

# Rationing Poor Consumers to Reduce Prices

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## Abstract

We study how rationing in the public sector influences prices in the private sector. A private firm uses consumers' cost information for cream-skimming. Only rationed consumers consider purchasing from the private firm. Rich consumers are more willing to pay for an indivisible good, such as a health treatment, than poor consumers. The public supplier decides on a rationing rule, and then the private firm reacts by setting prices. In equilibrium, the public supplier must ration both rich and poor consumers. Supplying all poor consumers leaves only rich consumers to the private market. The firm reacts by setting high prices because all available consumers have high willingness to pay. Rationing some poor consumers provides an incentive for the firm to reduce prices because some consumers with low willingness to pay are potential customers.

# 1 Introduction

Rationing of health and other services (such as education and housing) by a public supplier is very common. Public supply may coexist with active private markets. We study the effect of rationing on prices in these private markets. Rationing and public supply are two sides of the same action. When the public supplier provides a good for free to some consumers, rationed consumers may consider purchasing at the private sector at their own expenses. Rationing therefore determines the portfolio of consumers in the private market. A private firm must recognize this, and choose its pricing strategy accordingly.

In our model, consumers are heterogenous in two dimensions. The costs of providing the good to them differ and they have different wealth levels. Consumers wealth heterogeneity is of course natural, and more wealthy consumers may be more willing to pay for services. In the health market, cost heterogeneity is common. Patients with higher severity levels are more costly to treat. However, we let the good provide a standardized unit of improvement in well-being, and assume that its benefit is the same to all consumers. As an example, consider a hip replacement procedure. Treating patients with higher levels of disability require high costs. A hip replacement will allow a consumer to walk about without pain, which we regard as a unit of health improvement. Consumers who are more wealthy may be more willing to pay for the hip replacement.

We consider a public supplier which observes consumers' wealth level and can credibly commit to a rationing scheme based on consumer wealth. The public supplier may well have access to cost information but we consider the case where it is unable to use this information. In the health care market, for example, clinicians may decide on medical services based on needs rather than costs. In the education market, school districts are committed to provide education to all eligible students. In both cases, the rationing issue concerns who are entitled to public provision. In our model, this implies that rationing policy must be based only on wealth.

The firm does not observe consumers' wealth levels, but it does observe consumers' cost characteristic and knows the wealth-based rationing scheme implemented by the public supplier. If poor consumers are supplied by the public while rich consumers are rationed, only the rich consumers will consider the private market. Anticipating that consumers available in the market are rich and have high willingness to pay, a

firm tends to raise prices. High prices will still be optimal even if these rich consumers are good risks or have low costs of provision. This can be recognized as a form of cream-skimming.

In the health literature, cream-skimming usually refers to providers' selection of low-cost and, hence, profitable patients under a prospective payment system. This conception misses the possibility that consumers have different willingness to pay. In our model cream-skimming means that the firm can price discriminate according to the cost of provision but also that the firm uses information on consumers' expected willingness to pay. The firm supplies high-cost consumers provided that it can set a high enough price but may also set a high price to low-cost consumers if these consumers have high willingness to pay.

The public supplier can mitigate cream-skimming by using wealth information and by choosing the proportion of poor and rich consumers in the market. If the public supplier rations some poor consumers, making them available to the private market, a private firm may then find it attractive to lower prices when consumers have good risks or low costs. The mixture of rich and poor consumers in the private market makes price reduction in the private sector an equilibrium response. Using total consumer utility as a welfare index, we show that in equilibrium, the public supplier must ration both poor and rich consumers, and implement some price reduction in the private sector.

One may be concerned with equity and find rationing the poor disagreeable. Nevertheless, we show that rationing some poor consumers will yield a first-order gain due to price reduction. Supplying *all* poor consumers will exacerbate cream-skimming, eliminating this gain. Unless social preferences are so extreme that only the welfare of poor consumers matters, the price reduction due to the rationing of *some* poor consumers will be beneficial.

