Network Size and Network Capture

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Abstract:

Most types of networks, over time, spawn the creation of complementary stocks that enhance network value. Computer operating systems, for example, induce the development of the complementary stock of software applications that increase the value of the operating system. In this paper, we challenge the conventional wisdom that a large network, which induces the creation of large complementary stocks, serves as a barrier to entry that protects the incumbent from competition or network capture. We show that a larger network may either deter or attract entry depending on the relation between the network quality and the cost of an innovator's network product. The probability of entry also depends on the level of compatibility between the potential entrant's technology and existing complementary stocks, which in turn is influenced by the strength of the intellectual-property-rights environment. Intellectual property rights and the associated threat of entry may affect an incumbent's choice of network size in counterintuitive ways.

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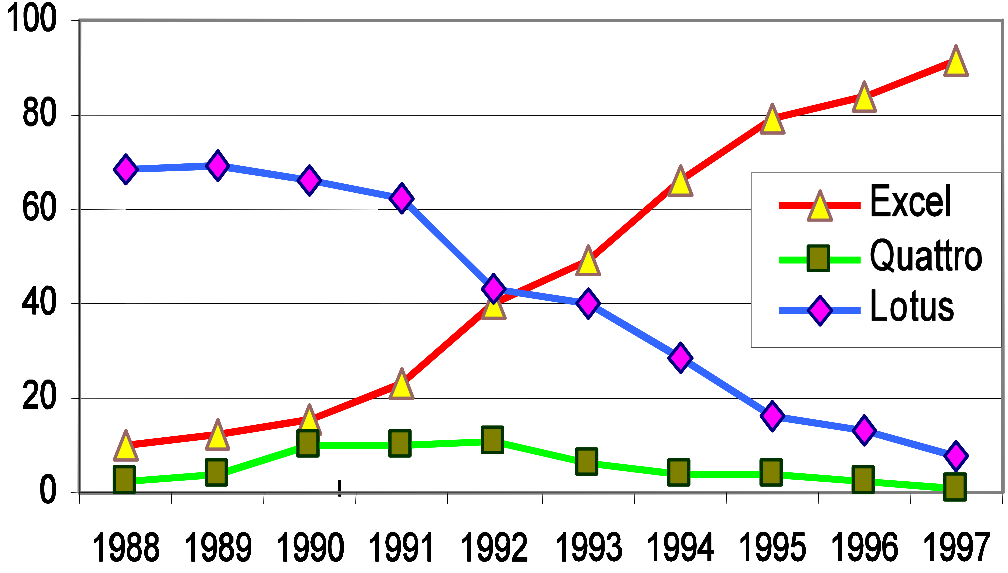
> ...business strategy has to go far beyond the usual adages of costs down, quality up, core competency. High tech adds a new layer of complication.... You want to build up market share; you want to build up user base. If you do, you can lock in that market.—*Joel Kurtzman, Strategy & Business (1998) cited by Liebowitz (2002).*

1 Introduction

A network product is a good or service whose value to one consumer is enhanced by other consumers' use of the same or similar products. The value enhancement is called a network effect. Similar network products that embody different technologies are often characterized by vertical product differentiation: most consumers tend to rank the quality of the products the same way. For that reason, network products of a given type tend to be dominated by a single technology. If the technology is proprietary, the associated network may well be controlled by a single firm. Moreover, technological innovations may allow an entrant to capture an entire network from an incumbent.

Figure 1. Spreadsheet Market Shares



Liebowitz and Margolis, 2001, p. 174

Spreadsheet applications provide a classical example of network capture. In the 1980's, Lotus 123 captured the spreadsheet market from Visicalc, the spreadsheet originator. Then in the early 1990's, Excel captured the market from Lotus, as Figure 1 from Liebowitz and Margolis (2001, p.174) so clearly demonstrates. In the recording industry, VHS (Matsushita *et al*) eliminated Betamax (Sony) and the compact disc (Phillips/Sony) took over from long-playing records (a standard not controlled by a single firm). More recently, in the internet-browser market, the

Microsoft Internet Explorer pushed out Netscape, and now it remains to be seen whether or not Mozilla Firefox will make inroads against the Internet Explorer.

The rapid transition between one network and another is usually referred to as "tipping." Tipping is produced in part by vertical network-product differentiation, but network effects add positive feedback to the tipping process and intensify it. This is because the value of a network product increases as more and more users adopt it. Tipping behavior is well documented in network markets. Evans and Schmalensee (2001) distinguish tipping-induced competition *for* the market in dynamic industries from ordinary competition *in* the market. Liebowitz and Margolis (2001) provide many examples of tipping behavior aside from the spreadsheet market: they document the rapid transitions in markets for word processors, personal-finance applications and internet browsers. The tipping phenomenon is consistent with the idea developed in Farrell and Katz (2001) that firms in these markets are "temporary monopolists."

Incumbents may use many devices to reduce the probability of network capture by an entrant: intellectual-property rights, marketing, market leverage (bundling), pricing schemes for differentiated services and long-term contracts with consumers (Aghion and Bolton, 1987). In this paper, however, we focus on the incumbent's use of his network size as a means to influence entry and avoid tipping. We also analyze the role played by compatibility between a potential entrant's technology and the incumbent's. Finally, we examine the welfare consequences of network size and the means that a social planner might use to control it.

Sometimes, as with telephone service, network effects operate contemporaneously. In other cases, as with many software applications, network effects appear with a lag. A lag is created when the network product spawns the creation over time of what we call *complementary stocks,* which may be understood as network-related capital that increases network value. Computer operating systems, for example, induce the development of the complementary stock of software applications that increase the value of the operating system. Likewise, franchise networks induce the growth of a base of loyal customers, and credit-reporting networks gradually develop a database of information about borrowers. Both the customer base and the database serve as value-enhancing complementary stocks. Complementary stocks are often the primary source of positive network effects.

In this paper we focus on competition between incumbents and potential entrants for the control of networks whose network effects are entirely mediated by complementary stocks. The incumbent can influence the rate of complementary-stock creation by varying the size of his network. The availability of complementary stocks, in turn, modifies the incentives of potential entrants and alters the probability of entry. Analysis of this mechanism and resulting social welfare forms the core of the model presented below. In general, complementary stocks have social value, because they increase the utility that consumers derive from a network product. Consequently, a regulator will be concerned with how the threat of entry affects the size of the stock that an incumbent creates. Do complementary stocks protect an incumbent from entry? Or do they instead yield a valuable asset to a competitor that makes entry more likely? The level of protection that a regulator ought to grant the incumbent depends on the answer to these questions.

We shall view network products as having two distinct quality characteristics, *stand-alone quality* and *network quality*. Stand-alone quality determines the value of a network product to a consumer when the product is used in isolation from its complementary stocks. For example, the stand-alone quality of a spreadsheet application measures the value that the application creates for a consumer, independent of his access to the spreadsheets of others. Network quality measures how effectively a product utilizes its complementary stock. The network quality of a spreadsheet application measures the value created for the consumer when he uses his application to access the existing stock of spreadsheets of another users of the same network product. The larger the complementary stock of spreadsheets, the greater the value that can be obtained from a spreadsheet application of a given network quality.

The cost characteristics of a network product are also important in the competition for network control. A higher stand-alone quality of a network product, on the one hand, and a lower cost of producing it, on the other, will have similar effects in its competitive position. Indeed, in our model, the effects will be precisely the same. Consequently, the stand-alone quality variable that is defined in the model should be viewed as reflecting a combination of quality and costs attributes.

Different network products are likely to engender the development of complementary stocks with different characteristics. The concept of network quality reflects the relation between a network product and the corresponding complementary stock. But we shall also be concerned with the relation between a network product with a new embodied technology and the complementary stock engendered by the previous technology. This is captured by the notion of *compatibility*.

In order to compete with an incumbent, a potential entrant's network product must embody innovations that yield either a higher stand-alone quality or a higher network quality. In the spreadsheet case, for example, new software might have advantages in stand-alone use, or might facilitate collaboration between different users, or both. However, a potential entrant may have an important disadvantage as compared to the incumbent in that the new network product may be less than fully compatible with the existing complementary stock. Limitations on compatibility may arise from technological hurdles or from intellectual property rights, among other things. If entry is to occur, the quality improvements in the potential entrant's product will have to be sufficient to compensate for limited compatibility. Indeed, in some cases, a new network product may be completely incompatible with the existing complementary stock however high its network quality may be.

