Public Sector Rationing and Private Sector Selection

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Abstract

We study the interaction between a public sector and a private sector in the provision of a private good. Under a limited budget, the public supplier uses a rationing policy. A private firm may supply the good to those consumers who are rationed by the public system. Consumers have different amounts of wealth, and costs of providing the good to them vary. We consider two information regimes: first, the public supplier observes only wealth information; second, the public supplier observes both wealth and cost information. The public supplier chooses a rationing policy based on its information; simultaneously, the private firm, observing only cost but not wealth information, chooses a pricing policy. In the first information regime, there is a continuum of equilibria; in each, rich consumers are rationed, and the private firm sells to these rationed consumers at high prices. In the second regime, there is a unique equilibrium. The public supplier allocates the good to consumers according to a cost-effectiveness rule. In the equilibrium, rationed consumers have high costs relative to the benefit, and the rationing rule is the same as if the private market were inactive.

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1 Introduction

Many governments and public organizations provide or subsidize goods and services such as education and health care. Public provision at subsidized or zero charge to consumers often coexists with a private market. In this paper, we model the interaction between the public and private sectors in the provision of an indivisible good. In our model, the private sector reacts to public supplies, and simultaneously, the public sector reacts to prices in the private market. We derive equilibria of the game between the two sectors under various information regimes.

Our model is set up to address a number of issues. First, the private sector may react to public supply by selecting or cream-skimming profitable consumers. How does the public sector react to cream-skimming? Second, consumers enjoy different surpluses from public and private supplies. How does this affect the equilibrium allocation by the public sector and prices in the private sector? Third, the allocation in the public sector and the supply in the private sector may be based on different information. How do equilibria change when information structures change?

Consumers differ in two dimensions. They have different wealth levels. The costs of providing the good to them also differ. We use wealth heterogeneity to model consumers' differences in valuations of the good. Rich consumers are more willing to pay for the good than poor consumers. The cost heterogeneity dimension arises because consumer characteristics may determine how much it costs to supply the good to them. For example, in the education market, the cost of helping a student to achieve an academic standard depends on the student's ability and aptitude. In the health care market, the cost of a treatment depends on illness severity. Variations in these characteristics affect the costs of provision.

We use a monopoly model for the private sector, but the analysis for a Cournot private sector remains the same; the analysis also extends to a perfectly competitive private sector. Our goal is to study the effect of a price-reactive private sector, so the monopoly setup is convenient. The private firm observes a consumer's cost but not wealth. Because consumers have different willingness to pay, the firm faces a downward-sloping demand function. By lowering the price, the firm sells to consumers with lower wealth and lower willingness to pay. As consumers' costs change, the firm changes its profit-maximizing prices.

Public supply is assumed to be free of charge to consumers but the available budget cannot cover all consumers. The public supplier must use a rationing rule. We consider two information regimes. In the first, consumers' wealth information is available to the public supplier, and the rationing rule is based on it. In the second, consumers' wealth and cost information is available, and the rationing rule is based on both.

The assumption that a firm knows its cost seems natural. We also think that the government should have good information about consumers' wealth or income through tax returns. This is the first information regime. Cost information also may be available to the public supplier. Afterall, the public supplier actually may provide the good to consumers. This is the second information regime.

We have included the first information regime because often a public supplier may not be able to use cost information. In the health and education markets, the government policy may commit itself to supply services at reasonable costs. Furthermore, there may be some decentralization in the implementation of provision. In the health care market, for example, clinicians may decide on medical services based on needs rather than costs. In the education market, school districts are committed to provide education to all eligible students.

The game between the public supplier and the private firm is described as follows. First, consumers' wealth and cost variables are drawn from their independent distributions. Second, in each information regime, the public supplier sets a rationing rule; simultaneously, the private firm sets prices as a function of consumers' costs. Third, those consumers who are rationed by the public supplier may choose to buy the good from the private firm at the offered prices. The public supplier maximizes the sum of consumer utility given the budget and the pricing rule in the private sector. The private firm maximizes profit given the public supplier's rationing rule.

Our game is used to study the three issues raised above. Cream-skimming and selection are captured by the private firm's pricing strategy. Prices are generally above marginal costs, and responses against the public provider's rationing policy. Consumers who obtain the good from the public supplier are no longer potential customers for the private firm. For example, if the public supplier serves all consumers with wealth below a threshold, the private firm now realizes that there are no poor consumers in the market. Even as costs drop to very low levels, the private firm has no incentive to reduce prices, because all available consumers have high willingness to pay.

The private firm's price schedule depends on consumers' costs. Among consumers who are willing to purchase, wealthy ones obtain higher inframarginal surplus. Because the public supplier attempts to maximize total consumer utility, it tends to supply poor consumers and ration wealthy consumers. Letting wealthy consumers buy from the private market generates more surplus. This trade-surplus effect is the public supplier's basic incentive when reacting against the private sector's pricing strategy.

In the first information regime, when rationing is based on wealth information, the private firm's pricing strategy and the trade-surplus effect reinforce each other. There is a continuum of equilibria; in each, the public supplier rations wealthy consumers. The private firm sets a price at least as high as one it would set if the public supplier did not have any budget, and often may set a higher price. In the equilibrium that generates the highest consumer utility and expected profit, consumers obtain the good from the public supplier if and only if their wealth is below a threshold, and the private firm's price will remain high even when the consumer's cost is low. Wealthy consumers who have low costs will be hurt by the public supply.

In the second information regime, rationing is based on both wealth and cost information. Here, absent the private sector, a cost-effectiveness principle applies. The public supplier allocates the good to consumers when the benefit is above the cost (adjusted by the shadow price of the budget). This cost-effectiveness principle continues to apply when the private market is reactive. In the unique equilibrium, consumers are allocated the good by the public supplier if and only if their costs are below the same threshold. The private firm only sells to remaining consumers, at prices it would have charged absent the public supply. It is as if the public supplier had ignored the private firm. While low-cost consumers are served by the public sector irrespective of their wealth, only rationed and wealthy consumers buy from the private firm. Low-cost consumers are better off as a result of the public supply because they get the good for free; high-cost consumers are not hurt by the public supply.

The public supplier's policy instrument is a rationing rule. Nonprice rationing is ubiquitous. Many governments set negligible prices for publicly provided education and health care, and ration these services in various ways. Presumably, this may be due to fairness or political considerations. There are also economic reasons for favoring rationing. For the health market, insuring consumers' financial risks due to illness is a fundamental goal. Under social insurance, consumers should not be exposed to too much financial risk upon becoming sick. For the education market, a government may encourage the investment of human capital, which may enhance economic growth and create externalities. Again, reducing costs of education may be a sensible policy.

The strategic interaction between the public supplier and the private firm is set up as a simultaneous-move game. Most other papers assume that the government is a first mover. Generally, a public supplier enjoys being a Stackelberg leader because being able to commit to a rationing rule is valuable. However, we show in a companion paper, Grassi and Ma (2008), that the public supplier's Stackelberg rationing rule is time inconsistent.

The commitment issue is this. If the public supplier precommits to ration some poor consumers, both poor and wealthy consumers are in the private market. When the firm observes a low-cost consumer, it sells at a low price, capturing both wealthy and poor consumers. Prices would then fall as costs drop. This benefits rich consumers, and raises total consumer surplus. Given the strictly increasing price schedule, however, the public supplier would like to ration wealthy consumers and supply poor consumers. Rationing the poor is never a best response given a strictly increasing price schedule.

In a simultaneous-move game, the public supplier and the private firm have symmetric commitment power. Facing an imperfectly competitive private market, a public supplier must want to choose its policy to react against existing market conditions (such as prices above marginal costs). Likewise, a private firm must want to react to public supplies. A sequential-move model, the usual approach in the literature, would not allow these mutual reactions, and our model departs from the literature by studying mutual best responses.

