk}(z_k)|^2} \cdot \frac{\partial |\varepsilon_{jk}(z_k)|}{\partial z_k}. \] (12)

Since the elasticity of demand and optimal prices are functions of the firm’s cost \( z_k \), so is pass-through. Similarly, firm sales and market share in each country are also (decreasing) functions of \( z_k \).

Consider first extremely productive firms, for which the unit cost \( z_k \) approaches zero. Those firms have large sales and market shares in each country they sell to. For those firms, the term \([1 - F_{j, \sim k}(p_{jk}(z_k))] = \text{prob}\{p_{jk}(z_k) \leq p_{j, \sim k}(z_k)\}\) approaches one, and demand approaches the one in a standard model with monopolistic competition and CES preferences. Hence the elasticity of demand tends to a constant: \( |\varepsilon_{jk}| \to \eta \), prices are characterized by the CES constant mark-up, and pass-through is complete. Consider now extremely unproductive firms, for which the unit cost \( z_k \) tends to infinity. Those firms have the smallest sales and market shares in each country they sell to. For those firms, the term \([1 - F_{j, \sim k}(p_{jk}(z_k))] = \text{prob}\{p_{jk}(z_k) \leq p_{j, \sim k}(z_k)\}\) approaches zero, the elasticity of demand tends to infinity, and prices tend to the perfectly competitive ones. With prices equal to marginal costs, pass-through is also complete. Finally, for firms such that \( z_k \) is at an intermediate range, \([1 - F_{j, \sim k}(p_{jk}(z_k))] = \text{prob}\{p_{jk}(z_k) \leq p_{j, \sim k}(z_k)\} \in (0, 1), |\varepsilon_{jk}| \in (\eta, +\infty)\) and – from Theorem 1 – is increasing in \( z_k \) (and in \( p_{jk}(z_k) \)), so pass-through is strictly between 0 and 1. As a result, pass-through is a U-shaped function of firm sales and market share in a country. This result is analogous to the one in Feenstra et al. (1996), extended to consider a continuum of goods, heterogeneous firms, and endogenous firm market shares. Notice that the analysis applies to any change in the unit cost of the firms: productivity shocks, changes in wages or transportation costs, and exchange rate shocks, which will be the focus of the empirical analysis. The numerical analysis in Section 3 illustrates the relationship between pass-through and firm size that is implied by the model. The empirical analysis in Section 4 shows that this prediction of the model finds support in the data.
2.3 Effects of Exchange Rate Changes on Domestic Prices

An implication of the model is that exchange rate fluctuations have an effect not only on the prices of traded goods, but also on the prices of goods produced and sold domestically. This result follows from the fact that (8) defines a system of two equations in the two unknowns \( p_{jh}(z_h), p_{jf}(z_f) \) that need to be solved jointly to determine the prices that producers in the two countries charge in a common market \( j (j = h, f) \).

Suppose - without loss of generality - that the currency of the Home country depreciates. Then the analysis of the previous section implies that the prices of \( h \)'s imports \( p_{hf}(z_f) \) increase, albeit not proportionally due to incomplete pass-through. As a consequence, the probability that prices of domestic goods are lower than prices of imported goods \( [1 - F_{hf}(p_{hh}(z_h))] \) increases, and equation (8) implies that \( p_{hh}(z_h) \) increases as well. The intuition behind this result is the following: Home firms can (at least slightly) increase their mark-ups on domestic sales, given the expectation on the higher import prices charged by foreign firms. As a result, a depreciation of the exchange rate induces a non-proportional increase not only on import prices, but on domestic prices as well. Since this is a second-order effect, we also expect the increase in domestic prices to be smaller than the corresponding increase in import prices. Section 4 shows that this prediction is consistent with the data.

2.4 Pricing to Market

Price adjustments following exchange rate shocks also depend on country-level characteristics. Firms’ reductions in mark-ups are affected by variables linked to the relative competitiveness in the countries under study. The expression “pricing-to-market” refers to the possibility that firms charge different prices (and different mark-ups) in countries with different characteristics.

It is clear from equation (8) that the model exhibits variable mark-ups, decreasing in firm-level unit costs. The multiplicative nature of iceberg transportation costs and exchange rates implies
that a) firms set different mark-ups for domestic sales and exports (lower in the export market if the productivity distributions across countries are not too different); b) firms set different mark-ups in different export markets (depending on exchange rates, trade costs, and differences in the productivity distributions across countries); c) the price differential between export prices and domestic prices varies across firms. As a result, the extent of pass-through of exchange rate fluctuations into import prices and its variation across firms should vary across country-pairs (pricing-to-market). The average productivity in a country also matters for the extent of pricing-to-market, as it is a measure of the “toughness” of competition in that market. Following a depreciation in $h$’s exchange rate, exporters from $f$ adjust their prices less the higher the average productivity in the destination market ($h$). The rationale behind this result is that foreign exporters tailor their price adjustments to the expected productivity of the firms they have to compete against in country $h$. As a result, we should expect lower pass-through in destination countries with high average productivity.