In the quotation that introduces this paper, Joel Kurtzman (1998) states that when it comes to high-tech network products, producers can lock in the market by building up their user base (a complementary stock). This is a reflection of the widely-held view that a large network serves as a powerful barrier to entry. A primary goal of this paper is to demonstrate that this view is frequently incorrect—the truth is far more complex. We explore two forces that work in the opposite direction, and which, at times, may make a small complementary stock desirable for an incumbent. First, if the entrant's network-quality improvement is sufficiently high, a large complementary stock may well attract the entrant rather than discourage him. In the case that this eventuality is probable, an incumbent may desire to have a small network and a small complementary stock. Second, the incumbent's development of a large complementary stock is an investment whose return is dependent on the incumbent's continued domination of the market. A substantial threat of network capture by a potential entrant lowers the expected return to investment in the complementary stock and makes it less profitable. In fact, these two forces acting together may induce a forward looking monopolist to create a complementary stock that is even smaller than what he would create if network capture by an entrant were certain.

A change in the compatibility level of an entrant's network product might also have counterintuitive implications. A reduction in the entrant's compatibility increases the incumbent's ability to appropriate rents from the stock he generates. One might think that the increased appropriability would induce the incumbent to create a larger network and associated complementary stock. This is not always true. In situations described above in which the incumbent reduces network size in order to deter entry, reduced compatibility makes that deterrence more effective. Therefore, as we demonstrate below, reduced compatibility might motivate the incumbent to deter entry even more by further reducing network size.

This paper builds on two strands of the literature on network effects. Starting in the 1980s, a number of papers explored the decision of consumers to adopt new technologies for products subject to network externalities. Farrell and Saloner (1985,1986) and other papers showed that under reasonable conditions consumers tended to inefficiently delay adoption of new and incompatible technologies, behavior referred to as "excess inertia." At about the same time, Katz and Shapiro (1985) studied the static competition among producers of partially incompatible products.

A substantial number of papers have considered the consequences of lagged network effects on the pricing policies of monopolists. For example, Bensaid and Lesne (1996) showed that network effects helps to mitigate the Coase conjecture when a monopolist cannot commit not to lower the price in the future. Similarly, Cabral et al.(1999) provided conditions under which a monopolist wants to provide an introductory price for his product.

2 Complementary Stocks

Networks with complementary stocks form an important segment of modern economies. A computer operating system is a case in point. A large inventory of software applications is a complementary stock that adds great value to the use of the operating system itself. Moreover, the more clients who use an operating system, the greater the incentive to write applications for it. Operating systems have a second complementary stock as well: the stock of expertise among computer professionals and technically minded computer users. When a user of an operating system needs help, he can often find colleagues to advise him, provided that the operating system has been widely used for a sufficient period of time.

The network effects generated by complementary stocks tend to be linked to past rather than current network size. With respect to the network of a computer operating system, the availability of both software applications and expertise is an increasing function of past network size, which means that current network effects and past network size are positively correlated. This is not to deny that an operating system may have contemporaneous network effects as well: if two people are using the same operating system, it may be easier for them to pass documents back and forth. But there can be little doubt that contemporaneous network effects in this case pale in importance in comparison with those linked to the past. As Judge Thomas P. Jackson notes:

> The main reason that demand for Windows experiences positive network effects ... is that the size of Windows' installed base impels ISVs [Independent Software Vendors] to write applications first and foremost for Windows, thereby ensuring a large body of applications from which consumers can choose. The large body of applications thus reinforces demand for Windows, augmenting Microsoft's dominant position and thereby perpetuating ISV incentives to write applications principally for Windows.[1]

In the credit-reporting-agency example, member financial institutions use the agency network to obtain reports on loan applicants, but they also submit information on their experience with applicants that have received loans. The larger the membership, the faster the database will grow in size and the more valuable network membership will be in the future. A member financial institution will not be concerned with current membership size, but rather with the size of the database (the complementary stock), which, in turn, correlates with past membership size.

Credit cards and other payment cards provide a somewhat different example of networks associated with complementary stocks. In this two-sided market, we have two interacting networks: the network of credit-card users, managed by the issuing banks, and the network of merchants who accept the cards, managed by the acquiring banks. Each network represents a complementary stock for the other. When a merchant decides to accept a credit card, she is interested in the size of the complementary stock of users of that card. When a consumer decides to carry a credit card, she is interested in the number of merchants who accept it. The size of the complementary stock of credit card users is associated with the previous size of the merchant network; likewise for the complementary stock of merchants.

When a new network product accesses the complementary stock created by a pre-existing network product, its effective network quality may be reduced by incompatibility with the pre-existing product. Varian and Varian (2003) provide a good practical example of what we have in mind in this regard. In order to measure the compatibility of open-source office applications with the stock of Microsoft Office documents, they imported a sample of office documents from the

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[1] Judge Thomas Penfield Jackson, *Findings of Fact*, the State of New York et al. v. Microsoft (Civil Action No. 98-1233, 1999).

internet and attempted to open them with appropriate applications. Analysis of resulting errors enabled them to classify each of the applications by their degree of compatibility, which ranged from a low of 71 percent to a high of 99 percent.

3 The Model

The primary goal of the paper is to analyze how an incumbent monopoly with a network product can use its control of the complementary stock to restrict entry by competitors with improved technologies. To that end, we model two firms: the incumbent (Firm i) and the potential entrant (Firm e), and we confine economic activity to two periods, $t = 1, 2$. Consumers of the network product are uniformly distributed with density 1 along a line $[0, \infty)$, where $s \in [0, \infty)$ denotes a consumer's location. Consumers of unit mass buy at most one unit of the product every period. Network products are located at 0, and a consumer at s must pay a transportation cost s to obtain a unit of the product.

In period 1, Firm i is a monopolist with 1 unit of the complementary stock, stand-alone quality 0 and network quality 1. The utility in period 1 of a consumer who buys the incumbent's network product is

$$U_1^i(s, p) \equiv 1 - s - p, \tag{1}$$

where p is the incumbent's first-period price and the constant term 1 is the product of the network quality and the complementary stock. Consumers who do not buy from the monopolist have utility 0. Therefore, demand (and output) in period 1 is given by $x = 1 - p$. The marginal consumer is located at $s = x$.

In period 2, the network product will have a complementary stock y_2 given by

$$y_2 = \rho x, \tag{2}$$

where ρ describes the relationship between past production and current complementary stocks. If a consumer buys from the incumbent in period 2, his utility will be

$$U_2^i(s, y_2, p_2^i) \equiv y_2 - s - p_2^i, \tag{3}$$

where p_2^i is the incumbent's second-period price. Measurement of the complementary stock is normalized so that each unit of y_2 adds one unit of utility to the incumbent's product.

Firm e, a potential entrant, can offer a technology in period 2 that is improved in two respects as compared to that of the incumbent: the new product incorporates a network advantage v and a stand-alone advantage u. The stand-alone advantage should be interpreted to include not only improvements in stand-alone quality in the narrow sense, but also decreases in unit production costs. (One can think of a drop in production costs as a stand-alone advantage that is passed on to consumers in the form of a discount subtracted from the Firm e's official price.)

Notwithstanding the quality improvements, the entrant's network product may not be completely compatible with the complementary stock created in conjunction with the incumbent's technology. The contribution of the existing complementary stock to a consumer's valuation of the entrant's network product is defined by a compatibility parameter $\delta \in [0,1]$. If a consumer buys the entrant's network product in period 2, his utility is

$$U_2^e(s, y_2) \equiv u + (1+v)\,\delta y_2 - s - p_2^e, \tag{4}$$

where p_2^e is the entrant's second-period price.[2] As in the first period, a consumer gets 0 utility if he doesn't buy at all.

A utility-maximizing consumer at s who has the choice between the incumbent's and the entrant's product would always be indifferent between them when the price difference is equal to the overall quality difference, as given by

$$p_2^e - p_2^i = u + (\delta v - (1-\delta))\,\rho x. \tag{5}$$

The consumer would prefer to buy from the entrant when the price difference is smaller than the quality difference and would prefer to buy from the incumbent when the price difference is larger. Notice that the preference of the consumer is independent of his location (identity) s. This means that all consumers have the same relative ranking among products, a market characteristic called vertical product differentiation. Moreover, our model has the stronger built-in feature that the difference in the willingness-to-pay for the two products is the same for all consumers. As a result, when the incumbent and entrant attempt to sell in the same market, all consumers will tend to buy from the same one of them. This captures the tendency that leads to tipping.