Many papers have studied selection. The models usually assume a competitive private market, or exogenous pricing rules. Barros and Olivella (2005) focus on public physicians referring patients to their own private practices. They let the public sector use waiting-time rationing, and physicians refer patients when the patients' costs are low. Hoel (2007) derives the optimal cost-effectiveness rule when patients have access to a private market, where prices are competitive, and do not respond to the public sector's allocation rule. In both models, the prices in the private sector are fixed.

A large literature is concerned with waiting-time rationing. Beginning with Lindsay and Feigenbaum (1984), economists have looked at waiting time as a disutility or reduction in quality. Iversen (1997) considers the effect of a private sector on waiting time in the public sector. In Iversen's model the price in the private sector is fixed, and there is no consumer-cost heterogeneity. Brekke, Siciliani and Straume (2008) study the effect of competition on waiting time, but do not consider the public sector. Hoel and Sæther (2003) consider supplementary private health care when public health care is subject to waiting time rationing. Again, the price in the private market is fixed.

The research on optimal education policy is related to our model. De Fraja (2002) derives optimal education policies in a general model of taxes and subsidies and intergenerational transfer. In his model, public policies can be based on wealth, as in one of our information regimes. Consumers differ in wealth and abilities, and the optimal policies based on wealth treat the wealthy and poor consumers differently. Richer consumers receive more subsidies. We do not consider taxes and subsidies; nor do we have intergenerational transfers. In our equilibria, richer consumers are rationed when rationing is based on wealth. Epple and Romano (2002) consider cream skimming by private schools when students differ in wealth and ability. Their focus is on how a voucher system may alleviate the problems.

Rationing is similar to transfers in kind. The literature has concentrated on redistribution effects. In Besley and Coate (1991), the government uses a poll tax and low quality in public education to transfer wealth from the rich to the poor. Rich consumers pay poll taxes but optimally choose high-quality education from the private market. Again, the private supply of quality is exogenously given. Public supply of education does not change prices in the private market. Blackorby and Donaldson (1988) show how transfers in kind may solve asymmetric information problems. In Ma (2003), public rationing aligns cost incentives in the private sector by releasing potential consumers to private firms operating under increasing returns. The private sector outcome is modelled as a contestable market equilibrium, in which prices equal average costs. The price reaction in the private sector against rationing is limited to changes in average costs due to differences in mass of consumers available to the private sector.

By taking the private sector price as exogenous, the above literatures, in fact, focus on how a private sector affects the public sector and consumer decisions. Our formal model is like a common agency model. The public supplier and the private firm are two principals whose actions will affect the consumer, who is the agent; see Bernheim and Whinston (1986). In line with the common agency model, we use a symmetric setup, so that both sectors react against each other's strategy. This is the innovation in our paper. We are unaware of a paper that models how public sector rationing and private sector pricing strategies mutually react. Our model also makes explicit the private sector reaction mechanism. Rationing alters the portfolio of consumers available to the private market, and private firms set prices accordingly.

Section 2 and its subsections lay out the model. In Section 3, we describe the private firm's price responses against rationing rules, and consumer welfare. In Sections 4 and 5, we derive equilibrium rationing and pricing schemes for the two information regimes, respectively. Section 6 considers alternative assumptions and robustness. The last Section draws some conclusions. An Appendix contains all proofs.

2 The model

We begin with the description of consumers. Next, we introduce a private sector and then a public sector. We also derive the optimal prices and the optimal rationing policies when the public sector or the private sector is inactive. We complete the model by describing the extensive-form games between consumers, the public supplier, and the private firm.

2.1 Consumers and their willingness to pay

There is a set of consumers. Each consumer may consume at most one unit of an indivisible good. We let there be a continuum of these consumers, with a total mass normalized to 1. Each consumer is indexed by two parameters, w and c. The variable w denotes the consumer's wealth. The variable c denotes the cost of supplying this good to the consumer. The cost of provision c is identical whether the good is supplied by the public or private sectors; we do not consider any productive comparative advantage between the private and public sectors in order to focus on information problems. We often use the term consumer (w, c) to refer to one who has wealth w and cost parameter c.

Let $F : [\underline{w}, \overline{w}] \to [0, 1]$ be the distribution function of w. We assume that F is differentiable, and the corresponding density f strictly positive. Similarly, let $G : [\underline{c}, \overline{c}] \to [0, 1]$ be the distribution function of c. We also assume that G is differentiable, and the corresponding density g strictly positive. Threrefore, the distribution functions F and G are both strictly increasing. The domains of both distributions are strictly positive and bounded. The variables w and c are assumed to be independently distributed. In Section 6 we will discuss the independence assumption.

For a general specification of preferences, we can let a consumer's utility be $U(w, 0)$ when he does not consume the good, and $U(w - p, 1)$ when he consumes the good at a price $p \geq 0$. The utility function U is strictly increasing, and strictly concave in w, and $U(w, 1) > U(w, 0)$. It saves on notation and simplifies the analysis if we let the utility function U be separable in the two arguments. The separability assumption says that a unit of the good generates the same utility increment, independent of the consumer's wealth. If the consumer with wealth w pays a price p to consume the good, his utility is $U(w - p) + 1$. The consumer's utility is $U(w)$ if he does not consume the good. We discuss the nonseparable utility function in Section 6.

If a consumer who has wealth w is indifferent between paying a price τ to consume the good and the status quo, we have:

$$U(w - \tau) + 1 = U(w). \tag{1}$$

This equation implicitly defines a willingness-to-pay function $\tau : [\underline{w}, \overline{w}] \to \mathbb{R}_+$ for consumers with various wealth levels. Because U is concave, hence almost everywhere differentiable, the willingness-to-pay function is differentiable. From total differentiation of (1), we have:

$$\frac{\mathrm{d}\tau}{\mathrm{d}w} = 1 - \frac{U'(w)}{U'(w - \tau)} > 0. \tag{2}$$

A consumer's willingness to pay for the good is strictly increasing in wealth due to the strict concavity of U.

We illustrate our description of consumer preferences and costs with examples in the health market. The good may refer to a surgical procedure (for example, a hip replacement). Patients differ in their illness severity levels (some hip replacements are more difficult than others). For a fixed amount of improvement in health, interpreted as a unit increment of utility (for example, the ability to walk about without pain), sicker patients require more resources, and wealthy patients are more willing to pay.

In our setup, consumer preferences do not directly depend on the provision cost c. In the health care example, this means that patients with different severity levels obtain the same incremental utility from the good. One interpretation is that the good provides a standardized unit of improvement in well-being. In other situations, consumers obtain different incremental utilities depending on their severity levels. Consumer preferences then may depend on cost, and we will discuss this alternative assumption in Section 6.

2.2 The private sector and profit-maximizing prices

We let there be a single firm in the private sector; the cases of a Cournot private sector as well as a perfectly competitive private sector will be considered. The selection issue is this. For consumer (w, c), the private firm observes c the cost of providing a unit of the good to the consumer, but not his wealth w.

The monopolist maximizes profit by setting a price to sell to consumers, given the cost c. For now, assume that the monopolist may have access to the entire mass of consumers. Not knowing the consumer's wealth, the monopolist does not know the willingness to pay. By setting a price, the monopolist sells to those consumers with willingness to pay higher than the price. Obviously, the monopolist will not set a price outside the range of willingness to pay τ. Setting a price p is equivalent to selecting the wealth level of the *marginal* consumer w, where $p = \tau(w)$. By the strictly monotonicity of τ, consumers with $w' > w$ have $\tau(w') > \tau(w)$, hence are willing to pay p to purchase the good. The function τ is like a demand function; we simply restate the common principle that a monopolist may choose equivalently between a price and a quantity while respecting the demand function.