I illustrate these properties of the model via numerical examples in Section 3 and test their consistency with the data in Section 4.

3 Numerical Examples

In this section I use numerical examples to quantify the extent of pass-through generated by the model and its variation across firms. I start by considering a scenario in which the two countries are identical and compute pass-through on both import prices and domestic prices following an exchange rate depreciation. Using an asymmetric example, I quantify the differences in the extent of pass-through induced by differences in transportation costs and country-level productivity.

3.1 Parameterization and Baseline Example

I start by considering two identical countries, and compute changes in import prices following a depreciation of the domestic currency. The two countries have the same technology ($G_h = G_f$),
the same wage level \((w_h = w_f)\), no trade costs \((t = 1)\), and start with a one-to-one exchange rate. I assume firms’ unit costs are distributed according to a Weibull law with shape parameter \(\vartheta\) and location parameter \(T\): \(G(z) = 1 - e^{-Tz^{\vartheta}}\), for \(T, \vartheta > 0\). I chose the Weibull distribution to be consistent with recent quantitative Ricardian models of trade.\(^{13}\) Notice that the Weibull distribution satisfies Theorem 1 for its entire parameters range. For these calculations, I set \(\eta\) to a standard value of 2. The shape parameter of the cost distribution is set to \(\vartheta = 4\),\(^{14}\) and the location parameter is normalized to \(T = 1\).

By solving the system of equations described by (8), I compute the prices of imported goods and the associated mark-ups across the entire firms’ cost distribution.\(^{15}\) I am interested in quantifying how prices and mark-ups react after a change in the exchange rate. Since the one described here is a purely real, partial equilibrium economy, exchange rate shocks are isomorphic to any exogenous shock changing relative marginal costs in the two countries, like shocks to productivity or transportation costs. I denote with \(e\) the real exchange rate between the two countries, expressed in units of domestic consumption per units of foreign consumption. I consider a 1% depreciation (appreciation) of the Home (Foreign) currency \((e' = 1.01)\), and compute its effect on import prices in the Home country. The model implies that – in response to the depreciation – foreign firms increase their export prices less than 1%. On aggregate, the model implies an increase in the import price index of 0.7%.\(^{16}\) This sluggish price adjustment is covered by a reduction in mark-ups: average sales-weighted mark-ups on imports drop of 1.1%. The extent of the adjustment following the depreciation varies across firms with different unit costs. Following the increase in import prices, domestic prices in the Home country rise as well: aggregate pass-through on domestic prices is 0.3%, positive and lower than the one on import prices.

\(^{13}\)See most notably Eaton and Kortum (2002).
\(^{14}\)Within a similar Ricardian framework with Bertrand competition, Bernard et al. (2003) estimate \(\vartheta = 3.6\).
\(^{15}\)The algorithm to solve for the optimal pricing rules is described in Appendix B, and is available upon request to the author.
\(^{16}\)The baseline example generates a level of pass-through that is comparable with empirical studies: Campa and Goldberg (2005) report an average long-run exchange rate pass-through coefficient of 64% among OECD countries. Gagnon and Knetter (1995) center the range of pass-through estimates found in various studies around 60% . Appendix C contains details on how individual price changes are aggregated.
Figure 1 shows the firms’ cost distribution, optimal import prices and mark-ups, and the extent of pass-through as functions of firm unit cost $z$.

Figure 1: Prices, mark-ups and pass-through as functions of unit costs (baseline example).

Firms with “low” $z$ are the most productive ones. These firms sell the largest quantities and have the largest market shares per destination country.\(^{17}\) Firms with “low” $z$ are also the ones with the highest mark-ups (up to 100% in this example) and they completely pass an exchange rate appreciation into a price increase (pass-through tends to one for $z \to 0$): since these firms are “infinitely” productive, the probability that buyers switch option (i.e., that they start buying from domestic producers instead) after they increase their price is almost zero. On the other hand, firms with “high” $z$ are the least productive ones. These firms sell the tiniest quantities and have the smallest market shares per destination country. Firms with “high” $z$ have almost zero profits and

\(^{17}\)CES preferences imply that the quantity sold by a firm is proportional to the term $p(z)^{-\eta}$, hence the market share of a firm in a country is given by $[p(z)/P]^{1-\eta}$. 

19
very little margin to shrink their mark-ups, so they have to pass through completely an exchange rate appreciation into a price increase (pass-through is one for \( z \) “high”). Finally, pass-through is strictly less than one for firms that lie in the middle of the cost distribution: these firms strategically reduce mark-ups and do not adjust fully their prices in order to induce buyers to keep buying from them and not to switch to domestic producers after the depreciation.

![Graph showing the relationship between exchange rate pass-through and firm market share.](image)

**Figure 2**: Pass-through as a function of firm market share (baseline example).

Figure 2 shows the model-based relationship between exchange rate pass-through and firm market share in the baseline example, by plotting pass-through \( \frac{\partial \log(p; e)}{\partial \log(e)} \) as a function of log-firm market share in a country. The figure displays a U-shaped relationship linking pass-through to firm market share. In Section 4, I use firm-level data from the European car market to estimate the shape of the empirical pass-through function.