The entrant's quality-advantage parameters u and v are drawn from jointly distributed random variables with density function $f(u, v)$, where u can take any real value and v can take any value with $v \geq -1$ (negative network effects are ruled out). At the beginning of period 2, u and v are realized an become common knowledge. Both the incumbent and the entrant have zero fixed costs. Without loss of generality, the incumbent's marginal cost is normalized to zero, and the entrant has constant marginal cost that is embodied in u.[3]

We construct a three-stage game between the incumbent and a potential entrant, and we search for a subgame-perfect equilibrium. The game proceeds as follows:

Stage 1. The incumbent chooses a first-period price p for his network product. (This determines

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[2] If u represents a cost decrease (passed on as a discount) rather than a stand-alone quality increase, the same utility function may be more suggestively expressed by

$$U_2^e(s, y_2) \equiv (1+v)\,\delta y_2 - s - (p_2^e - u)\,.$$

[3] If the entrant has a marginal-cost advantage over the incumbent, then, because the incumbent's cost is normalized to 0, the entrant's cost would be negative. However, precisely the same results are obtained with this normalization as would be obtained by setting the incumbent's cost to a positive value.

his first-period network size x and the size of the complementary stock y_2 in the second period.)

Stage 2. At the start of the second period, the potential entrant observes y_2 and realizations of his stand-alone (or cost) advantage u and his network advantage v. He then decides whether or not to enter. For simplicity, we assume he does not enter when he is indifferent between entering and not entering.[4]

Stage 3. If entry has not occurred, the incumbent, still a monopolist, chooses p^i_{2m} without the pressure of competition. Otherwise, the entrant and incumbent engage in Bertrand competition.

3.1 When Entry Has Not Occurred

In this subgame, the incumbent remains a monopolist. He chooses a second-period price that maximizes profits for that period. Demand is generated by the set of consumers that would obtain positive utility in the second period from buying the product. The location x_2 of the marginal consumer, or equivalently, the quantity demanded, is given by the solution of $U(s, y_2) = 0$ for s. Applying (2), we have

$$x_2 = \rho x - p_2.$$

Therefore, given that marginal cost is 0, second-period profits are

$$\pi^i_{2m}(x) = \max_{p_2}\ p_2(\rho x - p_2) \tag{6}$$

so that optimal monopoly price is $p^i_{2m} = (\rho x)/2$, the optimal quantity is $x^i_{2m} = (\rho x)/2$, and profits are $\pi^i_{2m}(x) = (\rho x)^2/4$. As is easy to see, the output of the monopolist in the second period increases both with his first period production and with the intensity of the network effects.

3.2 When Entry Has Occurred

Suppose x, u and v have been observed and entry has occurred. A new subgame is defined. Bertrand competition between the two firms will determine the equilibrium price. There are three possible regimes depending on the overall quality differential.

Case 1: Positive Overall-Quality Differential. If the right-hand side of (5) is positive, then the entrant's product has higher overall quality, and Bertrand competition yields a unique equilibrium in which the incumbent sets a price of 0 and all consumers buy from the entrant. The entrant's equilibrium price is constrained by the right-hand side of (5), and that expression defines the penetration price:

$$p^e_{2p} \equiv u + (\delta v - (1-\delta))\,\rho x.$$

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[4] This is equivalent to postulating the existence of an arbitrarily small entrance fee, which is then ignored.

If the entrant's monopoly price

$$p^e_{2m} \equiv (u + (1+v)\,\delta\rho x)\,/2, \tag{7}$$

is less than the penetration price, then in equilibrium he charges p^e_{2m}, sells the quantity

$$x^e_{2m} = (u + (1+v)\,\delta\rho x)\,/2$$

and accrues profits

$$\pi^e_{2m} = \left(\frac{u + (1+v)\,\delta\rho x}{2}\right)^2.$$

Otherwise the entrant charges his penetration price in equilibrium, sells the quantity

$$x^e_{2p} = \rho x,$$

and accrues profits

$$\pi^e_{2p} = u\rho x + (\delta v - (1-\delta))\,\rho^2 x^2. \tag{8}$$

The equilibrium choice between monopoly and penetration pricing in this case depends on the size of u and v. If u and v satisfy

$$v \geq \left(\frac{2-\delta}{\delta}\right) - \frac{u}{\delta\rho x}, \tag{9}$$

then the entrant has what Arrow (1962) labelled a "drastic innovation," which allows him to ignore the presence of the incumbent in setting his profit-maximizing price.

Case 2: Negative Overall-Quality Differential. If the right-hand side of (5) is negative, Bertrand competition also yields a unique equilibrium in which the entrant sets a zero price, has zero sales and earns zero profits. The incumbent charges either his limit price or his monopoly price, whichever is lower, and earns positive profits.

Case 3: Equal Overall Quality. Finally, if the right-hand side of (5) is zero, Bertrand competition yields a unique equilibrium in which both firms charge 0 and earn zero profits.

3.3 The Entry Decision

By construction, entry occurs if and only if the potential entrant can earn strictly positive profits in the second-period equilibrium. Therefore, in any subgame-perfect equilibrium with entry, Case 1, above, will prevail, the right-hand side of (5) will be positive, and only the entrant will have sales in Period 2.

Firm e's potential profits depend on u and v as follows:

$$\pi^e_2(v,u) = \begin{cases} u\rho x + (\delta v - (1-\delta))\,\rho^2 x^2 & \text{for} \quad v + \dfrac{1}{\delta\rho x}u < \dfrac{2-\delta}{\delta} \\[2ex] (u + (1+v)\,\delta\rho x)^2\,/4 & \text{for} \quad v + \dfrac{1}{\delta\rho x}u \geq \dfrac{2-\delta}{\delta} \end{cases},$$

where the first line of the function definition (lower product quality) corresponds to penetration pricing and the second line (higher product quality) to monopoly pricing.

Given u, entry will occur if and only if $v > \bar{v}$, where $\bar{v}$ satisfies $\pi_2^e(\bar{v}, u) = 0$. But it is easy to verify that $\rho^2 x^2 > 0$ forms a lower bound for profits in the monopoly pricing range whereas 0 forms a lower bound for profits in the penetration-pricing range. This means that marginal entrants must lie in the region of penetration pricing. It follows that $\bar{v}$ must be the solution for v of

$$0 = u\rho x + (\delta v - (1-\delta))\,\rho^2 x^2,$$

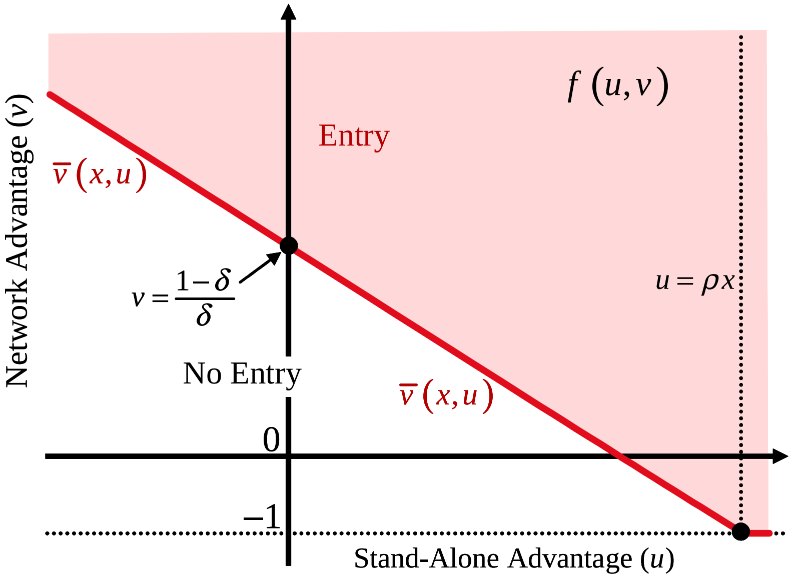
which yields

$$\bar{v}(x,u) = \frac{1-\delta}{\delta} - \frac{u}{\delta \rho x}. \tag{10}$$

We have:

Lemma 1 *For any past network size $x > 0$, the innovating firm will choose to enter if and only if $v > \bar{v}(x,u)$ defined by (10). Furthermore, the entrant will employ penetration pricing if $\bar{v} < v < \frac{2-\delta}{\delta} - \frac{u}{\delta\rho x}$ and monopoly pricing otherwise.*

Figure 2. Entry Decision



We shall be interested in how the probability of entry is affected by the first-period network size and the degree of compatibility. The probability Φ of entry is the function of x given by

$$\Phi(x) = \Pr_{v,u}[v > \bar{v}(x,u)]\,.$$

If $f(v,u)$ represents the joint density of continuous random variables v and u, then $\Phi(x)$ is given by