Given a cost c, if the monopolist intends to sell to consumers with wealth w or higher, it sets a price $\tau(w)$, and its profit is

$$\pi(w; c) \equiv \int_{w}^{\overline{w}} f(x)\mathrm{d}x \ \ [\tau(w) - c] = [1 - F(w)]\, [\tau(w) - c]\, . \tag{3}$$

Let the profit-maximizing choice $\widehat{w}^m : [\underline{c}, \overline{c}] \to [\underline{w}, \overline{w}]$ be

$$\widehat{w}^m(c) \equiv \underset{w}{\operatorname{argmax}}\, [1 - F(w)]\, [\tau(w) - c]\, . \tag{4}$$

The marginal consumer, one who pays the price equal to his willingness to pay, has wealth $\widehat{w}^m(c)$, and the profit-maximizing quantity is $1 - F(\widehat{w}^m(c))$. Although $\widehat{w}^m(c)$ denotes the marginal consumer, we also call $\widehat{w}^m(c)$ a quantity function when there is no possibility of confusion.

We assume that $\widehat{w}^m(c)$ is single-valued. By the Maximum Theorem $\widehat{w}^m(c)$ is continuous. We further assume that as c varies over $[\underline{c}, \overline{c}]$, the marginal consumers vary over a proper subset of $[\underline{w}, \overline{w}]$ with $\underline{w} < \widehat{w}^m(\underline{c}) < \widehat{w}^m(\overline{c}) < \overline{w}$. This requires that variation in wealth is sufficiently large relative to variation in costs.

Lemma 1 *The quantity function $\widehat{w}^m(c)$ is strictly increasing. If the monopolist has access to all consumers, the monopolist raises its price and sells to less consumers as cost increases.*

Lemma 1 does make use of the assumption $f(w) > 0$ or equivalently $F(w)$ is strictly increasing. Indeed, when all consumers are available, the private firm may always sell to more consumers by lowering its price. When cost falls, it must sell to more consumers. Later we will see that if some consumers are supplied by the public sector, there may not be consumers around to accept a price reduction. The profit-maximizing price may stop falling even when cost decreases.

2.3 The public sector and rationing

The public sector has a fixed budget $B > 0$, but the budget is insufficient to supply the good to all consumers for free. We consider two information regimes. First, only consumers' wealth information is available to the public supplier, and second, consumers' wealth and cost information is available. In each case, nonprice rationing will be used to allocate the budget for providing the good to consumers.

In the first regime, the public supplier's rationing rule is a function $\theta : [\underline{w}, \overline{w}] \to [0, 1]$. For $w \in [\underline{w}, \overline{w}]$, the public supplier provides consumers with wealth below w a total of $\int_{\underline{w}}^{w} (1 - \theta(x)) f(x) \, \mathrm{d}x$ units of the good at zero cost, but not the remaining consumers. The rationing rule θ modifies the density f so that at w, $[1 - \theta(w)] f(w)$ of consumers are supplied by the public sector at zero price, and $\theta(w) f(w)$ of consumers are available to the private market. Because wealth and cost are independently distributed, the cost c among rationed consumers is distributed according to G.

In the second regime the rationing rule is a function $\phi : [\underline{w}, \overline{w}] \times [\underline{c}, \overline{c}] \to [0, 1]$. It has the same interpretation as in the first regime. For consumer (w, c), the density $\phi(w, c)f(w)g(c)$ is available to the private firm.[1] In each regime, the supplier's objective is the sum of consumer utility, which will be defined below.

We can restrict the public provider to supply to either all or none of the consumers within a wealth class or a wealth-cost class. Rationing schemes are then functions that map $[\underline{w}, \overline{w}]$ to $\{0, 1\}$ and $[\underline{w}, \overline{w}] \times [\underline{c}, \overline{c}]$ to $\{0, 1\}$. The general rationing functions can now be interpreted as mixed strategies. For ease of exposition, we do not use the mixed strategy interpretation.

The rationing schemes θ and ϕ correspond to random rationing, but can be implemented by waiting times. We can add to the consumer preference specification a new parameter, say δ, a random variable whose distribution depends on wealth or cost. The utility of a consumer is now $U(w) + 1 - \delta t$ if he gets the good after a delay of t units of time. The parameter δ describes the consumer's marginal waiting cost An impatient consumer (one with a high value of δ) may decide against the public system if he expects a long delay. By setting the delay t, the public supplier determines the fraction of consumers within a wealth group or a wealth-cost group who choose to get the good.

2.4 Optimal rationing with an inactive private market

For now suppose that the public sector is the sole provider of the good. Consider the first information regime where rationing is based on wealth. Let $\gamma \equiv \int c \, \mathrm{d}G$ denote the expected cost. For a rationing rule θ, total consumer benefit from the public supply is $\int_{\underline{w}}^{\overline{w}} (1 - \theta(w))f(w) \, \mathrm{d}w$ as each unit of consumption increases a consumer's utility by one unit. The consumer welfare index, which the public supplier maximizes, is

$$V(\theta) \equiv \int_{\underline{w}}^{\overline{w}} U(w) \, \mathrm{d}F + \int_{\underline{w}}^{\overline{w}} [1 - \theta(w)] \, f(w) \, \mathrm{d}w. \tag{5}$$

The rationing rule must satisfy the budget constraint

$$\gamma \int_{\underline{w}}^{\overline{w}} (1 - \theta(w))f(w) \, \mathrm{d}w \leq B, \tag{6}$$

which says that the expected cost must not exceed the available budget.

[1] We restrict rationing rules to those that leave the functions $\theta(w)f(w)$ and $\phi(w, c)f(w)$ integrable, so that $\int_{\underline{w}}^{w} \theta(x)f(x) \, \mathrm{d}x$ and $\int_{\underline{w}}^{w} \phi(x, c)f(x) \, \mathrm{d}x$ are well defined for $w \in [\underline{w}, \overline{w}]$.

The determination of a rationing rule that maximizes (5) subject to (6) is rather trivial. Any rationing rule that exhausts the budget is optimal. The public supplier allocates the good to consumers without collecting any payment. Due to the separable utility function, the utility increment is independent of w. Any rationing scheme that exhausts the budget results in the same level of the welfare index, and is optimal.

Now we consider the second information regime, where rationing can be based on wealth and cost. For a rationing rule ϕ, the welfare index is

$$V(\phi) \equiv \int_{\underline{c}}^{\overline{c}} \int_{\underline{w}}^{\overline{w}} \{U(w) + [1 - \phi(w, c)]\} f(w)g(c) \, \mathrm{d}w \, \mathrm{d}c. \tag{7}$$

The rationing rule must satisfy the budget constraint

$$\int_{\underline{c}}^{\overline{c}} \int_{\underline{w}}^{\overline{w}} (1 - \phi(w, c))c \, f(w)g(c) \, \mathrm{d}w \, \mathrm{d}c \leq B. \tag{8}$$

By pointwise optimization with respect to ϕ, the optimal rationing rule is given by[2]

$$\phi(w, c) = 0 \quad \text{for } c < c^s \quad \text{and} \quad \phi(w, c) = 1 \quad \text{otherwise,}$$

where $\int_{\underline{c}}^{c^s} c \, \mathrm{d}G(c) = B$.

The supplier has perfect information, and the optimal rationing rule is based on a cost-effectiveness measure. Each unit of the good yields a fixed increment of utility. The optimal rationing policy therefore supplies the good to consumers if and only if their costs are below a threshold.

2.5 Interaction between the public and private sectors

We study the following game and look for its (subgam-perfect) Nash equilibria:

Stage 1: Nature draws (w, c) according to the distributions F and G. The private firm observes c. The public supplier observes either w, or both w and c.

Stage 2: In each information regime, the public supplier chooses a rationing rule, θ or ϕ, and the private firm chooses a quantity function $\widehat{w}$.