### 3.2 Pricing-to-Market

The model I presented in Section 2 exhibits pricing-to-market in the sense that trade costs and differences in productivities across countries affect prices and mark-ups. The left panel of Figure 3
shows the extent of pass-through on import prices following a 1% depreciation of the Home country currency depending on the trade costs separating the two countries. The figure shows that higher trade barriers induce more (less) productive firms to pass-through a smaller (larger) portion of their cost increase.

The intuition behind this result is as follows. Large trade barriers make exporters less competitive with respect to domestic firms in a market. The largest exporters, who have consistent profit margins, react to this lower competitiveness by reducing prices and mark-ups. Conversely, the smallest exporters cannot afford to reduce their already tiny markups much, and pass through a larger portion of the trade cost.

The relative competitiveness of domestic versus foreign firms may depend on trade barriers but also on cross-country productivity differences. The right panel of Figure 3 shows the extent of pass-through on import prices following a 1% depreciation of the Home country currency depending on the relative average productivity of the two countries. The figure shows that more (less) productive firms reduce their mark-ups more (less) when exporting to a country whose firms are on average more productive. Since the most productive firms in the market are the ones with the largest market shares, on aggregate we should observe smaller price increases into export destinations that are on average more productive. I report evidence giving partial support to this prediction in

Figure 3: Pricing-to-Market: changes in pass-through depending on trade costs and relative productivity across countries.
Section 4. The empirical analysis overall shows that the extent of pass-through indeed depends on the characteristics of the countries involved.

4 Empirical Evidence: Incomplete Pass-Through and Pricing-to-Market in the Cars Industry

In this section I use data from the European car industry to estimate the shape of the pass-through function as a function of firm size. In line with the predictions of the model, pass-through estimates are provided for both import prices and domestic prices. The cross-country structure of the data also allows me to test the predictions of the model regarding pricing-to-market.

4.1 Data

I use a panel data set of car prices assembled by Pinelopi Goldberg and Frank Verboven.\textsuperscript{18} I argue that the car industry is a good laboratory to test the predictions of my model. The small number of competitors in the car industry makes imperfect competition and strategic complementarity in pricing very plausible assumptions.

The data set contains detailed product-level information for car sales in 5 European markets (Belgium, France, Germany, Italy and the UK) over the period 1970-2000, before the introduction of the euro. For each product, or car model, the data record both the selling price and the quantity sold in each destination market and a list of car characteristics, which allow me to disentangle price changes that are not due to quality changes. Moreover, the data include information on both the country of incorporation of the producing firm and the country where the model was effectively produced. I define the origin country as the country where production effectively took place (independently on the country of incorporation of the firm). This is the relevant definition to consider shocks to the exchange rates as shocks that actually distort the relative cost of production

\textsuperscript{18}For a more detailed description of the data, see Goldberg and Verboven (2001, 2005).
between two countries. Based on this definition, there are 14 origin countries: the 5 destination markets plus Spain, Netherlands, Sweden, Japan, Korea, Czech Republic, Yugoslavia, Poland, and Hungary. I concentrate the attention on prices of imported cars in the five destination markets, keeping track of the origin and destination countries of each sale.

4.2 Exchange Rate Pass-through on Import Prices: Specification and Results

Pass-through in the model is a function of both aggregate and firm-level characteristics. At the firm level, as shown in the previous sections, the extent of pass-through depends on a firm’s size in the destination market. I use market share in the destination market as measure of size.\textsuperscript{19} Prices also depend on the aggregate productivity level of the destination country. As a proxy for it, I use real GDP per capita in the destination country (import demand shifter).

In the model, each firm only produces one good, and is identified with it. In the cars industry, however, most firms sell more than one product, so I need to take a stand on what is the relevant level of observation. Anecdotal evidence seems to suggest that a firm may be more or less competitive in a foreign market for some products with respect to others.\textsuperscript{20} For this reason, I identify a firm in the model with a firm-product pair in the data, and run the regressions at the product level.\textsuperscript{21,22}

Based on these considerations, I run the following reduced-form pass-through regression:

\[
\ln(p_{icdt}) = \alpha + \beta_1 \ln(q_{icdt}) + \beta_2 \ln(gdp_{dt}) + \gamma_0 \ln(e_{cdt}) + \gamma_1 \ln(q_{icdt}) \times \ln(e_{cdt}) + \ldots \quad (13)
\]

\[+ \gamma_2 [\ln(q_{icdt})]^2 \times \ln(e_{cdt}) + \delta_t + \delta_{cd} + \delta_{firm} + \delta_i + \delta_{char} + \varepsilon_{icdt}\]

\textsuperscript{19}The results are robust to using quantity sold as a measure of size. With measures of employment and cost of intermediates, one could construct measures of firm productivity such as output per worker or value added per worker. Unfortunately, the dataset does not include this information.