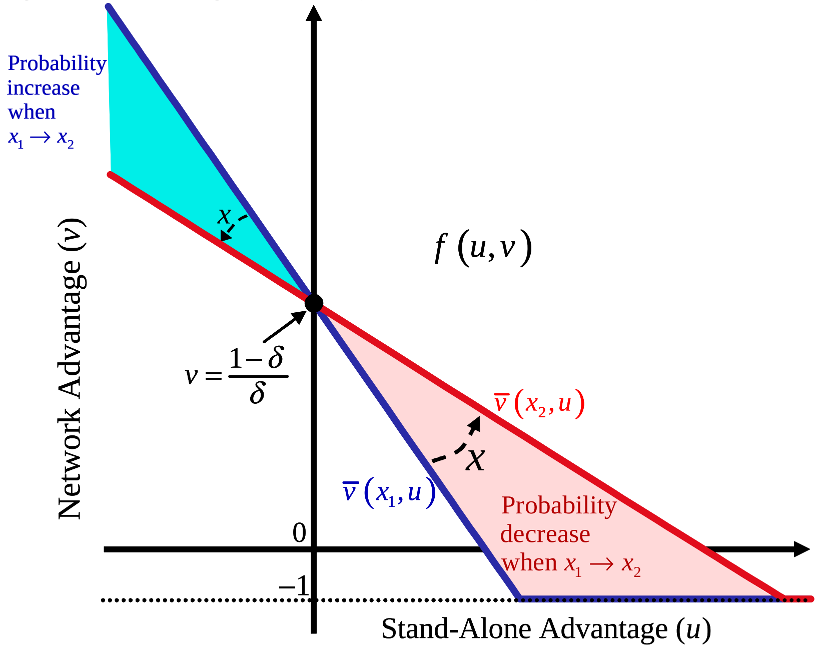
$$\Phi(x) = \int_{-\infty}^{\infty} \int_{\bar{v}(x,u)}^{\infty} f(v,u)\, dv du, \tag{11}$$

which yields

$$\Phi'(x) = -\frac{1}{\rho\delta x^2}\int_{-\infty}^{\infty} u\ f\left(\bar{v}(x,u)\,,u\right)du. \tag{12}$$

The range of integration in (11) is the area above the $\bar{v}$-line, as illustrated in Figure 2, whose axes measure u and v. From (10) it is clear that the line is negatively sloped and rotates through $\frac{1-\delta}{\delta}$ on the v-axis, going from the vertical when $x = 0$ towards the horizontal as x increases. As shown in Figure 3, the value of $\Phi'(x)$ depends on the rate that area is swept out by the $\bar{v}$-line as x changes and by the probability density $f(u,v)$ near the line.

Figure 3. Change in Entry Probability with Network Size



We will analyze the probability of entry in more detail below, but from the previous expression it is evident that if sufficient probability weight is on positive values of u, the probability of entry decreases as the previous network size x is increased. This is because a preponderance of marginal entrants have a stand-alone or cost advantage over the incumbent and an overall network disadvantage. Conversely, if enough probability weight is on negative values of u, for example when increasing network quality entails increasing cost, the probability of entry increases with x. Here the preponderance of marginal entrants have an overall network advantage but a stand-alone or cost disadvantage. Therefore, to the extent that a positive stand-alone or cost advantage is a likely outcome of innovation, large network size is likely to serve the incumbent as a barrier to entry; in the opposite case, though, large network size actually tends to induce entry—the large complementary stock forming a target of opportunity.

We next turn to the first period decision that the incumbent will make regarding the first-period price p and the corresponding network size x.

3.4 The First Period

In the first period, the incumbent is a monopolist in the market with a complementary stock normalized to 1. The firm chooses p so as to maximize the sum of first and second period profits.

Network size or consumer demand in the first period is $x = 1-p$, but for convenience we consider incumbent's first-period choice variable to be x rather than p. The incumbent is a monopolist in the first period, and remains in business (as a monopolist) in the second period with probability $1 - \Phi(x)$. Therefore, the sum of expected profits for the incumbent in both periods as a function of the first-period network size is

$$\pi_i(x) = x(1-x) + (1-\Phi(x))(\rho x)^2/4. \tag{13}$$

The incumbent's problem is now

$$\max_x \pi_i(x). \tag{14}$$

The derivative of profits with respect to first period network size is

$$\pi_i'(x) \equiv (1-2x) + (1-\Phi(x))\frac{\rho^2 x}{2} - \Phi'(x)\left(\frac{\rho x}{2}\right)^2, \tag{15}$$

where $1-2x$ represents the incumbent's marginal revenue in the first period (marginal cost is zero), $1-\Phi(x)$ is the probability that the incumbent will retain the network in the second period, and $\Phi'(x)$ represents the sensitivity of the probability of entry to the first-period network size.

The incumbent's first-order condition for the optimal first-period network size is

$$\pi_i'(x) = 0. \tag{16}$$

Below we solve this equation for specific distributions of u and v.

4 Network Size and the Threat of Entry

How does the threat of entry affect the incumbent's choice of first-period network size? We analyze two effects that are conceptually distinct: the *entry discount effect* and the *entry-deterrence effect.* A positive probability of entry reduces the expected value of the complementary stock to the current incumbent in the following period, because the incumbent may not be present to utilize it. The incentives created, which we call the discount effect, tend to reduce the size of the first-period network.

In addition, changes in the size of the complementary stock tend to change the probability of entry. The direction of that change will depend on the probability distribution of possible innovations. If a large stock increases the probability of entry, the incumbent will have an incentive to reduce first-period network size; if a large stock reduces the probability of entry, the incentive will go in the opposite direction. We call this the deterrence effect.

4.1 The Discount Effect

Suppose now that the joint distribution $F(u,v)$ is such that the probability of entry is a constant given by $\Phi(x) \equiv \alpha$ for all relevant x. This happens, for example, when the support of F is restricted to $u = 0$. The incumbent's problem becomes

$$\max_x \left\{ x(1-x) + (1-\alpha)(\rho x)^2 /4 \right\},$$

and his profit-maximizing value of first-period network size is

$$x_\alpha = \frac{2}{4 - (1-\alpha)\,\rho^2}.$$

When $\alpha = 1$, entry is certain and the incumbent behaves myopically and maximizes first-period profits by setting

$$x = x_s \equiv 1/2. \tag{17}$$

When $\alpha = 0$, entry will not occur, so the incumbent maximizes his two-period profits by setting

$$x = x_m \equiv \frac{2}{4-\rho^2}. \tag{18}$$

Here, the incumbent is sacrificing short-term monopoly profits in order to create a larger complementary stock for use in the second period.

As the probability of entry increases parametrically, α increasing from 0 to 1, x_a falls from x_m to x_s. The change in x is entirely induced by the changing discount effect of potential entry on optimal network size: a positive probability of entry causes the incumbent to discount future profits and put more weight on his first-period profits. As a result the incumbent reduces his network size further below the perfectly competitive level than his two-period monopolistic position would indicate. We will see that this effect can change markedly when the probability of entry is sensitive to network size.

4.2 The Deterrence Effect

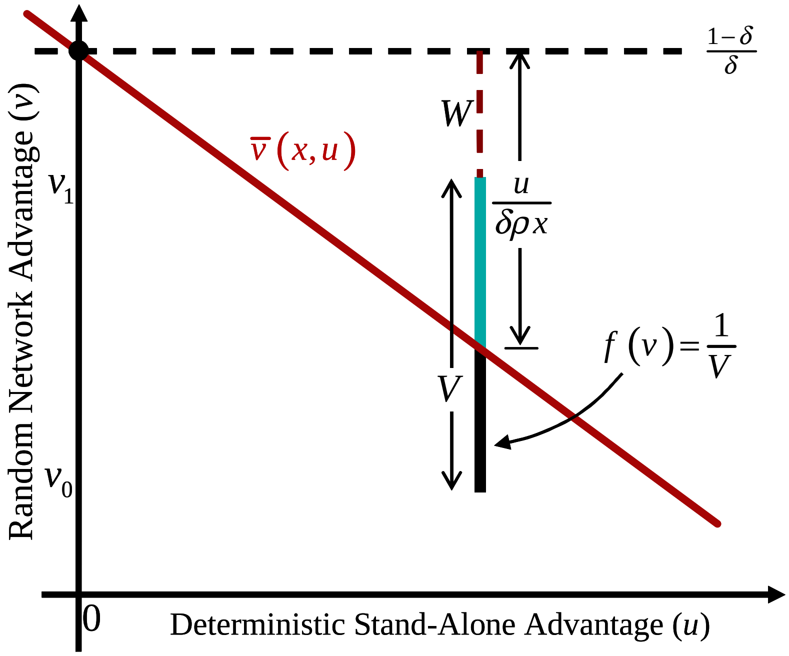
We have already noted that when positive stand-alone advantage is sufficiently likely, an increase in the size of the incumbent's first-period network will tend to deter entry. In general, the incumbent will be induced to modify the first-period network size in order to reduce the probability of entry. We call this modification the *deterrence effect.*

As we show below, the deterrence effect in some cases may be strong enough to counteract the discount effect and induce the incumbent to push first-period network size above the two-period monopoly optimum x_m. In other cases, the deterrence effect tends to reinforce the discount effect and may push the value of x to a level even below that of the single-period monopoly x_s.