[2] The Lagrangean is $U(w) + [1 - \phi] + \lambda[B - (1 - \phi)c]$, and its first-order derivative with respect to ϕ is $-1 + \lambda c$, which is strictly positive if and only if c is larger than a threshold, say, c^s.

Stage 3: Consumers supplied by the public sector get the good for free, and consumers not supplied by the public sector may purchase from the private firm at prices set in Stage 2.

3 Price responses and consumer welfare when rationing is based on wealth

In this section we derive the private firm's best response against the public supplier's rationing policy when rationing is based on wealth. Given a rationing rule θ, the private firm's profit from selling to consumers with wealth higher than w is

$$\pi(w; c, \theta) \equiv \int_{w}^{\overline{w}} \theta(x) f(x) \, \mathrm{d}x \quad [\tau(w) - c], \tag{9}$$

which differs from the expression in (3) in that at w only a fraction $\theta(w)$ of consumers with wealth w would consider buying from the private firm. Let $\widehat{w}(c)$ be the optimal quantities, and $\widehat{\pi}(c)$ the maximum profit:

$$\widehat{w}(c) = \arg\max_{w} \pi(w; c, \theta), \tag{10}$$

$$\widehat{\pi}(c) = \pi(w'; c, \theta), \quad w' \in \widehat{w}(c). \tag{11}$$

For some rationing rules, there may be multiple quantities that maximize profit, so $\widehat{w}(c)$ is a correspondence. According to the Maximum Theorem, the correspondence $\widehat{w}(c)$ is upper semicontinuous. An equilibrium is a selection from such a correspondence. We present some monotonicity results on the firm's profit-maximization problem.

Lemma 2 *The maximum profit is strictly decreasing in c. Any selection from the profit-maximizing prices,* $\widehat{w}(c) = \arg\max_{w} \pi(w; c, \theta)$, *is increasing in c; that is, if* $c_1 < c_2$, *then* $w_1 \leq w_2$, *where* $w_1 \in \widehat{w}(c_1)$ *and* $w_2 \in \widehat{w}(c_2)$.

The profit-maximizing price may not be strictly increasing in cost c, although the maximum profit is strictly decreasing. For some rationing rules, the profit-maximizing prices at many cost levels may be identical. For example, suppose that the rationing rule specifies that $\theta(w) = 0$ for $w < \widetilde{w}$, and $\theta(w) = 1$ for $w > \widetilde{w}$. This scheme supplies consumers (at zero price) if and only if their wealth is below a threshold $\widetilde{w}$. At a low cost, the firm already may sell to all available consumers, setting the price at the willingness to pay $\tau(\widetilde{w})$. The optimal price will not reduce further even when cost falls; see Figure 1. There, two quantity functions are graphed. First, the quantity function $\widehat{w}^m(c)$ is the profit-maximizing quantity function when the firm has access to all consumers, while the quantity function $\widehat{w}(c)$ maximizes profit when consumers with wealth less than $\widetilde{w}$ are supplied by the public sector. The quantity function $\widehat{w}(c)$ coincides with $\widehat{w}^m(c)$ for cost levels above a threshold, and it becomes the horizontal, dotted line when the cost falls below that.

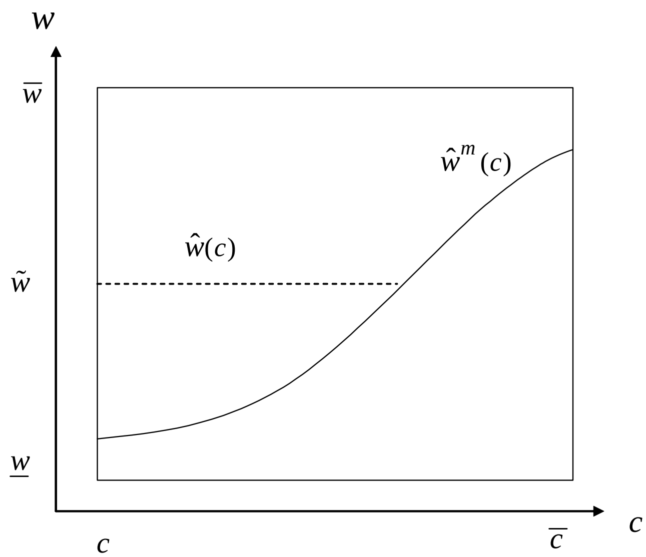

Figure 1: Quantity functions $\widehat{w}^m(c)$ and $\widehat{w}(c)$.

For some rationing rules, there may be multiple quantities that maximize profit. For example, suppose that $\theta(w) = 0$ for $w \in [w_1, w_2]$ where $\underline{w} < w_1 < w_2 < \overline{w}$, and $\theta(w) = 1$ otherwise. The public sector supplies only to consumers with wealth levels in a medium range. Figure 2 illustrates the density of consumers available to the private firm. The profit-maximizing quantity function $\widehat{w}(c)$ is illustrated in Figure 3. For $c < c_1$ or $c > c_2$, the profit-maximizing quantity is unique. For $c \in (c_1, c_2)$, the price remains constant. As the cost falls below c_2, the firm does not lower its price because all consumers with wealth in $[w_1, w_2]$ are supplied by the public sector. At cost c_1, the firm makes equal amounts of profit whether it charges a price $\tau(w_2)$ or $\tau(w_0)$. The profit from selling to consumers with w between w_0 and w_1 and those with $w > w_2$ at a lower price $\tau(w_0)$ is exactly the same as selling only to those with wealth above w_2 at the higher price $\tau(w_2)$. Notice that the value of w_0 must be strictly below w_1. Finally, we note that some quantities may never be chosen. If $\theta(w) = 0$ for w in an interval $[w_1, w_2)$ as in Figure 3, the firm never sets $\widehat{w}(c)$ to any

$w \in [w_1, w_2)$; setting the quantity at w_2 does strictly better.

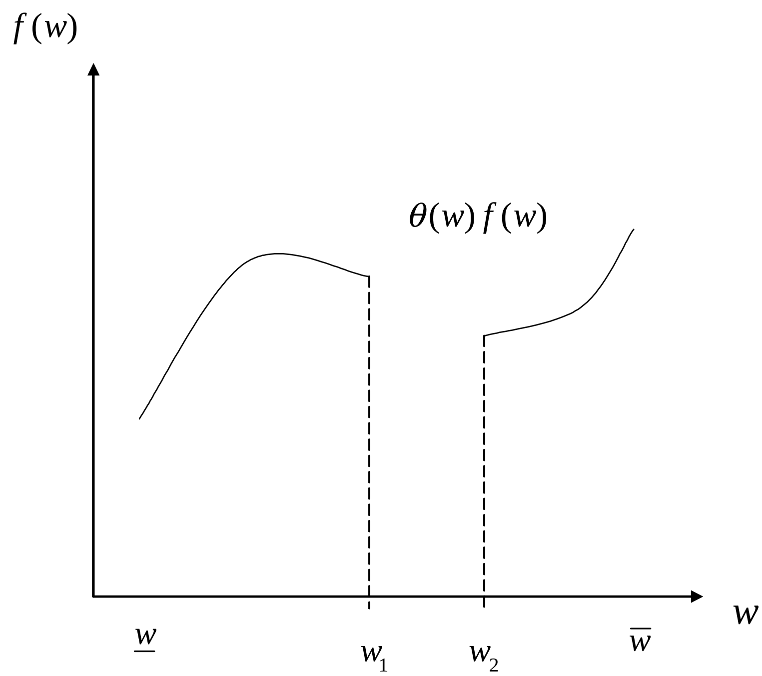

Figure 2: Consumer density under rationing scheme θ.