\textsuperscript{20}See The Economist (2008)’s survey on cars in the emerging markets.

\textsuperscript{21}The dataset allows to identify car models that have changed name over time but retained more or less constant characteristics. I treat different denominations of the same model over the years as the same product in the analysis.

\textsuperscript{22}Berman et al. (2012) also run pass-through regressions using firm-product pairs. Chatterjee et al. (2013) extend the framework in Berman et al. (2012) to consider multiproduct firms, and study within-firm, across-products price adjustments following exchange rates appreciations.
where $p_{icdt}$ denotes the price of product $i$, produced in country $c$ and sold in country $d$, in year $t$ and in local currency (importer’s currency), $q_{icdt}$ is the market share in country $d$ of the same product, $e_{cdt}$ is the exchange rate (importer’s currency $d$ per unit of exporter’s currency $c$) in year $t$, $gdp_{dt}$ is the GDP per capita of the destination country in year $t$, $\delta_t$ are year fixed effects (to interpret the results as a pure within estimation), $\delta_{cd}$ are country-pair fixed effects, $\delta_{firm}$ are firm fixed effects, $\delta_i$ are product fixed effects, and $\delta_{char}$ are fixed effects related to cars characteristics. $\varepsilon_{icdt}$ is an orthogonal error term. The role of the fixed effects in the regressions is to control as much as possible for supply-side determinants of prices, so that the estimation of equation (14) can be interpreted as the estimation of a demand function.

Regression (14) generates the following empirical counterpart to the pass-through function shown in the model:

$$\frac{\partial \ln(p_{icdt})}{\partial \ln(e_{cdt})} = \gamma_0 + \gamma_1 \ln(q_{icdt}) + \gamma_2 [\ln(q_{icdt})]^2,$$

where the inclusion of a linear and a quadratic interaction term follows Feenstra et al. (1996) and is meant to capture the nonlinearities in the relationship between pass-through and size that the model predicts.

Table 1 displays the results. Column (I) reports the results of the regression without interaction terms. The coefficient on size is negative, indicating that larger firms tend to charge lower prices, in line with the predictions of the model. The coefficient on GDP per capita is negative, indicating that firm charge lower prices in richer, more productive countries. As expected, the pass-through coefficient $\gamma_0$ is positive and smaller than one. All three coefficients are significant at the 1% level. Figure 4 plots the empirical pass-through function (14) that is implied by the estimates reported in column (I), together with the distribution of firm market shares that is observed in the data.

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23 I use car class (or segment) as the characteristic defining these fixed effects. In the data, cars are grouped in five classes: subcompact, compact, standard, intermediate, and luxury. Previous studies (most notably Feenstra et al. 1996) treated cars as a homogeneous product category. The presence of car characteristics in the dataset I use allows me to compare cars that belong to the same market segment. Moreover, controlling for car characteristics ensures that changes in prices of individual products do not reflect changes in product quality.

24 The results are robust to the use of lagged market share as a measure of size.
Table 1: Pass-through regressions with size-exchange rate interactions (standard errors in parentheses). All specifications include country-pair fixed effects.

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Figure 4: Plot of the empirical pass-through function.

Column (II) reports the results of the regression adding linear and quadratic interaction terms. The common coefficients are significant and similar in size to the previous specification. The
estimates of $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are positive and significant at the 1% level, indicating that pass-through is indeed a U-shaped function of firm size in the destination market. Column (III) shows the results of the same regression adding car class and firm fixed effects, to control for the effects of car characteristics on prices. The signs of all the coefficients are preserved, but the interaction terms lose significance. Column (IV) shows the results of the same regression without firm and car class fixed effects but with product fixed effects. The signs of the coefficients are unchanged with respect to the previous specifications, and the linear and quadratic interaction terms are significant at the 1% level. The results of these different specifications strongly support the prediction of the model about the U-shape of the firm-level pass-through function.

4.3 Exchange Rate Pass-through on Domestic Prices: Specification and Results

In the model, shocks to exchange rates affect not only import prices, but also the prices of goods produced and consumed domestically. A depreciation of the domestic currency implies a non-proportional increase in the price of imported goods. The increase in import prices is correctly anticipated by domestic producers, whose expectations about import prices increase. As a consequence, domestic producers can charge relatively higher prices in their domestic market, as the probability that their prices are lower than the ones charged by their competitors has increased. Hence, a depreciation of the domestic currency determines an increase in domestic prices. Since this is a second-order effect, we expect the increase in domestic prices to be less than proportional to the exchange rate shock, and smaller than the increase in import prices (pass-through on domestic prices is “more incomplete” than pass-through on import prices).

Analytically, let $e_{jct}$ denote the exchange rate between country $j$ and country $c$ (importer’s

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25 The empirical analysis in Feenstra et al. (1996) treats cars as homogeneous goods, due to the unavailability of car characteristics consistently defined across countries. The data assembled by Goldberg and Verboven overcome this problem, allowing to control for car characteristics to disentangle effective mark-up changes from quality differences across different models.