4.3 Deterministic Positive Stand-Alone Advantage with Uniformly Distributed Network Advantage

Suppose now, as in Figure 4, u takes a deterministic value u and v is uniformly distributed on the interval $[v_0, v_1]$. In this subsection, we are assuming that u is positive, as when innovation yields a cost reduction.

Figure 4. Uniformly Distributed Network Advantage



When $0 \le \bar{v}(x,u) \le 1$, the probability of entry is given by

$$\Phi(x) = \frac{1}{V}\left(\frac{u}{\delta\rho x} - W\right),$$

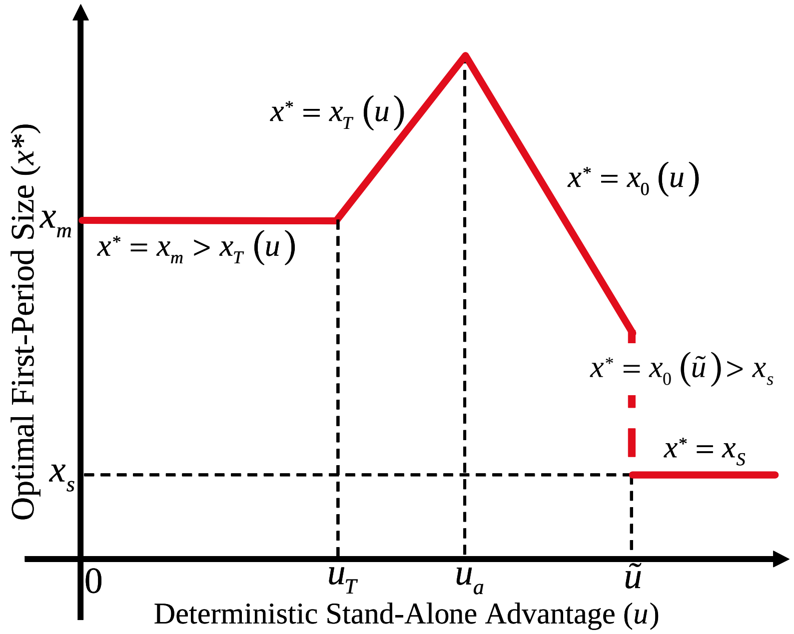
where $V \equiv v_1 - v_0$ and W is the vertical distance between the intercept of $\bar{v}(x,u)$ and the support $[v_0, v_1]$, defined by

$$W = \frac{1-\delta}{\delta} - v_1.$$

Although not illustrated in Figure 4, W can be zero or negative, as well as positive. The ratio W/V lies in the interval $(-\infty, -1)$, $[-1, 0]$ or $(0, \infty)$ depending on whether the support of v is entirely above, partly above and below, or entirely below the intercept formed by $\delta/(1-\delta)$. In the first of these situations, entry is certain, and in the first period, the incumbent will behave as a one-period monopolist. So we will analyze the model with $W/V \in (0, \infty)$, and then explain why $W/V \in [-1, 0]$ can be understood as special case of that result.

It turns out that the optimal first-period network size as a function of u is a discontinuous piecewise linear function with four regions as illustrated in Figure 5. In the appendix, we derive the function segment by segment. The discontinuity and the kinks in the graph are an artifact of the uniform distribution of v that we have adopted—the transitions would be gradual given a distribution with full support. However, the descriptions of the regions presented below would remain applicable.

Figure 5. Optimal First-Period Size with Vertical Support



For u in a neighborhood of 0 and $W > 0$, v_1 is not large enough to overcome the entrant's lack of perfect compatibility with the existing complementary stock, so entry can never occur and the incumbent can maximize profits simply by producing x_m in the first period. However, as u increases past u_T, the entrant's positive stand-alone or cost advantage enables him to compensate in part for his network disadvantage. The incumbent can defend against this by increasing his first-period network size—and thus his complementary stock—which magnifies the importance of his own overall network advantage as compared with the stand-alone advantage of the entrant.

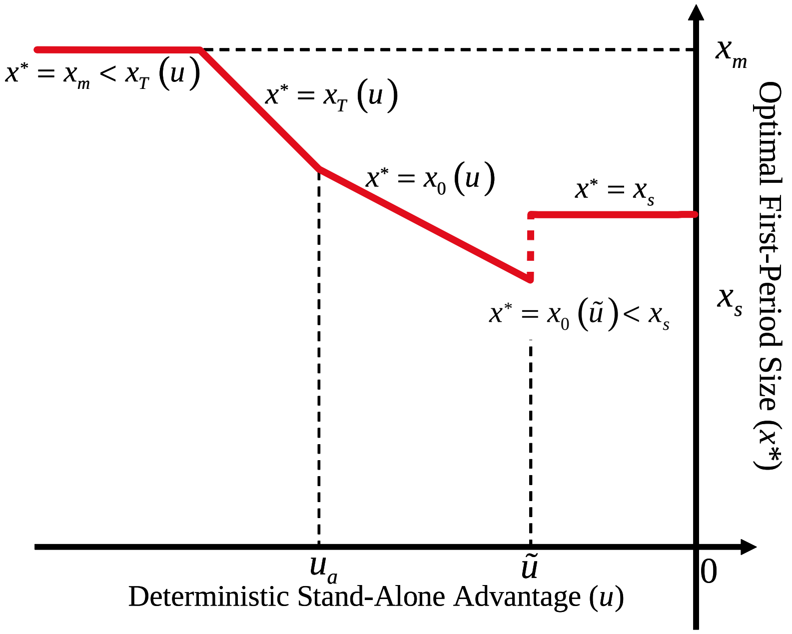
But when u becomes greater than u_a it no longer pays the incumbent to deter entry completely. Instead he partially accommodates to a positive probability of entry and avoids an even higher probability of entry by setting x larger than he otherwise would. Here the deterrence effect and the discount effect coexist. However, when u increases past $\tilde{u}$, it no longer pays to sacrifice first-period profits to achieve limited deterrence. At this point, the incumbent discontinuously reduces x to x_s, the one-period monopoly output level, and entry becomes certain.

In the foregoing, we assumed $W > 0$. But for $W/V \in [-1, 0]$ (that is for $W \leq 0 < V + W$) the description in Figure 5, still applies, but beginning beyond u_a. There is a positive probability of entry for any $u \geq 0$, because there may be entrants with a technology that dominates that of the incumbent in both dimensions.

4.4 Deterministic Negative Stand-Alone Advantage with Uniformly Distributed Network Advantage

Suppose now that u takes a deterministic negative rather than a positive value u, as when the increased network quality of the entrant comes at an additional cost. Here, the network advantage v of the potential entrant has to overcome the cost disadvantage implied by $u < 0$. Figure 6 illustrates the optimal network size when v remains uniformly distributed on an interval $[v_0, v_1]$ but where $v_0 > (1-\delta)/\delta$ so that $W < -V < 0$. By comparing this figure to Figure 5, it is evident that as u moves from left to right on the horizontal axis (algebraically increasing) the regions of u and x encountered change in parallel to those for $u > 0$. We explain each segment of the graph but omit supporting calculations for the sake of brevity. As before, the discontinuity and kinks are the result of the uniform distribution.