A best response is a selection from the profit-maximizing quantity correspondence. Such a selection may not be continuous. Nevertheless, because it must be increasing, any point of discontinuity of $\widehat{w}(c)$ must be an upward jump, as in Figure 3. Because there is no risk of confusion, we also denote such a selection by $\widehat{w} : [\underline{c}, \overline{c}] \to [\underline{w}, \overline{w}]$. By Lemma 2, we only need to consider those quantity functions $\widehat{w} : [\underline{c}, \overline{c}] \to [\underline{w}, \overline{w}]$ that are increasing.

For a given quantity function $\widehat{w}(c)$ and the corresponding price function $\tau(\widehat{w}(c))$, consumer (w, c) buys from the private firm if and only if $w \geq \widehat{w}(c)$. Let this set of consumers be denoted by $\Omega \equiv \{(w, c) : w \geq \widehat{w}(c)\}$. In Figure 3, this is the set above the graph of $\widehat{w}(c)$. If we integrate the utilities of consumers in Ω, we obtain the total consumer benefit.

It is more convenient to view the set Ω as one indexed by a function $\widehat{c} : [\underline{w}, \overline{w}] \to [\underline{c}, \overline{c}]$ that is like an "inverse" of $\widehat{w}$. Define $\widehat{c}(w) = \sup\{c : w \geq \widehat{w}(c)\}$; if there is no $c \in [\underline{c}, \overline{c}]$ such that $w \geq \widehat{w}(c)$, set $\widehat{c}(w) = \underline{c}$. Such a function $\widehat{c}(w)$ is illustrated in Figure 3. While the function $\widehat{w}$ gives the wealth of the marginal consumer in terms of his cost, the function $\widehat{c}$ gives the threshold cost level below which a consumer with

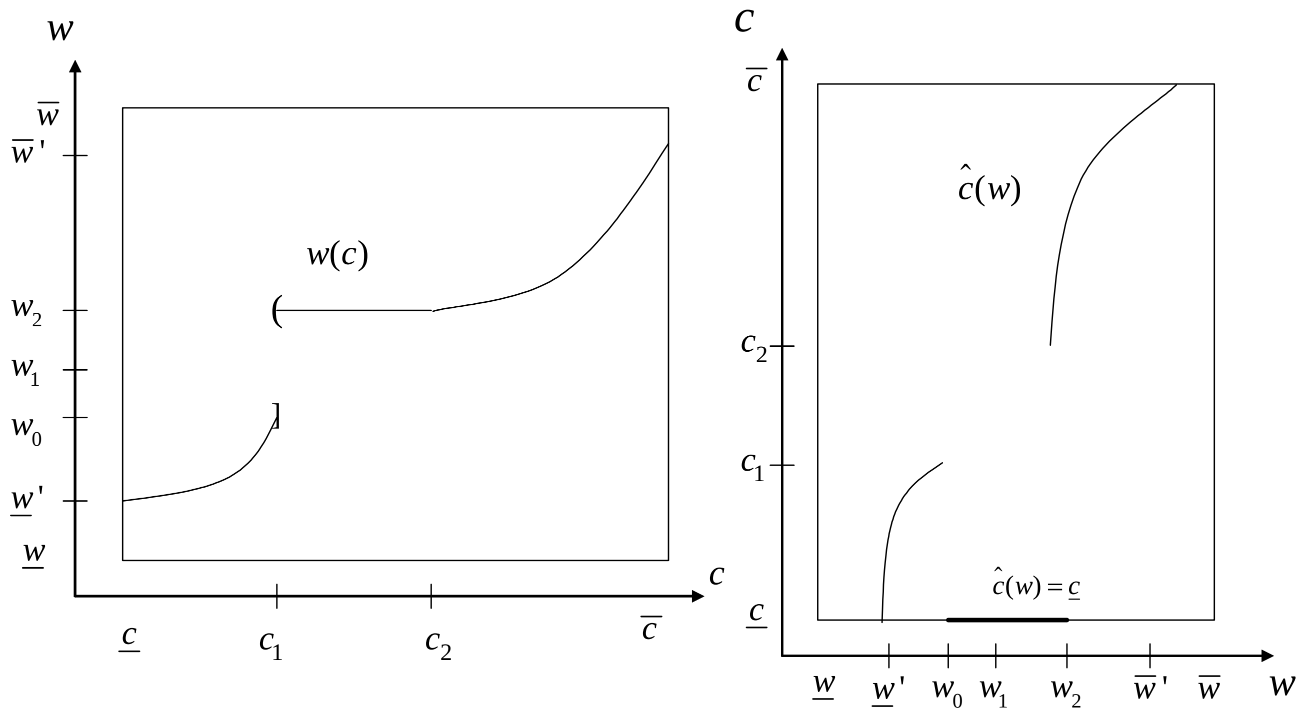

Figure 3: The quantity function $\widehat{w}(c)$ and its "inverse" $\widehat{c}(w)$

wealth w will buy from the firm at price $\tau(\widehat{w}(c))$. Whenever $\widehat{w}$ is strictly increasing and continuous, the function $\widehat{c}$ is its inverse. When $\widehat{w}$ is constant on an interval, then $\widehat{c}$ exhibits discontinuties at the two ends of the interval. Finally, $\widehat{c}(w)$ becomes $\underline{c}$ when the firm does not sell to consumer (w, c). Clearly $\widehat{c}(w)$ is increasing whenever its value is not $\underline{c}$. The set $\Omega' \equiv \{(w, c) : c \leq \widehat{c}(w)\}$ differs from Ω at most for a set of measure zero. Functions $\widehat{w}$ and $\widehat{c}$ are two equivalent ways of keeping track of consumer types who purchase from the private firm.

Given a quantity function $\widehat{w}$ (and its equivalent $\widehat{c}$), if the public supplier chooses a rationing scheme θ, the welfare index $V(\theta)$ is

$$\int_{\underline{w}}^{\overline{w}} \theta(w) f(w) \left[\begin{array}{c} \int_{\underline{c}}^{\widehat{c}(w)} \{U(w - \tau(\widehat{w}(c))) + 1\} g(c) \, \mathrm{d}c \\ + \int_{\widehat{c}(w)}^{\overline{c}} U(w) g(c) \, \mathrm{d}c \end{array} \right] \mathrm{d}w + \int_{\underline{w}}^{\overline{w}} [1 - \theta(w)] f(w) \left[U(w) + 1\right] \, \mathrm{d}w. \qquad (12)$$

In this expression, the first term inside the square brackets is the utility of consumers who buy from the private firm while the other terms refer to utilities of consumers who are either given the good for free or refuse to buy from the private firm after having been rationed. Consumer (w, c) pays the price $\tau(\widehat{w}(c))$ when he buys from the private firm, and this price is always lower than his willingness to pay $\tau(w)$. The welfare index can be simplified to

$$V(\theta) = \int_{\underline{w}}^{\overline{w}} \theta(w)f(w) \left[\int_{\underline{c}}^{\widehat{c}(w)} \{U(w - \tau(\widehat{w}\,(c))) + 1 - U(w)\}\, g(c)\ \mathrm{d}c \right] \mathrm{d}w + \int_{\underline{w}}^{\overline{w}} [U(w) + (1 - \theta(w))]\, f(w)\ \mathrm{d}w, \tag{13}$$

where the first term is the expected inframarginal surplus consumers obtains from the private market.

4 Equilibrium rationing and prices when rationing is based on wealth

An equilibrium is a pair of rationing and quantity schemes $(\theta, \widehat{w})$ such that θ maximizes the welfare index (12) subject to the budget constraint (6) given a quantity scheme $\widehat{w}$, and $\widehat{w}$ maximizes profit (9) for every c given θ. That is, the rationing and pricing schemes are mutual best responses against each other.