The literature on rationing and the private sector usually assumes an exogenous supply in the private market. Barros and Olivella (2005) consider doctors who self-refer patients in the public sector to the private sector. Prices paid by patients in the private sector are fixed, while doctors only refer low-cost patients. Iversen (1997) studies waiting-time rationing when there is a private market. Hoel and Sæther (2003) consider the effect of competitive supplementary insurance on a national health insurance system. Hoel (2007) derives the optimal cost-effectiveness rule when patients have access to a competitive private

market. The common assumption has been a competitive private market. This may well be relevant in many settings, but is unlikely to be true always. In fact, when the private market pricing rule is fixed, one only can study how the private market influences public policies. By contrast, we study the case where public policies influence private market responses. In Grassi and Ma (2008) we use a similar model to study the strategic interaction between the public supplier and the private firm, but we consider a simultaneous-move game where the public supplier cannot credibly commit to a rationing policy.

## 2 The model

There is a set of consumers. Each consumer's wealth is either  $w_1$  or  $w_2$ , with  $w_1 < w_2$ . Let  $m_i > 0$  be the mass of consumers with wealth  $w_i$ ,  $i = 1, 2$ . We sometimes call consumers with wealth  $w_1$  poor consumers, and consumers with wealth  $w_2$  rich consumers.

Each consumer may consume at most one unit of an indivisible good. If a consumer pays a price  $p$  for the good, his utility is  $U(w_i - p) + 1$ , while if he does not consume the good (and pays 0), his utility is  $U(w_i)$ . The function  $U$  is strictly increasing and strictly concave. The good gives a unit utility increment to a consumer. We can use a general utility function where the utilities from consuming the good at price  $p$  and from not consuming the good are  $U(w - p, 1)$  and  $U(w, 0)$ , respectively. A separable utility function simplifies the analysis.

A consumer's willingness to pay for the good  $\tau_i$  is defined by the following expression

$$U(w_i - \tau_i) + 1 = U(w_i), \quad i = 1, 2, \tag{1}$$

so  $\tau_i$  is the maximum price a consumer with wealth  $w_i$  is willing to pay. Because  $U$  is strictly concave,  $\tau_1 < \tau_2$ ; a rich consumer is more willing to pay for the good than a poor consumer. Similarly, by the strict concavity of  $U$ , for any  $p > 0$ , we have  $U(w_2) - U(w_2 - p) < U(w_1) - U(w_1 - p)$ . It follows that a rich consumer enjoys more surplus than a poor consumer when the good is sold at price  $p$

$$U(w_1 - p) + 1 - U(w_1) < U(w_2 - p) + 1 - U(w_2).$$

The cost of providing the good to a consumer is random. Let  $c$  denote this cost,  $G : [\underline{c}, \bar{c}] \rightarrow [0, 1]$  its

distribution function, and  $g$  the corresponding density, both defined on a positive support, and  $g > 0$ . The distribution  $G$  is independent of wealth and is uncorrelated to the utility benefit from consuming the good which is fixed at 1. In the health care example, this means that patients with different severity levels obtain the same incremental utility from the good. Furthermore, we assume that the hazard rate  $G(c)/g(c)$  is increasing. Let  $\gamma$  be the expected value of  $c$ . We assume that  $\underline{c} < \tau_1 < \bar{c} \leq \tau_2$ . The last weak inequality involves no loss of generality because a firm facing any consumer with cost higher than  $\tau_2$  will not make any sale.

A public supplier has a budget  $B < (m_1 + m_2)\gamma$ , which is insufficient to provide the good for free to all consumers. We consider nonprice rationing based on wealth. A rationing rule is given by  $(\theta_1, \theta_2)$ ,  $0 \leq \theta_i \leq 1$ , where the regulator supplies  $(1 - \theta_i)m_i$  consumers with wealth  $w_i$ ,  $i = 1, 2$ . The public supplier's payoff is the sum of consumer utilities.

There is a private market and we model it as a monopoly. The private firm does not observe consumers' wealth but does observe their costs. Rationed consumers, those  $\theta_i m_i$  of consumers with wealth  $w_i$ ,  $i = 1, 2$ , may purchase from the private firm. Given the rationing rule, the private firm chooses prices as a function of costs to maximize profits.

We study the subgame-perfect equilibrium of the following extensive-form game:

**Stage 0:** Nature draws a cost realization for each consumer according to the distribution  $G$ ; the private firm observes a consumer's cost realization, but not his wealth; the public supplier observes a consumer's wealth level, but not the cost realization.

**Stage 1:** The public supplier sets a rationing policy  $(\theta_1, \theta_2)$ ,  $0 \leq \theta_i \leq 1$ , supplying  $(1 - \theta_i)m_i$  of consumers with wealth  $w_i$ ,  $i = 1, 2$ .