Figure 6. Optimal First-Period Size with $u < 0$



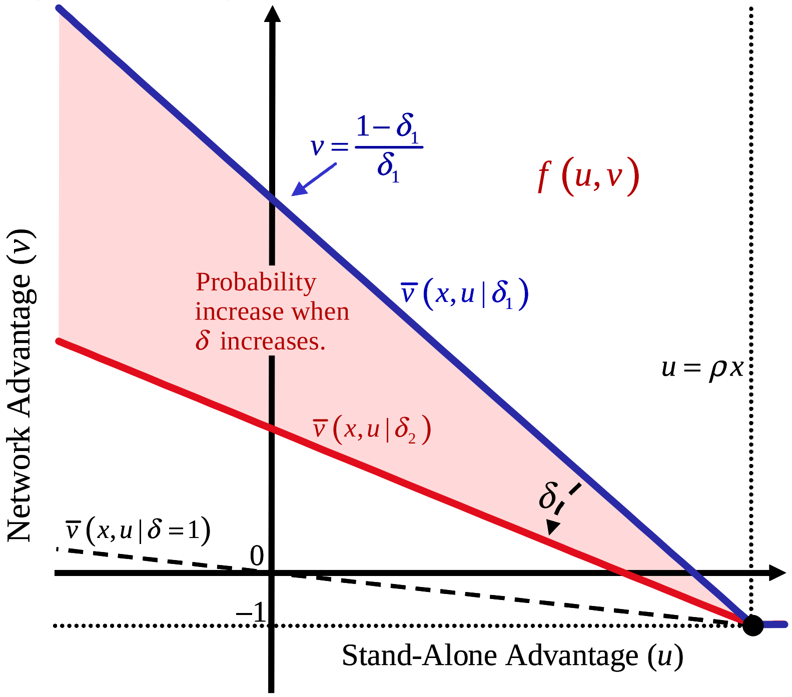
At the left of Figure 6, for u sufficiently negative, v_1 is not large enough to overcome the entrant's cost disadvantage, so entry can never occur, and the incumbent can maximize profits by producing x_m in the first period. However, as u increases past u_T, the entrant's network advantage may be sufficient to compensate in part for his higher cost. The incumbent can defend against this by decreasing his first-period network size—and thus his complementary stock—which reduces the importance of the entrant's overall network advantage.

But when u becomes greater than u_a it no longer pays the incumbent to deter entry completely. As opposed to the previous case, the deterrence effect (as well as the discount effect) motivates the incumbent to decrease the size of his first-period network. For $u > u_a$, the discount effect decreases x and the deterrence effect forces x down further, even below the incumbent's optimum for a one-period monopoly. However, when u increases past $\tilde{u}$, it no longer pays to sacrifice first-period profits to achieve limited deterrence. At this point, the incumbent discontinuously increases x to x_s, the one-period monopoly quantity, and entry becomes certain.

4.5 The Degree of Compatibility: Comparative Statics

If the complementary stock is completely incompatible with entrant's network product ($\delta = 0$), then the entrant will not be able to break into the market without sustaining losses. The only exception to the rule occurs when stand-alone utility of the entrant's product is as large as the utility provided by incumbent's product and complementary stock (i.e. when $u \geq \rho x$). With perfect compatibility, however, the entrant will be able to capture the market whenever $u, v \geq 0$.

Figure 7. Change in Entry Probability as δ Increases



From (10) and Figure 7 we see that the probability of entry increases monotonically as δ increases parametrically, and in most cases, the discount effect will be sufficiently strong so as to decrease incumbent's first-period size. But this is not always true. For positive values of u associated with a zero probability of entry, the discount effect may not operate, and the incumbent's response to an increase in δ, would be to increase network size as a way to deter all entry; in other words, only the deterrence effect would operate. It is also true that for $u < 0$, if $x < x_s$ as a result of the deterrence effect, then an increase in δ might cause the incumbent to accommodate certain entry and increase x to x_s.

5 Welfare

In this section we explore the social-welfare implications of the competition between the incumbent and a potential entrant. We apply numerical methods to formulas presented in previous sections to calculate expected producer and consumer surplus as a function of the compatibility level δ. Because δ is directly influenced by the intellectual-property law and other economic-policy measures, we treat δ as the policy parameter of interest for welfare determination.

In the first period welfare arises only from the production of the incumbent monopolist. In the second period, given the size of the complementary stock generated during the first period, expected welfare depends on the producer and consumer surplus that would be generated by each firm, weighted by the probabilities of incumbent survival and entry.

Figure 8 displays the values of four different variables as a function δ. For this graph, we set the other parameters as follows: $\rho = \frac{4}{5}$, a deterministic $u = \frac{1}{3}$ and v uniformly distributed in the $-\frac{1}{2}$ to $\frac{1}{2}$ interval. When δ is varied parametrically with u held fixed, the determination of the entry probability passes through four regimes (as with varied u and fixed δ). The regimes change in the following sequence as δ goes from 0 to 1:

1. **Entry blocked.** The incumbent ignores the threat of entry and chooses the dynamic monopoly output x_m, and as a consequence, the probability of entry is in fact 0. When the level of compatibility δ is sufficiently small, the entrant cannot compete with the incumbent dynamic monopolist at any v in the support. Welfare is independent of δ in this range.

2. **Entry fully deterred**. It pays the incumbent to modify his behavior in the first period so to force the probability of entry to 0. With $u > 0$, the incumbent accomplishes that by increasing first period size beyond that of dynamic monopoly. When δ is somewhat larger, entry would occur for sufficiently large v unless the incumbent modifies his behavior in this fashion to benefit from his network advantage over the marginal would-be entrant. In this range, with positive u, welfare increases with δ. This is not because a more compatible entrant is ever realized, but rather because the incumbent employs a more efficient first-period network-size[5] as a reaction to the increased threat of entry.

3. **Entry partially accommodated.** The incumbent accepts a positive probability of entry; the first-period network size reflects both the discount and deterrence effects. As compatibility δ continues to increase, the incumbent is induced to reduce his first-period network size, since the incumbent finds decreasingly profitable to deter entry completely. The reduced first-period network size tends to lower welfare by decreasing the complementary stock. However, increasing compatibility increases the likely advantage of potential entrants, so that the probability of entry increases. The increased entry probability increases welfare for two

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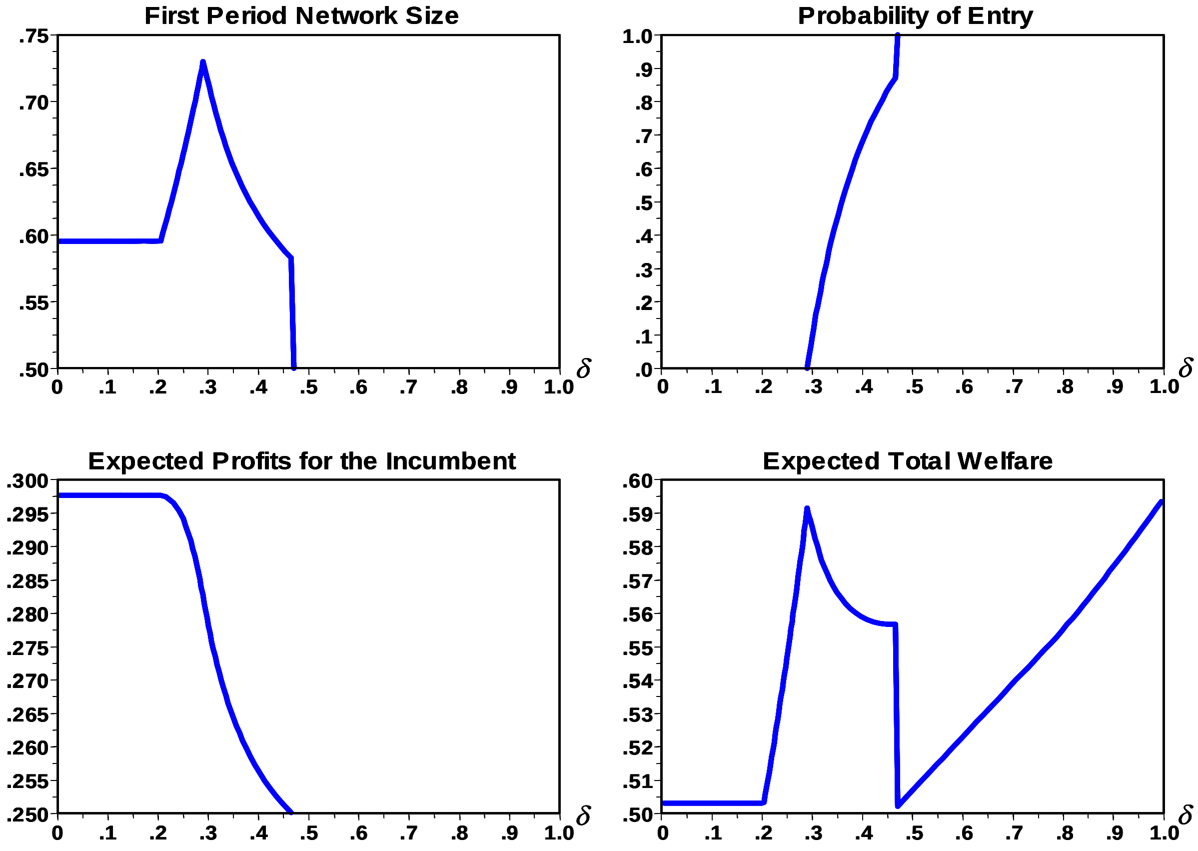
[5] We ignore the remote possibility that the incumbent would enlarge the network even beyond the first-best size.