We let the supplier pick the net density of consumers that will be made available to the private firm θf, and impose the requirement that $0 \leq \theta f \leq f$. The consumer welfare index (13) is linear in θf, and for each w its first-order derivative with respect to θf is

$$\frac{\partial V}{\partial \theta f} = \int_{\underline{c}}^{\widehat{c}(w)} \{U(w - \tau(\widehat{w}\,(c))) + 1 - U(w)\}\, g(c)\ \mathrm{d}c - 1. \tag{14}$$

This expression measures the welfare tradeoff between rationing one unit of consumer and one unit of public provision at wealth level w. In the private market, consumer (w, c) faces the price $\tau(\widehat{w}\,(c))$. He will buy from the private market at the price $\tau(\widehat{w}\,(c))$ if his wealth w is above $\widehat{w}\,(c)$. The term inside the integral in (14) is the expected incremental surplus from such transactions for a consumer with wealth w. Against this, the welfare index is reduced by 1, the incremental utility of the good if it is supplied by the public sector at zero charge. The following key lemma establishes a monotonicity in the supplier's preferences.

Lemma 3 *The first-order derivative* $\dfrac{\partial V}{\partial \theta f}$ *(14) is increasing in* w*. It is strictly increasing in* $w \in [w_1, w_2]$ *unless* $\widehat{c}(w) = \underline{c}$ *for each such* w*.*

Lemma 3 says that the public supplier favors rationing the consumer over supplying him the good as the wealth level increases. This is a basic principle in our model. The price in the private sector depends only on cost c. Consumer (w, c) gets more surplus from a trade at price $\tau(\widehat{w}(c))$ as w increases: $U(w - \tau(\widehat{w}(c))) + 1 - U(w)$ is increasing in w.

The last part of Lemma 3 says that the derivative (14) at w is constant if and only if consumers with wealth lower than w do not purchase from the private market. Again, if consumer (w, c) gets to purchase from the private market for some levels of cost, then as w increases, the incremental surplus increases. The derivative of (14) with respect to w must be strictly positive. Figure 4 illustrates a situation where the quantity function $\widehat{w}$ becomes constant as cost falls below $\widetilde{c}$, and the price remains at $\tau(\widetilde{w})$. Consumers with wealth below $\widetilde{w}$ do not buy, and the derivative of (14) with respect to w vanishes. We will show that every equilibrium quantity function is like the one in Figure 4.

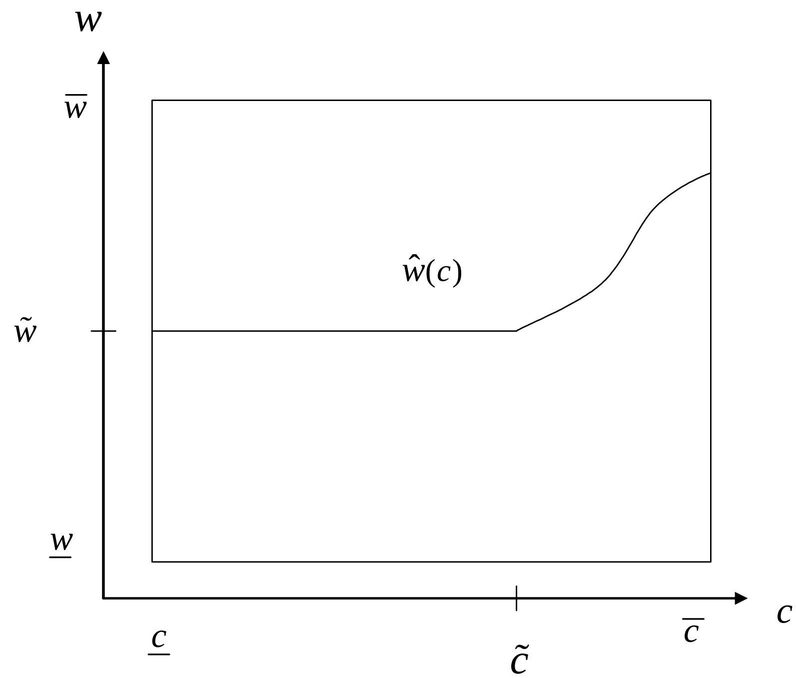

Figure 4: Equilibrium quantity function.

Against a quantity function $\widehat{w}(c)$ (and the corresponding $\widehat{c}(w)$), the public supplier chooses θf to maximize (12) subject to the budget constraint (6). Using pointwise optimization, we consider the Lagrangean

$$\theta(w)f(w)\left[\begin{array}{c}\int_{\underline{c}}^{\widehat{c}(w)}\{U(w-\tau(\widehat{w}(c)))+1\}\,g(c)\;\mathrm{d}c\\ \\ +\int_{\widehat{c}(w)}^{\overline{c}}U(w)g(c)\;\mathrm{d}c\end{array}\right]+[1-\theta(w)]f(w)\,[U(w)+1]-\lambda[\gamma(1-\theta(x))f(x)-B],$$

where λ is the multiplier. The first-order derivative of the Lagrangean with respect to θf is

$$\frac{\partial V}{\partial \theta f} + \lambda\gamma = \int_{\underline{c}}^{\widehat{c}(w)} \{U(w - \tau(\widehat{w}(c))) + 1 - U(w)\} g(c)\, \mathrm{d}c - 1 + \lambda\gamma. \tag{15}$$

From Lemma 3, the first-order derivative of the Lagrangean is strictly increasing in w whenever some consumers with wealth less than w purchase from the private market.

Lemma 4 *In an equilibrium, the public sector rations consumers with wealth above a threshold $\widetilde{w}$. That is, in an equilibrium there is $\widetilde{w} < \overline{w}$ such that $\theta(w) = 1$ for $w > \widetilde{w}$.*

Lemma 4 follows from the monotonicity of the supplier's preferences. If it is optimal for the supplier to ration a consumer at some wealth level, then it must also be optimal to ration all consumers with higher wealth. In any equilibrium, there must exist $\widetilde{w}$ such that $\theta(w) = 1$ if $w > \widetilde{w}$. Lemma 4 does not assert that there is a unique equilibrium.; nor does it say that in an equilibrium, $\theta(w) < 1$ for any $w < \widetilde{w}$

Lemma 5 *In an equilibrium, the private firm sets a constant price when cost falls below a threshold $\widetilde{c}$. That is, $\widehat{w}(c)$ is constant for $c < \widetilde{c}$.*

We already know that an equilibrium quantity function is increasing. Lemma 5 says that it cannot be strictly increasing as cost decreases. The reason is that such a strictly increasing quantity function will make the public supplier allocate the good to consumers with low wealth. Nevertheless, a strictly increasing quantity function when costs are low is not a best response against that. When there are no consumers with low wealth and hence low willingness to pay to accept the price reduction, the private firm will leave the price unchanged even as cost falls. Figure 4 illustrates an equilibrium quantity function. Symmetric to Lemma 4, Lemma 5 does not assert that there is a unique equilibrium.

The last two lemmas establish the form of an equilibrium. The public sector must ration consumers with high wealth, and the private firm must not sell to consumers with wealth below a threshold. The basic economic principle is the following. Because of the limited budget and the higher surplus for wealthy consumers in the private market, rationing of wealthy consumers must be part of an equilibrium. The public sector supplying the less wealthy consumers makes these consumers unavailable to the private firm. As cost decreases, the private firm has a corner solution in its profit-maximizing choice of prices. Prices will become constant even as cost drops further because there may be so few (even zero) consumers with lower willingness to pay to take any price reduction.