**Stage 2:** The private firm sets a price for each cost realization.

**Stage 3:** Consumers who are rationed by the public supplier may purchase from the private firm at prices set at Stage 2.

### 3 Equilibrium rationing and prices

#### 3.1 Continuation equilibrium prices

For a given rationing rule  $(\theta_1, \theta_2)$ , we derive the firm's continuation equilibrium prices in Stage 2. Because consumers are either poor or rich, the firm will set its price to either  $\tau_1$  or  $\tau_2$ . Clearly at any cost above  $\tau_1$ , the firm must set the price at  $\tau_2$ , selling only to rich consumers. Suppose that the cost  $c$  decreases below  $\tau_1$ . The firm may set a low price  $\tau_1$ , selling to both rich and poor consumers, or a high price  $\tau_2$ , selling only to rich consumers; these profits are respectively:

$$\pi(\tau_1; c \leq \tau_1) \equiv (m_1\theta_1 + m_2\theta_2)[\tau_1 - c] \quad (2)$$

$$\pi(\tau_2; c \leq \tau_1) \equiv m_2\theta_2[\tau_2 - c]. \quad (3)$$

The profits in (2) and (3) are linear in  $c$ , and (2) decreases in  $c$  at a faster rate than (3); these functions are illustrated in Figure 1. For some rationing policies  $(\theta_1, \theta_2)$  there may be a cost level  $c_1 < \tau_1$  in  $[\underline{c}, \bar{c}]$  such that  $\pi(\tau_1; c_1) = \pi(\tau_2; c_1)$ , as in Figure (1). The value of  $c_1$  is given by

$$c_1 \equiv \tau_1 - \frac{m_2\theta_2}{m_1\theta_1}(\tau_2 - \tau_1). \quad (4)$$

As the cost drops below  $\tau_1$ , a price reduction is worthwhile only if there are enough poor consumers relative to rich ones. The value of  $c_1$  is the cost threshold at which a price reduction occurs and is decreasing in the ratio of available rich to poor consumers,  $\frac{m_2\theta_2}{m_1\theta_1}$ . If there are few poor consumers in the market, the cost has to be much lower than  $\tau_1$  for a price reduction to occur. In an extreme, if only the rich consumers are rationed and all the poor are supplied by the public sector, the firm will not reduce the price at all. We summarize the firm's continuation equilibrium prices by the following:

**Lemma 1** *Given a rationing scheme  $(\theta_1, \theta_2)$ , in a continuation equilibrium if  $c_1$  in (4) is greater than  $\underline{c}$ , the firm sets the high price  $\tau_2$  if  $c > c_1$ , and the low price  $\tau_1$  if  $c < c_1$ ; if  $c_1$  in (4) is less than  $\underline{c}$ , the firm always sets the high price  $\tau_2$ .*

We should rule out any situation where the cost has no influence on prices when the budget is zero. For this, we assume that when all consumers are rationed because  $B = 0$ , the firm switches from low to high