Figure 8. Welfare Simulation



reasons. First, the average entrant has a higher overall technological quality than the incumbent. Second, entry creates competition in the second period which gives rise to an increase in the number of consumers served. With the parameters specified for Figure 8, the welfare-decreasing effect is dominant for the lower values of δ in the range, but the welfare-increasing effect catches up for the higher values of δ, which creates the convex segment in the welfare graph.

4. **Entry certain.** The incumbent accepts certain entry, and adjusts his first-period network size to that of a one-period monopoly. The downward adjustment of the first-period network size creates an initial fall in welfare, which, in the case of the uniform distribution of v, is abrupt. Unlike the previous regime, the probability of entry, already at 1, is not increasing; however, increasing compatibility is increasing the average overall technological improved offered by entrants. The latter effect causes welfare to increase to its maximum after its initial drop.

The numerical example we provided illustrates an important general principle. To understand it, we must first note that because of network effects, the functions that relate network characteristics to welfare tend to be convex, so that optimal outcomes tend to be extreme. In this paper, we have focussed on two sources of network-related welfare changes connected to the level of compatibility, both of which are convex in network attributes.

First, the level of compatibility foreseen by the incumbent, affects the size of the complementary stock he creates. The larger the complementary stock, the more valuable the network product will be in the second period and the more of it will be sold. Together, these two factors make welfare a convex function of the compatibility level.

The second source of welfare modification arises from the link between compatibility and quality. The higher the level of compatibility, the higher will be the effective quality of an entrant's product and the greater the probability that entry will occur. Here, too, welfare is a convex function of the compatibility level.

As our model demonstrates, the social planner will sometimes be in a position of trading off the two effects of compatibility, one against the other. The fact the welfare effects are both convex implies that the resultant net effect will be convex as well, though the welfare result would smooth out if we had used distribution functions with full support. The important point is that the convexity of the welfare as a function of compatibility could lead a social planner to take compatibility to one extreme or the other. At one extreme the social planner promotes welfare by choosing a low level of compatibility, which deters entry and induces the incumbent to create a larger complementary stock. At the other extreme, the planner promotes welfare by choosing a high level of compatibility, which increases the probability of entry and the effective quality of the entrant's product. Which of these two extreme compatibility options the social planner selects will depend on the distribution of the quality attributes of potential entrants and on the extent of network effects.

These principles ought to have empirical implications. If we look at different jurisdictions around the world, we would expect to see some with weak intellectual property rights that permit a high level of compatibility and a high rate of network capture. In such jurisdictions networks would tend to be small. Elsewhere, we would expect to see jurisdictions with strong intellectual property rights, which restrict the compatibility of entrants' products and yields a low rate of network capture. Here, networks would tend to be large.

6 Is a Complementary Stock an Essential Facility?

The analysis of the preceding sections can be reinterpreted in light of what is known in legal circles as the "doctrine of essential facilities." That doctrine was established by the courts in order to prevent monopolies from using their control of facilities required for one stage of a production process as a weapon to stifle competition in upstream or downstream markets.

Consider the following example. A textile firm owns the only bridge crossing a nearby river. The capacity of the bridge is many times larger than that needed by the textile firm to transport its own inputs and outputs. For that reason, the firm allows many other businesses to use the bridge upon payment of a toll. But when a new competitor in the textile business asks to use the bridge, the owner denies it permission. The courts in the US or Europe might well declare the bridge to be an essential facility and enjoin the owner to allow use of the bridge on a nondiscriminatory basis.[6] Similarly, ports, pipelines and power grids are often cited examples of essential facilities.

Not all monopoly-controlled facilities can be deemed essential facilities. If an entrant can procure or construct an alternative facility at a cost comparable to that paid by the incumbent, lack of access to the incumbent's facility would not place the entrant at a competitive disadvantage. The US 7th Circuit Court of Appeals (1983) summarized the necessary conditions for the application of the essential-facilities doctrine as follows:

> A monopolist's refusal to deal....may be unlawful because a monopolist's control of an essential facility (sometimes called a "bottleneck") can extend monopoly power from one stage of production to another, and from one market into another. Thus, the antitrust laws have imposed on firms controlling an essential facility the obligation to make the facility available on non-discriminatory terms.
>
> The case law sets forth four elements necessary to establish liability under the essential facilities doctrine: (1) control of the...facility by a monopolist; (2) a competitor's inability practically or reasonably to duplicate the...facility; (3) the denial of the use of the facility to a competitor; and (4) the [economic] feasibility [from the owner's point of view] of providing the facility.[7]

Although the essential-facilities doctrine is a controversial one among lawyers,[8] we think that it is a useful concept for understanding the economic implications of the control of complementary stocks. Application of the doctrine requires that a competitor cannot "practically or reasonably" reproduce the facility. Such inability is often linked to indivisibilities inherent in the construction of the facility in question, as would be the case with ports or power grids. However, in the case of complementary stocks from which network effects arise, there tends to be indivisibilities in the usage rather than in the construction of the stocks. This is because the stock as a whole contributes to the utility of the network product, rather than just a portion of the stock commensurate with the output of the product. A small credit-card issuer benefits from having a large stock of stores that accept its card.

The essential-facilities doctrine can be applied to intellectual property as well as to physical property. The European Commission, for example, held that information enabling the interoperability of Microsoft Windows with software applications should be treated as an essential facility

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[6] This example is a much simplified version of the U S v. Terminal Railroad Ass'n of St. Louis 224 U.S. 383 (1912), the case in which the US Supreme Court first enunciated the doctrine of essential facilities.

[7] MCI Communications Corp. v. AT&T Co., 708 F.2d 1081 (7th Cir. 1983), cert, denied, 464 U.S. 891 (1983). A similar doctrine was established by the European Commission after cases such as Magill TV Guide O.J. L 78/43 (1989), Tierce Landbroke v. Commisssion Case T-504/93 (1997), Bronner v. Mediaprint, Case C-7/97 (1998).

[8] For a review of the legal literature, see Kezsbom and Goldman (1996).

(see Lévêque 2005). In the case of NDC Health vs. IMS Health, however, the European Commission ultimately ruled against applying the doctrine to a proprietary database structure owned by the defendant.[9]

Although legally distinct, the "genericness doctrine" of trademark law embodies the same concept as the essential facilities doctrine. The owner of a trademark normally has the intellectual property right to prohibit its use by others. But when a trademark comes into common usage as the principal descriptor of a class of products rather than as simply a brand designation, the trademark becomes an essential facility for marketing any product in the class. Aspirin, brassiere, cellophane, cube steak, dry ice, escalator, gold card, gramophone, kerosene, lanolin, light beer, linoleum, mimeograph, nylon, raisin bran, shredded wheat, superglue, thermos, trampoline, yo-yo and zipper are all examples of former trademarks that lost their legal protection.

The model we have presented can be conceptualized as a description of the endogenous choice of the size of the complementary stocks for network products. In selecting the size of a complementary stock, its creator can be expected to concern himself with its influence as an essential facility on the entry of potential competitors. This dynamic aspect of essential facilities is less likely to arise with indivisibilities in construction, where cost efficiency rather than the entry probability tends to determine the facility's size.

Some facilities may be effectively controlled by a firm that doesn't own them. Control may be exercised through franchise contracts, warranty agreements or through the control of inputs. General Motors sells automobiles only through franchised dealerships, which are not permitted to sell the products of competing manufacturers. A competing manufacturer would be forced to find other automobile dealers. Equipment manufacturers are often able to maintain monopoly control over consumable inputs for their equipment, even though the equipment itself is owned by others.

The complementary stocks of a network product frequently consist of facilities that are controlled but not owned. In the example of General Motors above, the network of dealerships is a complementary stock for the network of new General-Motors car owners. This is because an increased density of dealerships increases the car owner's convenience in obtaining service.

Another important difference between complementary stocks and traditional essential facilities is that control of the traditional facilities is primarily determined by property rights or contractually conferred rights, whereas control of complementary stocks may also depend on their technological

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[9] IMS held a near monopoly in what it is called the *1860 brick structure*, a segmentation of Germany into 1860 geographical areas that has since served as the industry standard. IMS used this segmentation to provide data on prescriptions dispensed by pharmacies organized by these areas. This service induced other firms to develop auxiliary databases also keyed to the brick structure. Thus, the prescription data became a network product whose value was increased by the many compatible auxiliary databases, which then acted as a complementary stock. But IMS controlled access to the complementary stock through their ownership of the brick structure.