Proposition 1 *The following is an equilibrium. The public supplier rations all consumers with wealth above a threshold* $\widetilde{w}^s$ *and supplies all consumers with wealth below* $\widetilde{w}^s$*:* $\theta(w) = 1$, $w > \widetilde{w}^s$ *and* $\theta(w) = 0$, $w < \widetilde{w}^s$*. The value of* $\widetilde{w}^s$ *exhausts the budget and is given by* $F(\widetilde{w}^s)\gamma = B$*. The private firm sets a price equal to the monopoly price when cost is above a threshold, and keeps the price constant as cost falls below this threshold; that is, it sets a price* $\tau(\widehat{w}^m(c))$ *for* $c > \widetilde{c}^s$ *where* $\widehat{w}^m(\widetilde{c}^s) = \widetilde{w}^s$*, and a price equal to* $\tau(\widetilde{w}^s)$ *for* $c < \widetilde{c}^s$*.*

The equilibrium in Proposition 1 is illustrated in Figure 4 (set $\widetilde{c}$ to $\widetilde{c}^s$ and $\widetilde{w}$ to $\widetilde{w}^s$ there). The private firm sets its prices like it is the monopoly in the market except that it has no access to consumers with wealth below $\widetilde{w}^s$, so the prices in the monopoly quantity schedule $\widehat{w}^m(c)$ will stop falling at $\tau(\widetilde{w}^s)$ even as cost falls below $\widetilde{c}$. The public supplier allocates the good to all consumers with wealth below $\widetilde{w}$ and leaves all consumers with higher wealth to purchase from the private market.

This equilibrium is similar to many practical schemes in which poor consumers receive subsidies and free supplies from the government while the rich do not. The reason behind this equilibrium in our model, however, is not one of equity concern. The public supplier selects among consumers with different wealth levels to participate in the private market. Prices in the private market, however, are dependent on cost. Because of higher willingness to pay, wealthy consumers will realize larger gains in trade in the private market, and this is the basis for the equilibrium scheme. The private market fully anticipates this, so even when cost decreases, the equilibrium price may stop falling because consumers with low willingness to pay have already been supplied by the public sector.

Surprisingly, the equilibrium in Proposition 1 is not the only equilibrium in this information regime. Lemmas 4 and 5 do not pin down the supplier's or the firm's strategies. A pricing schedule that becomes constant for low costs may still be consistent with the public supplier rationing some consumers with very low wealth levels. If only a small mass of poor consumers are available to the private market, the private firm will ignore them. To sell to these consumers requires a big price reduction.

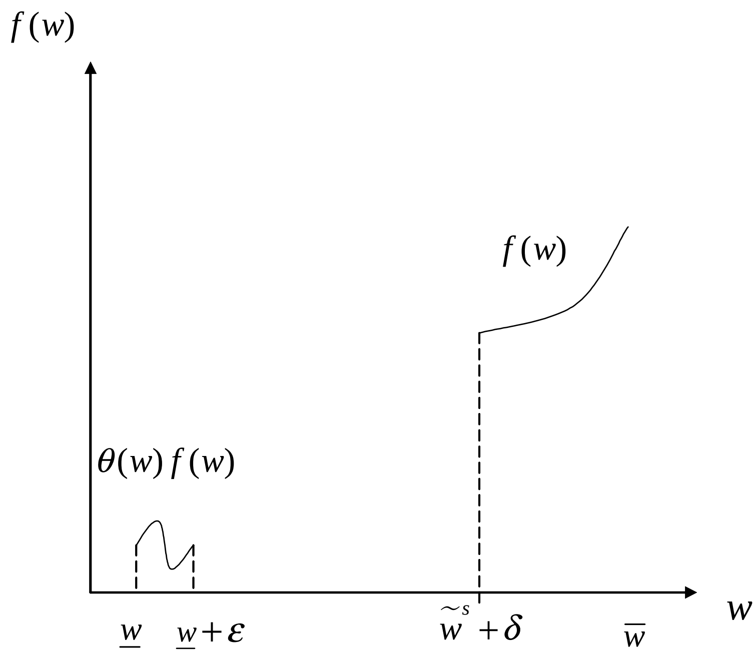

Figure 5: Equilibria in which some poor consumers are rationed.

The following is another equilibrium. Let $\epsilon > 0$ and $\delta > 0$ be both small numbers. The supplier's strategy is the following rationing scheme:

$$\begin{aligned}\theta(w) &= 1 \text{ for } \underline{w} \leq w < \underline{w} + \epsilon \\ \theta(w) &= 0 \text{ for } \underline{w} + \epsilon < w < \widetilde{w}^s + \delta \\ \theta(w) &= 1 \text{ for } \widetilde{w}^s + \delta < w \leq \overline{w},\end{aligned}$$

where $\widetilde{w}^s$ is defined in Proposition 1. In this rationing rule the supplier shifts some resources from those with wealth just above the lowest value $\underline{w}$ to those consumers with wealth just above $\widetilde{w}^s$, the equilibrium threshold in Proposition 1. Figure 5 shows the density of consumers that are available to the private firm in such an equilibrium.

The values of ϵ and δ can be so chosen that the new rationing scheme satisfies the budget. Against this rationing scheme, the private firm sets a quantity function equal to $\widehat{w}^m(c)$ for $c > \widetilde{c}^s + \eta$ and $\widehat{w}^m(\widetilde{c}^s + \eta)$ for $c < \widetilde{c}^s + \eta$, where $\widetilde{c}^s$ is the cost threshold in Proposition 1, and $\eta > 0$ satisfies $\widehat{w}^m(\widetilde{c}^s + \eta) = \widetilde{w}^s + \delta$.

In this equilibrium, the public supplier gives the good to some consumers with wealth slightly higher than $\widetilde{w}^s$, but rations consumers with wealth close to the lowest level. Rationing a small mass of consumers with wealth near $\underline{w}$ would not encourage the private firm to reduce price even when cost becomes very low. Furthermore, because consumers with wealth slightly higher than $\widetilde{w}^s$ are now supplied by the public, the private firm's price will not fall all the way to $\tau(\widetilde{w}^s)$. Both Lemmas 4 and 5 continue to hold. Figure 4 can still be used to depict this new equilibrium by setting $\widetilde{c}$ to $\widetilde{c}^s + \eta$ and $\widetilde{w}$ to $\widetilde{w}^s + \delta$.

Infinitely many equilibria can be constructed in a similar fashion. As long as the private firm does not find it profit-maximizing to reduce price in order to sell to consumers with low willingness to pay, a quantity function like the one in Figure 4 remains a best response. In all these equilibria the public supplier rations some consumers with low wealth, but must ration all consumers with wealth above a threshold.

The equilibrium in Proposition 1 is focal. This is the one that achieves the highest welfare index for the public supplier. This is because it has the widest range of price reduction as cost decreases. The equilibrium also allows the private firm to make the highest equilibrium expected profit, where the expectation is taken over cost parameters c. Any equilibrium different from the one in Proposition 1 would have fewer transactions in the private market, as the next Proposition shows.

Proposition 2 *The equilibrium in Proposition 1 achieves the highest equilibrium welfare for the public supplier, and the highest equilibrium expected profit for the private firm. In any other equilibrium, the public supplier sets* $\theta(w) = 1$, *for* $w > \widetilde{w}^e$, *where* $\widetilde{w}^e > \widetilde{w}^s$ *(defined by* $F(\widetilde{w}^s)\gamma = B$ *in Proposition 1) and the firm sets a price equal to* $\tau(\widehat{w}^m(c))$ *for* $c > \widetilde{c}^e$, *and a price equal to* $\tau(\widetilde{w}^e)$ *for* $c < \widetilde{c}^e$ *where* $\widehat{w}^m(\widetilde{c}^e) = \widetilde{w}^e$ *and* $\widetilde{c}^e > \widetilde{c}^s$.

How are consumers' utilities affected by the public supply and the price reaction in the private market? In the equilibrium in Proposition 1, poor consumers, those with wealth lower than $\widetilde{w}^s$, are better off since they get the good for free. Rich consumers, those with wealth higher than $\widetilde{w}^s$, are hurt by the public supply. If these rich consumers have costs above $\widetilde{c}^s$, they face the same price as if there were no public supply, but if their costs were lower than $\widetilde{c}^s$, they actually face a price increase, since prices will remain at $\tau(\widetilde{w}^s)$.