26 An even more restrictive specification of regression (14) would include product-year fixed effects. Unfortunately, the small number of data points in some of the groups that these fixed effects produce prevents me from running this specification.
currency \( c \) per unit of exporter’s currency \( j \) at time \( t \). If \( e_{jt} \) increases, \( c \)’s currency depreciates with respect to \( j \)’s currency. Hence import prices in \( c \), \( p_{jct} \), increase. As a consequence, domestic prices \( p_{ct} \) also increase less than proportionally.

In order to test this mechanism in the data, we need to define a depreciation of a country’s currency with respect to all the other currencies in the economy. To do so, we define an “average exchange rate” for country \( c \) as follows:

\[
e_c \equiv \sum_j \frac{sales_{jc}}{sales_c} e_{jct}. \tag{15}
\]

Each exchange rate \( e_{jct} \) is weighted by the “importance” that firms from country \( j \) have in \( c \)’s market (the share of sales in \( c \) of products from \( j \)).

Equipped with this measure of average exchange rate, we run the following regression:

\[
\ln(p_{icct}) = \alpha + \beta_1 \ln(q_{icct}) + \beta_2 \ln(gdp_{ct}) + \gamma_0 \ln(e_{ct}) + \gamma_1 \ln(q_{icct}) \times \ln(e_{ct}) + \ldots \tag{16}
\]

\[
\ldots + \gamma_2 [\ln(q_{icct})]^2 \times \ln(e_{ct}) + \delta_t + \delta_c + \delta_{firm} + \delta_i + \delta_{char} + \varepsilon_{icct}
\]

where \( p_{icct} \) denotes the domestic price of product \( i \), produced in country \( c \), in year \( t \) and in local currency, \( q_{icct} \) is the domestic market share of the same product, \( e_{ct} \) is the average exchange rate from equation (15) in year \( t \), \( gdp_{ct} \) is the GDP per capita of country \( c \) in year \( t \), \( \delta_t \) are year fixed effects, \( \delta_c \) are country fixed effects, \( \delta_{firm} \) are firm fixed effects, \( \delta_i \) are product fixed effects, and \( \delta_{char} \) are fixed effects related to cars characteristics. \( \varepsilon_{icct} \) is an orthogonal error term.

Table 2 shows the results. Column (I) reports the results of the regression without interaction terms. Like in the regression for import prices, the coefficient on size is negative, indicating that larger firms tend to charge lower prices, in line with the predictions of the model. The coefficient on GDP per capita is also negative. Consistently with the predictions of the model, the pass-through coefficient \( \hat{\gamma}_0 \) is positive and significant at the 1% level, smaller than one, and smaller than the

\footnote{Berman et al. (2012) construct a measure of “multilateral” exchange rate in the same way.}
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<td>2021</td>
</tr>
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</table>

Table 2: Pass-through of exchange rate changes on domestic prices (standard errors in parentheses). All specifications include country fixed effects.

The pass-through coefficient on import prices reported in Table 1, indicating that firms increase their domestic prices following a depreciation of the currency and consequent loss of competitiveness of foreign exporters. Columns (II) and (III) report the results of the regression adding linear and quadratic interaction terms, with and without year, firm and car class fixed effects. In these specifications, the pass-through coefficients are all non-significant. Column (IV) shows the results of the same regression without firm and car class fixed effects, but with product fixed effects. According to this specification, the three pass-through coefficients are all positive and significant, like the ones for pass-through on import prices.

In summary, the results are consistent with the prediction of the model according to which strategic complementarity in pricing implies that also the prices of domestically sold products are affected by exchange rate changes.
4.4 Pricing to Market: Specification and Results

The endogeneity of mark-ups implies that mark-ups differ depending on the countries involved in the trade relationship (pricing-to-market). Within each country pair, Theorem 1 establishes that pass-through should be incomplete, and varying with firm size. Moreover, the numerical examples in Section 3 show that the extent of pass-through is also correlated with productivity differences across countries.

This section addresses empirically the predictions of the model for pricing-to-market. Following Knetter (1989), Knetter (1993), and Goldberg and Knetter (1997), I quantify differences in pricing to market across firms by running the following pricing-to-market regressions:

\[
\ln(p_{icdt}) = \alpha_c + \beta_1 c \ln(q_{icdt}) + \beta_2 c \ln(gdp_{dt}) + \beta_3 c \ln(gdp_{dt}) \times \ln(e_{cdt}) + ... (17)
\]

\[...
+ \sum_{d \neq c} [\gamma_{0cd} \ln(e_{cdt}) + \gamma_{1cd} \ln(q_{icdt}) \times \ln(e_{cdt}) + \gamma_{2cd} (\ln(q_{icdt}))^2 \times \ln(e_{cdt})] + ...
\]

\[... + \delta_t + \delta_d + \delta_{firm} + \delta_{char} + \varepsilon_{icdt}, \forall c.\]