NDC, a competitor of IMS in the provision of prescription data, asserted in a hearing before the European Commission that the 1860 brick structure [and, consequently, the complementary stock] was an essential facility for the production of their services. The cost of adaption to a new data structure, they argued, would be too high for it to be economically viable. They asked the court to deny IMS the right of exclusive use of the brick structure (COMP D3/38044 - NDC Health/IMS Health: Interim Measures). Ultimately, the commission ruled against NDC Health.

compatibility with the entrant's network product.

When a facility becomes essential because of indivisibilities in construction, regulators must choose between allowing or denying access. Access to intellectual property tends to be more a matter of degree. When a complementary stock is an essential facility, a competitor's cost of access can be controlled by intellectual property-right policies. Existing websites, for example, constitute the complementary stock for web-browsing software. So, suppose that the regulator grants broad patent protection for an incumbent's internet protocols. This might make it difficult and expensive for a potential entrant to obtain a reasonable degree of compatibility with websites established according to the incumbent's specifications. Such considerations are at the core of the controversy over the application of patent protection rather than copyright protection to software.

Despite the differences between traditional essential facilities and complementary stocks, two things can be learned from the comparison of the two cases. First, network industries can be understood as posing a problem of vertical relationships, in the sense that complementary stocks are an input that nurtures a downstream product. This is quite similar to the example of the bridge that acts as an essential input for textile production.

Second, and also important, is that the standard remedy used in the case of essential facilities can be applied to complementary stocks as well. Courts often require the owners of essential facilities to access pricing: payments accepted by the owner of the facility in return for its use. Access pricing could also be applied to the use of complementary stocks. In the context of complementary stocks, access pricing would take the form of a royalty for each unit sold by the entrant in return for the right of full compatibility with the network built by the incumbent. Such royalties could be structured in the context of compulsory licensing.

The idea that a complementary stock should be treated like any other privately held asset is mistaken, we think. Complementary stocks typically have features that place them squarely in the camp of essential facilities. In the realm of monopoly-controlled networks, economic efficiency has two important aspects: the size of the network created by the incumbent and the scope for the development of improved technologies. Our model focused on the relation between policies governing the treatment of network complementary stocks and the size of the networks that incumbents create. Policies that address the essential-facilities aspect of complementary stocks are important to the creation of networks of efficient size.

7 Conclusion

This paper demonstrates that the economics of the strategic use of network size is complex. Simplistic notions, such as the idea that a large network gives the incumbent an almost insurmountable advantage over potential entrants, are often misleading or just plain wrong. Our results would be more dramatic if they led to strong generalities about welfare-maximizing policies for network regulation. Unfortunately, they do not. The model suggests only that policies that protect incumbents and promote large networks, on the one hand, and contrary policies that facilitate entry and technological advancement, on the other, may each increase welfare more than policies that compromise between these two objectives.

Economists normally conceive of network products as goods or services that have important contemporaneous externalities: the more widespread is the use of the product, the more valuable it is to each user. But we introduced the concept of complementary stocks, which highlights the importance of lagged network effects. Indeed, we would argue that because of the almost universal presence of stocks that complement network products, lagged network effects are more widespread and more significant economically than contemporaneous effects are. In the presence of lagged effects, the entry of a competitor depends not only on the degree of innovation, customer switching costs and coordination issues, but also on the size of the previously established network. This fact, in turn, gives rise to the strategic use of network size, a primary focus of the paper.

We have explained why the doctrine of essential facilities is frequently applicable to complementary stocks, most especially when those stocks have the form of intellectual property. As intellectual property, complementary stocks tend to be characterized by an indivisibility of use, which reduces economic efficiency when an entrant is required to create complementary stocks of his own. An important task of a regulating authority is to influence the ability of a potential entrant to appropriate the complementary stocks created by others. On the one hand, if an entrant can appropriate complementary stocks too easily, a forward looking incumbent would be reluctant to invest in their creation. On the other hand, if it is too costly or difficult for an entrant to make his product compatible with the incumbent-generated stocks, then potential entrants would be discouraged from appropriate investments in network innovation and may be reluctant to enter the market even when if would be efficient to do so.

We have introduced a number of concepts that relate to the incumbent's strategic use of network size. We characterized innovations in network products as a pair of improvements in stand-alone quality or cost and in network quality. The benefits that consumers derive from increments in network quality are positively correlated with the size of the complementary stock (the lagged network size) whereas benefits from increments in stand-alone quality or from cost reductions are independent of the complementary-stock size. In our analysis, the prospect of superseding innovations affects the behavior of the incumbent through two channels: the discount effect and the entry-deterrence effect. The former always induces the incumbent to strategically reduce current network size (and that of the complementary stock), but the latter can operate in either direction. To the extent that an incumbent expects the innovations of potential entrants to be relatively concentrated in the area of network quality, the two effects reinforce each other and will induce the incumbent to restrict network size, possibly even below the level that would be selected by a myopic monopolist. Instead, if the incumbent expects the innovations of potential entrants to be relatively concentrated in the area of stand-alone quality or cost, the deterrence effect will oppose and might even dominate the discount effect and induce the incumbent to expand the size of the network beyond the level that he would choose if there were no threat of entry.

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A Appendix: Characterization of the Optimal Network Size

Here we characterize the optimal first-period network size in the case of a positive stand-alone advantage, $u > 0$. The results are illustrated in Figure 5. A similar procedure can be used to characterize the optimal network size when $u < 0$ as displayed in Figure 6.

From Figure 4, we can see that the line defined by $\bar{v}(x, u)$ intersects the top of the support of v for x given by

$$x_T(u) = \frac{u}{\delta \rho W},$$

and the bottom of the support of v for x given by

$$x_B(u) = \frac{u}{\delta \rho (W + V)}.$$

This means that $\bar{v}(x, u)$ intersects the support of v whenever $x \in [x_B(u)\,,\; x_T(u)]$.

Let x^* denote the incumbent's optimal first-period network size. If $u \leq u_T \equiv \delta \rho W x_m$, then x_m, the optimal first-period network size for the two-period monopoly, exceeds $x_T(u)$ and completely deters entry, so that $x^* = x_m$.

Let u_a be given by

$$u_a \equiv \frac{2\delta \rho W}{4 - \rho^2 \left(1 + \frac{1}{2}\frac{W}{V}\right)}. \tag{19}$$

Then for $u \in (u_T,\; u_a)$, $\bar{v}(x_m, u)$ intersects the interior of the support of v and π_i defined by (13) has the property that $\pi_i'(x) > 0$ for all $x \in [x_B(u)\,,\; x_T(u)]$. This means that profits are maximized at $x^* = x_T(u)$ (because $x_m < x_T(u)$, the incumbent would not want to increase x beyond the support of v).

If $u \geq u_a$, then the incumbent will maximize profits at one of two values of x: either at x_0 defined by $\pi_i'(x_0) = 0$ or at $x_s = \frac{1}{2}$, the static monopoly value of x. The solution for u of the equation $\pi_i(x_0) = \pi_i(x_s)$ is given by

$$\tilde{u} = \frac{\delta V}{\rho}\left(4 - 2\sqrt{4 - \rho^2\left(1 + \frac{W}{V}\right)}\right). \tag{20}$$

Then for $u \in [u_a,\; \tilde{u})$, $\pi_i(x_0) > \pi_i(x_s)$ so that $x^* = x_0$, which is given by

$$x_0(u) = \frac{2 - \dfrac{\rho u}{2\delta V}}{4 - \left(1 + \dfrac{W}{V}\right)\rho^2}. \tag{21}$$

For $u > \tilde{u}$, $\pi_i(x_0) < \pi_i(x_s)$ and $x^* = x_s$ (for $u = \tilde{u}$ both x_0 and x_s are optimal).

We have the following proposition:

Proposition 2 *For a range of values of u the deterrence effect dominates the discount effect and the incumbent produces more in the first period than he would in the absence of a potential entrant. In particular, for $W > 0$, we have $x_m < x^* = x_T(u)$ when $u \in \left(\dfrac{2\delta\rho W}{4 - \rho^2}, u_a\right]$, and $x_m < x^* < x_T(u)$ when $u \in \left(u_a,\; \dfrac{4\delta\rho W}{4 - \rho^2}\right)$.*