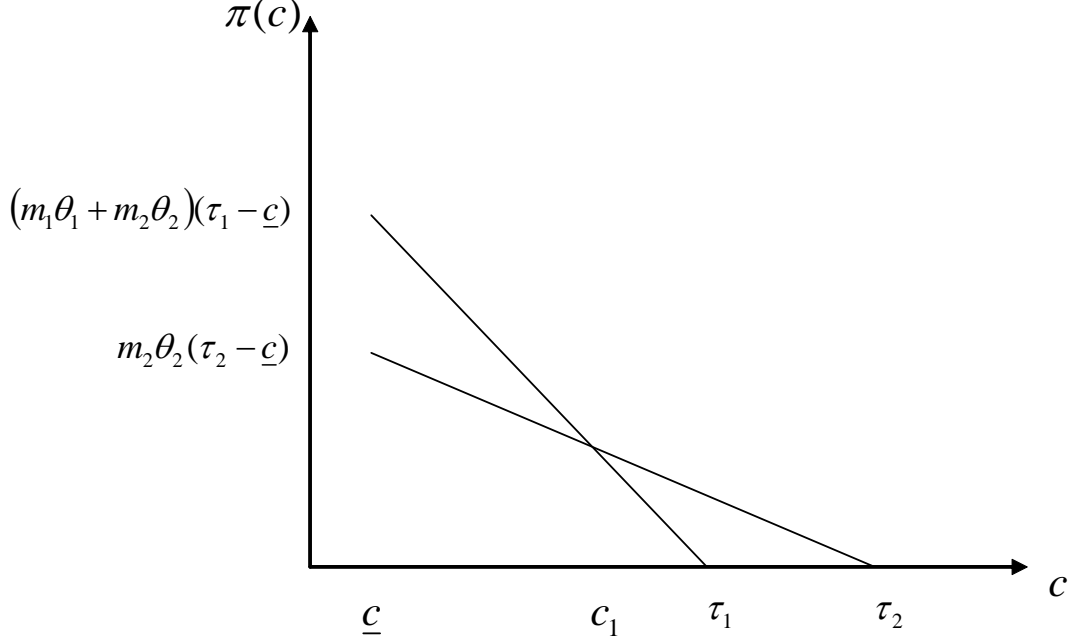


Figure 1: Comparison of profit between setting high and low prices

price at an interior cost threshold. That is, at  $\theta_1 = \theta_2 = 1$  in (4) there is  $c_m$  in the interior of  $[\underline{c}, \bar{c}]$  where

$$c_m = \tau_1 - \frac{m_2}{m_1}(\tau_2 - \tau_1). \quad (5)$$

### 3.2 Equilibrium rationing

We introduce a new notation  $\beta \equiv B/\gamma$ . Because  $B$  denotes the budget available to the public supplier and  $\gamma$  the expected cost, the term  $\beta$  is the number of consumers to whom the public supplier can provide the good. Given the continuation equilibrium prices, the total consumer utility is:

$$m_1 \left[ (1 - \theta_1) (U(w_1) + 1) + \theta_1 \left\{ \int_{\underline{c}}^{c_1} [U(w_1 - \tau_1) + 1] dG + \int_{c_1}^{\bar{c}} U(w_1) dG \right\} \right] +$$

$$m_2 \left[ (1 - \theta_2) (U(w_2) + 1) + \theta_2 \left\{ \int_{\underline{c}}^{c_1} [U(w_2 - \tau_1) + 1] dG + \int_{c_1}^{\bar{c}} [U(w_2 - \tau_2) + 1] dG \right\} \right].$$

In this expression, terms involving  $(1 - \theta_i)$  are consumers' utilities when they receive the public supply at no charge. Terms involving  $\theta_i$  are the market outcomes. For poor consumers, if their costs are below  $c_1$ , they purchase at  $\tau_1$ , which actually leaves them no surplus (see definition of  $\tau_i$  in (1)). Similarly, for rich consumers, if their costs are above  $c_1$ , they purchase at price  $\tau_2$ , earning no surplus. However, if rich

consumers' costs are below  $c_1$ , they earn a surplus  $U(w_2 - \tau_1) + 1 - U(w_2) > 0$  since the price  $\tau_1$  is lower than their willingness to pay  $\tau_2$ .

Using  $U(w_i - \tau_i) + 1 = U(w_i)$ ,  $i = 1, 2$ , we simplify the previous expression to

$$[m_1U(w_1) + m_2U(w_2) + m_1(1 - \theta_1) + m_2(1 - \theta_2)] + m_2\theta_2 \int_{\underline{c}}^{c_1} [U(w_2 - \tau_1) + 1 - U(w_2)] g(c)dc, \quad (6)$$

where  $c_1 \geq \underline{c}$  characterizes the firm's continuation equilibrium price strategy. The term  $m_1(1 - \theta_1) + m_2(1 - \theta_2)$  is the total number of consumers receiving public supply, while  $U(w_2 - \tau_1) + 1 - U(w_2)$  is the surplus a rich consumer enjoys when he purchases at price  $\tau_1$ . In an equilibrium the budget  $B$  must be exhausted. Hence, we replace  $m_1(1 - \theta_1) + m_2(1 - \theta_2)$  by  $\beta$ , and simplify (6) to

$$V(\theta_2, c_1) \equiv [m_1U(w_1) + m_2U(w_2) + \beta] + m_2\theta_2 \int_{\underline{c}}^{c_1} [U(w_2 - \tau_1) + 1 - U(w_2)] g(c)dc. \quad (7)$$

An equilibrium is a rationing rule  $(\theta_1, \theta_2)$  and the continuation equilibrium price strategy in Lemma 1, characterized by the threshold  $c_1$ , which maximize (7) subject to the cost threshold definition (4), the budget constraint

$$m_1(1 - \theta_1) + m_2(1 - \theta_2) \leq \beta \equiv \frac{B}{\gamma} (< m_1 + m_2), \quad (8)$$

and the boundary conditions  $\underline{c} \leq c_1$ , and  $0 \leq \theta_i \leq 1$ ,  $i = 1, 2$ .