The regression is run separately for each origin country \(c\). The dependent variable \(p_{icdt}\) is the price of product \(i\), produced in country \(c\) and sold in country \(d\), in year \(t\) and in local currency (importer’s currency). \(q_{icdt}\) denotes the market share in country \(d\) of the same product, \(e_{cdt}\) is the exchange rate (importer’s currency \(d\) per unit of exporter’s currency \(c\)) in year \(t\), \(gdp_{dt}\) is the real GDP per capita of the destination country in year \(t\), \(\delta_t\) are year fixed effects, \(\delta_d\) are destination country fixed effects, \(\delta_{firm}\) are firm fixed effects, and \(\delta_{char}\) are fixed effects related to cars characteristics.

The coefficients \(\gamma_{0cd}\) describe pricing-to-market, i.e. the differential response of prices following changes in the exchange rates for each country pair. The model predicts these coefficients to be between zero and one if condition (A.1) holds. Moreover, these coefficients should differ across destination countries \(d\) for every origin country \(c\). The coefficients \(\gamma_{1cd}\) and \(\gamma_{2cd}\) describe how pricing-to-market varies with firm size. Finally, the coefficient \(\beta_{3c}\) quantifies the dependence of
pass-through on the average productivity of the destination market. The model predicts that firms
selling to a relatively more productive (or richer) market should exhibit smaller price adjustments,

hence we expect $\hat{\beta}_{3c} < 0$.

The data include 14 origin and 5 destination countries, but I run the regression only for the
ones for which we have a significant amount of data: France, Germany, Italy, the U.K. and Japan.\textsuperscript{28}

Table 3 displays the results. As the statistic of the Wald test reports, the data confirm a
significant amount of pricing-to-market: the coefficients $\hat{\gamma}_{0cd}$ are all jointly different from zero for
all origin countries except the UK. Once we look at price adjustments by country pairs, the results
are mixed: 11 of the 16 pricing-to-market coefficients are between zero and one like the theory
predicts (and only 6 of those are significant). The remaining coefficients are negative, indicating
no pass-through for certain country pairs. Albeit not completely satisfactory, these results confirm
analogous findings in previous papers (see Knetter 1989). The prediction that pass-through depends
on size finds partial support in the country-pair analysis: 8 out of the 16 coefficients $\hat{\gamma}_{1cd}$ are positive
(and 3 of them significant), while 9 out of the 16 coefficients $\hat{\gamma}_{2cd}$ are positive (and only 2 of them
significant). Finally, the negative relationship between the extent of the price adjustment and the
GDP per capita of the destination country is satisfied in only one out of 5 origin countries. Overall,
the results of the pricing-to-market regressions provide partial support to the predictions of the
theory.

5 Conclusions

I presented a simple two-country model of trade and international price-setting where firms are het-
erogeneous and the world market of each good has the characteristics of an international oligopoly
with unobservable firm-level costs. I have shown that – for a wide range of parameterizations –
firms’ strategic price setting endogenously generates residual demands with an elasticity that is
increasing in the price charged, and hence incomplete pass-through of cost changes into prices and

\textsuperscript{28} The dataset contains more than 1000 observations for each of these origin countries.
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<td>(.013)**</td>
<td>(.018)</td>
<td>(.023)**</td>
<td>(.010)</td>
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Prob > F:
\[ H_0: \gamma_{0cd} = \gamma_{0cd}' \forall d, d' \neq c \]

\[
\begin{array}{cccccc}
\text{Prob} > \text{F} & 0 & .0001 & 0 & .6575 & .0038 \\
\end{array}
\]

\[ R^2 \]

\[
\begin{array}{cccccc}
.998 & .997 & .99 & .996 & .994 \\
\end{array}
\]

No. of obs

\[
\begin{array}{cccccc}
1975 & 1829 & 1308 & 670 & 1873 \\
\end{array}
\]

Table 3: Pricing-to-market regressions (standard errors in parentheses). All specifications include destination country fixed effects, year fixed effects, and firm and car class fixed effects.
Strategic behavior in price setting has three related implications. First, the extent of pass-through and pricing-to-market depends on a firm’s relative size compared to its competitors. The model predicts a U-shaped relationship, where very small and very large firms pass-through a larger portion of an exchange rate appreciation into their export prices. Second, exchange rate fluctuations affect not only the prices of imported goods, but also the prices of goods sold domestically. Third, the extent of pass-through at the firm level depends on the characteristics of the destination market. I tested the predictions of the model using a panel data set of cars prices in five European markets. The estimates broadly support the predictions of the theory.

I believe that this paper contributes to the literature on incomplete pass-through and pricing-to-market in two ways. First, the model deepens our understanding of the possible channels that drive these phenomena, by providing a novel structural framework where trade flows and market shares are endogenous and depend on firms’ strategic considerations. Second, by supporting the specific predictions of the structural model, the empirical analysis improves our understanding of the relationship between firm size, pass-through, and pricing-to-market.