**Proposition 1** *In the equilibrium, the public supplier rations consumers in each wealth class:  $\theta_1 > 0$  and  $\theta_2 > 0$ , while the firm charges the low price  $\tau_1$  when the consumer's cost is below a threshold  $c_1^*$ , where  $\underline{c} < c_1^* < \tau_1$ .*

Proposition 1 (whose proof is in an appendix) says that for any budget, the public supplier must ration some poor consumers and some rich consumers. Only rich consumers potentially gain from the private market, so rationing some rich consumers must be an equilibrium. The gain will not be realized if the firm does not reduce the price. To make sure that the potential gain is realized, the public supplier must leave enough poor consumers in the market so that the ratio between rich and poor consumers makes it attractive for the private firm to reduce price. Therefore, the equilibrium cost threshold  $c_1$  at which the price drops from  $\tau_2$  to  $\tau_1$  must be strictly above  $\underline{c}$ . We emphasize that rationing of consumers occurs in equilibrium

even if the budget is sufficient to provide for an entire class of consumers. The following characterizes the equilibrium cost threshold  $c_1$ , and the rationing rule  $(\theta_1, \theta_2)$ .

**Proposition 2** *If the budget  $B$  is sufficiently large, the value of  $c_1^*$  is given by the unique solution of*

$$\frac{G(c_1^*)}{g(c_1^*)} = \frac{(\tau_1 - c_1^*)(\tau_2 - c_1^*)}{\tau_2 - \tau_1},$$

$\theta_1 < 1$ , and  $\theta_2 < 1$ ; the public supplier supplies some consumers in each wealth class. If the budget is small, either  $\theta_1$  or  $\theta_2$  may be equal to 1, the public supplier may ration an entire wealth class; the budget constraint then can be used to solve for the optimal rationing rule.

Proposition 2 (whose proof is in the appendix) reports that there are three possible rationing outcomes. In the “interior” solution, the value of  $c_1$  is obtained by the appropriate first-order conditions, and the boundary conditions  $\theta_i \leq 1$  do not bind. In the “corner” solutions, either  $\theta_1 = 1$  or  $\theta_2 = 1$ .

The maximization of the objective function in (7) is equivalent to the maximization of

$$m_2\theta_2 [U(w_2 - \tau_1) + 1 - U(w_2)] G(c_1).$$

The expression describes the surplus earned by the fraction  $\theta_2$  of rationed rich consumers with cost below the threshold  $c_1$ . The inframarginal gain is constant, and the optimization problem is equivalent to the even simpler maximization of  $m_2\theta_2 G(c_1)$ . This objective function is increasing in both  $\theta_2$  and  $c_1$ , but the equilibrium price and budget constraints, respectively (4) and (8), and the boundary conditions  $\theta_i \leq 1$  limit how high  $\theta_2$  and  $c_1$  can be.

The constraints are graphed in Figure 2. The downward sloping line is the budget constraint (8), while the upward sloping line through the origin is the equilibrium cost threshold (4) for some  $c_1$ ; as the value of  $c_1$  increases, the line becomes flatter. The two dotted lines are the boundary constraints for  $\theta_i$ .

By solving for  $\theta_1$  and  $\theta_2$  with (4) and (8), and then substituting into the objective function, we have the objective function expressed in terms of  $c_1$  alone, and then after ignoring constants, we can write it as  $\frac{\tau_1 - c_1}{\tau_2 - c_1} G(c_1)$ . (Details are in the proof.) The equilibrium value  $c_1^*$  in the Proposition achieves the maximum of this objective function with the boundary conditions  $\theta_i \leq 1$  ignored.