References


33


Appendix

A Proofs

This section contains the proofs of Theorem 1 and Corollary 1.

**Theorem 1.** The elasticity of demand $|\varepsilon_{jk}|$ is increasing in the price charged if and only if the competitors’ cost distribution $G_{\sim k}(\cdot)$ satisfies:

$$g'(z) > -\frac{g(z)}{z} \left[1 + \frac{g(z)}{1 - G(z)} z \right] \quad \forall z. \tag{A.1}$$

**Proof:** The proof proceeds in two steps. I first use an auxiliary, simplified model to derive condition (A.1), and then show that condition (A.1) is also the necessary and sufficient condition for the full model.

Let us consider an auxiliary model where firms in one of the two countries (without loss of generality, say $h$) set prices equal to their marginal costs, while firms in the other country ($f$) set mark-up prices. The two countries are identical under every other characteristic, so wages are equalized and normalized to one. Goods are freely tradeable ($t = 1$). We prove that – for this auxiliary model – (A.1) is a necessary and sufficient condition for incomplete pass-through of changes of marginal costs into import prices. If $p_{jh} = z_h$, for $j = h, f$, the elasticity of import demand for the Home country ($|\varepsilon_{hf}|$) reduces to:

$$|\varepsilon_{hf}| = \eta + \frac{g_h(p_{hf}(z_f))}{1 - G_h(p_{hf}(z_f))} p_{hf}(z_f). \tag{A.2}$$

Then condition (A.1) follows from differentiation of equation (A.2) with respect to $p_{hf}(z_f)$.

Condition (A.1) holds with equality when the cost distribution $G(\cdot)$ is a Pareto.\(^{29}\) So to ensure that the elasticity of demand is increasing in the price charged we need the density $g(\cdot)$ to exhibit

\(^{29}\) $G(z) = 1 - \left(\frac{a}{z}\right)^{-\vartheta}$ for $z \geq a$, $a > 0$, and $\vartheta > 0$.  

36
a larger first derivative than a Pareto on its entire domain.

Now that I established the result for the auxiliary model, I move to consider the full model, where firms from both countries charge mark-up prices, there may be also arbitrary wage differences and possibly non-zero trade costs. By differentiating (9) with respect to $p_{jk}(z_k)$, we obtain:

$$f'(p(z)) > -\frac{f(p(z))}{p(z)} \left[ 1 + \frac{f(p(z))}{1 - F(p(z))} p(z) \right] \forall p(z) \quad \text{(A.3)}$$

where the country indexes have been suppressed to ease the notation. When the cost distribution $G(\cdot)$ is Pareto with shape parameter $\vartheta$ and location parameter $a$, one can solve analytically for the optimal pricing rule, which in this case is linear in the marginal cost:

$$p_{jk}(z_k) = \frac{\vartheta + \eta}{\vartheta + \eta - 1} c_{jk} z_k. \quad \text{(A.4)}$$

When unit costs are Pareto-distributed, optimal prices are also Pareto-distributed over the support $[amc_{jk}, \infty)$, where $m$ is the constant mark-up in (A.4): $m = \frac{\vartheta + \eta}{\vartheta + \eta - 1}$. Moreover, the elasticity of demand is constant ($|\varepsilon_{jk}| = \eta + \vartheta$) and condition (A.3) holds with equality. Hence also for the full model the Pareto distribution is the cutoff separating the set of distributions implying incomplete pass-through from the others. Conditions (A.1) and (A.3) are characterized by the same cutoff, hence (A.1) is sufficient to characterize the set of distributions implying incomplete pass-through also for the full model. \((q.e.d)\)

**Corollary 1.** If the survival function $[1 - G(z)]$ is log-concave $\forall z$, then condition (A.1) holds and the elasticity of demand is increasing in the price charged.

**Proof:** The survival function $[1 - G(\cdot)]$ is log-concave if and only if the following inequality holds:

$$g'(z) > -\frac{g(z)^2}{[1 - G(z)]} \forall z$$

which implies that inequality (A.1) also holds. \((q.e.d.)\)
B An Algorithm to Solve for the Pricing Rule

In this Section I illustrate how to compute optimal prices in the model.

The pricing rules $p_{jk}(z_k)$, for $j, k = d, f$, are the solution of the following system of first order conditions:\footnote{The system (B.1) is derived from problem (7).}

$$p_{jk}(z_k)(1 - \eta) + \eta c_{jk}z_k - p_{jk}(z_k)[p_{jk}(z_k) - c_{jk}z_k] \left[ \frac{f_{j, \sim k}(p_{jk}(z_k))}{1 - F_{j, \sim k}(p_{jk}(z_k))} \right] = 0, \quad (B.1)$$

for $j, k = d$. Notice that system B.1 can be solved in pairs, \textit{i.e.}, the two equations for $j = d$ are independent from the two equations for $j = f$. For each $j$, I solve numerically the system with the following algorithm:

1. Given an initial vector of productivities $z_k$ for each country $k$, guess an initial pricing rule $p^0_{jk}(z_k)$ for each origin country $k$ to destination country $j$.
2. Compute $F^0_{jk}(p^0_{jk}(z_k))$.
3. Given $F^0_{jk}(\cdot)$, find the pricing rules $p^1_{jk}(z_k)$ (one for each origin country $k$) that solve (B.1).
4. If $p^1_{jk}(z_k) - p^0_{jk}(z_k) < \varepsilon$, $\forall$ $k$, STOP.
   
   If $\exists$ $k$ such that $p^1_{jk}(z_k) - p^0_{jk}(z_k) \geq \varepsilon$, compute $F^1_{jk}(p^1_{jk}(z_k))$, and continue iterating until convergence.

If a solution exist, it must lie between the marginal cost and the monopolistic competition pricing rule. I use the marginal cost to initialize the algorithm.

C Aggregation

In this section I discuss how to aggregate individual prices to investigate the extent of incomplete pass-through of aggregate shocks into aggregate prices.
On this matter, it is important to observe that we cannot use ideal price indexes to measure the effect of aggregate shocks on aggregate prices. By construction, the CES ideal price indexes take into account the equilibrium expenditure shares in each of the goods, so a variation in the individual prices is accompanied by the corresponding equilibrium variation in demand. Moreover in this model a change in price may induce a change in the acquisition option: consumers may switch to imports after an increase in the domestic price, and this switch is taken into account by the ideal price index. In order to isolate the change in aggregate prices while keeping quantities and acquisition choices constant, I adopt an aggregation method that mimics the construction of the CPI index.\footnote{I refer to these new price aggregates as the “actual” price indexes.} I refer to these new price aggregates as the “actual” price indexes.

The actual price index for goods produced in country $k$ and sold in country $j$ ($j, k = h, f$) is:

\[
\hat{P}_{jk} = \frac{1}{\sum_{jk} p_{jk}(z_k) s_{jk}(z_k) g(z)} \int_{B_{jk}} p_{jk}(z_k) s_{jk}(z_k) g(z) dz
\]

(C.1)

where $B_{jk}$ is the set of goods that a consumer in country $j$ buys from a producer in country $k$:

\[
B_{jk} = \{(z_d, z_f): p_{jk}(z_k) = \min\{p_{jd}(z_d), p_{jf}(z_f)\}\} \text{ for } j, k = h, f.
\]

(C.2)

The term $s_{jk}(z_k)$ is the expenditure share in country $j$ of a good bought from a country $k$ producer

\footnote{The CPI index contains two levels of aggregation. At the lower level, individual prices are collected into item-area indexes through a geometric average:

\[
\pi_i^t = \prod_{j=1}^{N_i} \left( \frac{p_{jt}}{p_{j(t-1)}} \right)^{\frac{s_i}{N_i}}
\]

where $p_{jt}$ is the price of good $j$ at time $t$ in the item-area $i$ and $N_i$ is the number of collected prices in item-area $i$. At the upper level, item-area indexes are aggregated into the CPI index by:

\[
\Delta CPI_{b,t} = \sum_i s_i \pi_i^t
\]

where $s_i = E_i^t / \sum_i E_i^t$ is the expenditure share in item-area $i$ in the base year $b$. In my calculations, I abstract from the lower level aggregation and implicitly assume $N_i = 1 \forall i$, so that the item-area index is just equal to the individual price variation.}
with cost $z_k$:

$$s_{jk}(z_k) = \frac{P_{jk}(z_k)q_{jk}(z)}{\int_{B_{jk}} p_{jk}(z_k)q_{jk}(z)g(z)dz} \left[ 1 - F_{j, \sim k}(p_{jk}(z_k)) \right].$$

(C.3)

The expenditure share $s_{jk}(z_k)$ is given by the expenditure share conditional on consumers from country $j$ buying from country $k$ ($s_{jk}(z_k)|\Pi_{JK}$) times the probability that consumers from country $j$ buy from $k$ ($\text{prob}\{\Pi_{JK} = 1\}$). The term $S_{jk}$ in equation (C.1) is the total expenditure share in country $j$ on goods bought from country $k$:

$$S_{jk} = \frac{P_{jk}q_{jk}}{P_jq_j} = \frac{\int_{B_{jk}} p_{jk}(z_k)q_{jk}(z)g(z)dz}{P_jq_j}.$$ 

(C.4)

Changes in actual price indexes are computed aggregating individual price changes, but assuming that the price changes do not affect the expenditure shares and the acquisition option. I denote with $\hat{P}_{jk}$ the percentage variation in the actual price index $\tilde{P}_{jk}$:

$$\hat{P}_{jk} = \frac{1}{S_{jk}} \int_{B_{jk}} \hat{p}_{jk}(z_k)s_{jk}(z_k)g(z)dz$$

(C.5)

where $\hat{p}_{jk}(z_k)$ is the percentage variation in the individual price $p_{jk}(z_k)$.