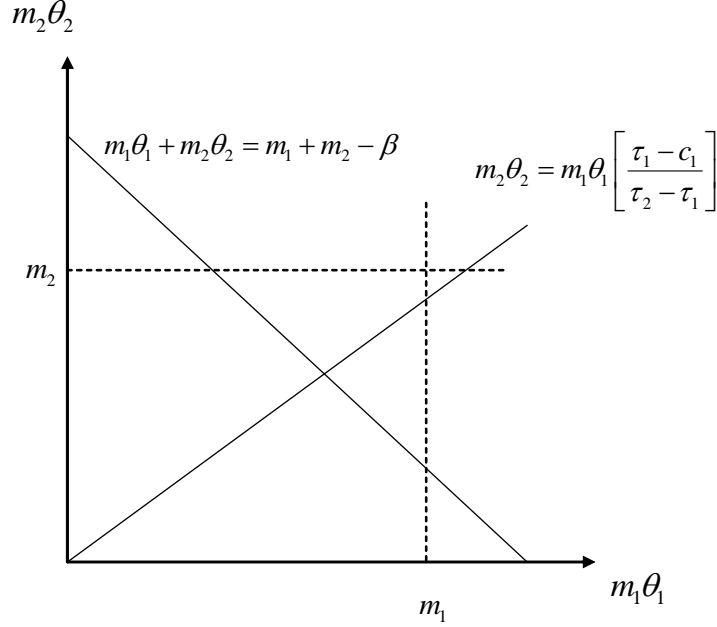


Figure 2: Budget and cost threshold constraints; boundary conditions.

The value of  $c_1^*$  balances the trade-off between rationing more rich consumers and rationing more poor consumers. Rationing a rich consumer allows him to realize a surplus if his cost is below the cost threshold. Rationing a rich consumer, however, implies supplying a poor consumer. With fewer poor consumers in the market, the cost has to fall below a lower  $c_1$  threshold before the private firm reduces its price. This then reduces the likelihood  $G(c_1)$  that a rich consumer will benefit from the private market.

Clearly,  $c_1^*$  is the equilibrium value when the boundary conditions  $\theta_i \leq 1$  are satisfied. In this case we can use the budget and cost threshold constraints to solve for  $\theta_i$  after setting  $c_1$  to  $c_1^*$ , and both  $\theta_1$  and  $\theta_2$  are smaller than 1:

$$\theta_1 = \frac{m_1 + m_2 - \beta}{m_1 \left[ 1 + \frac{G(c_1^*)}{g(c_1^*)} \frac{1}{\tau_2 - c_1^*} \right]} < 1 \quad \text{and} \quad \theta_2 = \frac{m_1 + m_2 - \beta}{m_2 \left[ 1 + \frac{g(c_1^*)}{G(c_1^*)} (\tau_2 - c_1^*) \right]} < 1. \quad (9)$$

(Details of the computation are in the proof.) Figure 2 illustrates such a case where the intersection of the budget and cost threshold constraints (at  $c_1 = c_1^*$ ) is in the interior of the area bounded by the boundary conditions represented by the two dotted lines.

The boundary conditions are unlikely to bind when the budget is big, so that in Figure 2, the budget line is located closer to the origin. If the budget is small, the budget line is located farther from the origin.

One of the two boundary conditions may be violated when  $c_1$  is set at  $c_1^*$ . In this case, the unconstrained maximization of  $\frac{\tau_1 - c_1}{\tau_2 - c_1}G(c_1)$  is infeasible. Instead, the intersection points between the budget line and the boundary conditions must be considered. Either  $\theta_1 = 1$  or  $\theta_2 = 1$ . Then the value of cost threshold  $c_1$  can be obtained by the constraint (4).

From (9), we can see that the total number of consumers  $m_i$  in a wealth class likely will determine whether that group will be completely rationed when the budget is small. We can also see this in Figure 2. If the value of  $m_1$  is small, then the vertical dotted line is closer to the origin. It is more likely that  $\theta_1$  becomes 1. That is, to implement a cost reduction, the public supply may have to let the private supplier potentially sell to all poor consumers. Conversely, if  $m_2$  is small, then the horizontal dotted line is closer to the origin. It is more likely that  $\theta_2$  becomes 1. In this case, the public supplier finds it optimal to let all rich consumers gain from trading in the private market.

## 4 Concluding remarks

We have presented a simple model where public rationing implements price reactions in the private market. Public policies should take into account market responses. We show how rationing may implement a price reduction in the private market.

The model has been simplified to make the analysis tractable. Extending the model and deriving the equilibrium rationing scheme for a general distribution of consumer wealth involve more complex computation, but may well be worthwhile. Including quality differences between the public and private sector may also be of interest.

# Appendix

**Proof of Proposition 1:** Because all terms in square brackets in the objective function (7) are constant, we alternatively can write the objective function as  $m_2\theta_2G(c_1)$ .

The boundary conditions  $\underline{c} \leq c_1$ , and  $0 \leq \theta_i$  do not bind. If either  $\theta_2 = 0$  or  $c_1 = \underline{c}$  at a solution, then the optimized value is  $m_2\theta_2G(c_1) = 0$ . We show that a rationing policy with  $\theta_1 = \theta_2 = k > 0$  does strictly better. This policy satisfies the budget constraint (8) for some  $0 < k < 1$ . Moreover, from (4) and (5), we have  $c_1 = c_m > \underline{c}$  by assumption. Therefore, this rationing rule,  $\theta_1 = \theta_2 = k$ , is feasible, and yields a payoff  $m_2kG(c_m) > 0$ . This implies that at a solution  $c_1 > \underline{c}$  and  $\theta_2 > 0$ . Because  $c_1 > \underline{c}$ , it follows from (4) that  $\theta_1$  must be bounded away from 0. ■

**Proof of Proposition 2:** From Proposition 1 we know that  $\theta_i > 0$  and  $c_1 > \underline{c}$ . For the time being, ignore the (remaining) boundary conditions  $\theta_i \leq 1$ . Rewrite the budget constraint (8) as  $m_1\theta_1 + m_2\theta_2 \geq m_1 + m_2 - \beta \equiv K > 0$ . Clearly, the budget constraint must bind at a solution. From constraint (4), we have  $m_2\theta_2(\tau_2 - \tau_1) = m_1\theta_1(\tau_1 - c_1)$  which yields

$$m_1\theta_1 = m_2\theta_2 \frac{\tau_2 - \tau_1}{\tau_1 - c_1}. \quad (10)$$

Substituting this into the modified budget constraint, we can solve for  $m_2\theta_2$ :

$$m_2\theta_2 = K \frac{\tau_1 - c_1}{\tau_2 - c_1}. \quad (11)$$

We next substitute (11) into the objective function  $m_2\theta_2G(c_1)$ . The constrained maximization problem (with the boundary conditions  $\theta_i \leq 1$  omitted) is the same as the unconstrained maximization problem:

$$\max_{c_1} K \frac{\tau_1 - c_1}{\tau_2 - c_1} G(c_1).$$

Ignoring the parameter  $K$ , after simplification we obtain the first-order derivative

$$\frac{g(c_1)}{\tau_2 - c_1} \left[ (\tau_1 - c_1) - \frac{G(c_1)}{g(c_1)} \frac{\tau_2 - \tau_1}{\tau_2 - c_1} \right].$$

Setting the first-order derivative to zero, we have

$$\frac{G(c_1)}{g(c_1)} = \frac{(\tau_1 - c_1)(\tau_2 - c_1)}{\tau_2 - \tau_1}. \quad (12)$$

The left-hand side of (12) is increasing in  $c_1$ . For  $c_1$  between  $\underline{c}$  and  $\tau_1$ , the right-hand side is decreasing. At  $c_1 = \underline{c}$ , the left-hand side of (12) is zero, while the right-hand side of (12) is strictly positive. At  $c_1 = \tau_1$ , the left-hand side of (12) is strictly positive, while the right-hand of (12) is zero. Therefore, there exists a unique  $c_1^*$  strictly between  $\underline{c}$  and  $\tau_1$  that satisfies (12).

To recover  $\theta_i m_i$ , we use (10) and (12) to get  $m_2 \theta_2 = m_1 \theta_1 \frac{G(c_1^*)}{g(c_1^*) (\tau_2 - c_1^*)}$ , which, together with the budget constraint (8), can be used to solve for the values of  $m_1 \theta_1$  and  $m_2 \theta_2$ :

$$m_1 \theta_1 = \frac{m_1 + m_2 - \beta}{1 + \frac{G(c_1^*)}{g(c_1^*)} \frac{1}{\tau_2 - c_1^*}} \quad \text{and} \quad m_2 \theta_2 = \frac{m_1 + m_2 - \beta}{1 + \frac{g(c_1^*)}{G(c_1^*)} (\tau_2 - c_1^*)}. \quad (13)$$

If  $\beta$  is sufficiently large, the right-hand side values in (13) will be less than  $m_1$  and  $m_2$ , and the omitted boundary conditions  $\theta_i \leq 1$  are satisfied. Otherwise, if  $\beta$  is small, one or both of the right-hand side values in (13) will be more than  $m_1$  or  $m_2$ . In this case, a boundary condition binds. ■

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