We have assumed a monopolistic private sector. The extension to an imperfectly competitive sector poses no conceptual problem. For our model of a homogeneous good, we consider a Cournot model. Let there be N firms in the private sector. Given a rationing scheme θ, let each firm choose a quantity function $\widehat{q}_i(c)$, where $i = 1, ..., N$. The total supply is $q(c) = \sum_{i=1}^{N} q_i(c)$. For the market to clear the marginal consumer is $\widehat{w}(c)$ where $\int_{\widehat{w}(c)}^{\overline{w}} \theta(w) f(w) \, \mathrm{d}w = q(c)$, and the price in the private sector is $\tau(\widehat{w}(c))$. All results derived above continue to hold for any given number of firms in the private sector.

Next, we can extend our model to the case of a perfectly competitive private sector. Here, the price in the private sector is marginal cost: $\tau(c) = c$. Given this pricing function, the corresponding quantity function $\widehat{w}(c)$ is implicitly defined by $U(\widehat{w}-c)+1 = U(\widehat{w})$. Lemma 3 can be applied to this quantity function. Because the perfectly competitive quantity function is strictly increasing, the derivative (15) is strictly increasing for all values of w. Lemma 4 continues to hold. In sum we have the following.

Corollary 1 *If the private market is perfectly competitive so that prices are equal to marginal costs, the public sector uses the entire budget on consumers with low wealth levels:* $\theta(w) = 0$, *for* $w < \widetilde{w}^s$, *and* $\theta(w) = 1$, *for* $w > \widetilde{w}^s$ *where* $F(\widetilde{w}^s)\gamma = B$.

5 Equilibrium rationing and prices when rationing is based on wealth and cost

In this section we let the supplier observe both wealth and cost information; the private firm observes only consumers' costs. A rationing policy is $\phi : [\underline{w}, \overline{w}] \times [\underline{c}, \overline{c}] \to [0, 1]$. Suppose that the firm observes that a consumer's cost of service is c, the density of consumer available to the private firm is $\phi(w, c)f(w)$. If it sets a price equal to $\tau(w)$, the total mass of consumers purchasing is $\int_{w}^{\overline{w}} \phi(w, c)f(w)\mathrm{d}w$, and the profit is

$$\int_{w}^{\overline{w}} \phi(w, c)f(w)\mathrm{d}w \quad [\tau(w) - c]. \tag{16}$$

We use the same notation and let $\widehat{w}(c)$ be the marginal consumer that maximizes profit (16), and $\widehat{\pi}(c)$ be the maximum profit.

Consider a rationing function $\phi : [\underline{w}, \overline{w}] \times [\underline{c}, \overline{c}] \to [0, 1]$ and a quantity function $\widehat{w} : [\underline{c}, \overline{c}] \to [\underline{w}, \overline{w}]$. Consumer (w, c) buys from the private firm if and only if $U(w - \tau(\widehat{w}(c))) + 1 \geq U(w)$. Therefore, when the supplier rations consumer (w, c), that consumer obtains a utility $\max\left[U(w - \tau(\widehat{w}(c))) + 1, \ U(w)\right]$ from the private sector. The consumer welfare index from a policy ϕ is

$$\int_{\underline{c}}^{\overline{c}} \int_{\underline{w}}^{\overline{w}} \{\phi(w,c) \max [U(w - \tau(\widehat{w}(c))) + 1, U(w)] + [1 - \phi(w,c)] [U(w) + 1]\} f(w) g(c) \, \mathrm{d}w \, \mathrm{d}c. \tag{17}$$

Our next result reports that in equilibrium, the supplier does not supply to consumers who have high costs, irrespective of their wealth levels.

Lemma 6 *In an equilibrium, consumer* (w, c) *is rationed when his cost is above a threshold,* $\widetilde{c} > \underline{c}$*. That is,* $\phi(w, c) = 1$ *for* $c > \widetilde{c} > \underline{c}$ *and any* w*.*

When the supplier observes wealth and cost information, a standard cost-effectiveness principle applies. The basic principle says that if the cost is too high relative to the benefit, the consumer should not be given the good. Now the availability of the private supply to consumers with high cost does not alter this principle. It remains cost effective to ration high cost consumers when they have opportunities to buy from the private market. Lemma 6 says that for high-cost consumers the rationing rule is the same as the optimal allocation without a private sector.

Now we characterize the equilibrium rationing scheme for low-cost consumers.

Lemma 7 *In an equilibrium, consumer* (w, c) *is given the good when his cost is below a threshold. That is,* $\phi(w, c) = 0$ *for* $\underline{c} < c < \widetilde{c}$*, any* w*.*

Due to cost effectiveness, the supplier will assign the good to low-cost consumers who do not purchase from the private sector. Now some wealthy consumers with low cost may obtain a higher surplus from the private sector if the price is low. Lemma 7 says that this cannot happen in an equilibrium. Consider the marginal consumer (w, c); he pays a price $\tau(w)$ in the private market and obtains a zero incremental surplus. Now because his cost is low, the public supplier prefers to allocate the good to him, giving him a positive incremental surplus. By continuity, the supplier also assigns the good to those with wealth slightly above w; this assignment, however, eliminates these consumers from the private market. The best response by the private firm is to raise the price and increase the wealth of the marginal consumer. This unravelling continues until the private firm raises the price to $\tau(\overline{w})$, and does not sell to consumers with low costs. As a result, in an equilibrium, the price is too high even for low-cost consumers, and the supplier provides the good to all these consumers.

Lemma 6 and Lemma 7 make minimal assumptions on the structure of prices set by the private firm. A simpler set of arguments suffices for equilibrium characterization in the regime where rationing is based on wealth and cost rather than wealth alone. To summarize, we present the following.

Proposition 3 *If the public sector rations consumers based on wealth and cost information, the equilibrium rationing function is identical to the optimal rationing function when the private sector is inactive. That is,* $\phi(w, c) = 0 \quad$ *for* $c < c^s$ *and* $\phi(w, c) = 1$*, otherwise. The private firm chooses the monopoly quantity scheme* $\widehat{w}^m(c)$ *for* $c > c^s$*, and* $\widehat{w}^m(c) = \overline{w}$ *for* $c < c^s$*.*

In contrast to the regime when rationing is based on wealth information only, here consumers are not hurt by public supply. Obviously, low-cost consumers, those with cost below c^s, get the good for free. High-cost consumers, those with cost above c^s, get the good at the same price as if there were no public supply.

Equilibria in the two information regimes are very different. When rationing is based on wealth, in the focal equilibrium in Proposition 1, consumers with low wealth levels get the good, but when cost information is also available, the equilibrium rationing scheme becomes independent of wealth, and assigns the good to low-cost consumers. In both information regimes, the market is segmented, and selection occurs in the private market with the private firm selling only to rationed consumers with higher willingness to pay.

There is not a clear welfare comparison between equilibria across the two information regimes. Equilibria in Propositions 1 and 3 refer to rationing of very different consumer types. In both regimes, equilibrium prices in the private market always follow the monopoly schedule $\widehat{w}^m$ for costs that are high. When public rationing is based on wealth, the price there becomes constant when costs are low. The extent of the fall in prices depends on consumers' willingness-to-pay function $\tau(w)$ as well as the distribution of wealth $F(w)$.

Clearly, Proposition 3 applies directly to imperfectly competitive firms in the private market. Equilibrium rationing when the private market is perfectly competitive is a little different. Here, some low-cost and wealthy consumers may be rationed. When the private market is competitive, prices are given by marginal costs, $\tau(c) = c$. The consumer welfare index is

$$\int_{\underline{c}}^{\overline{c}} \int_{\underline{w}}^{\overline{w}} \{\phi(w,c) \max[U(w-c)+1, U(w)] + [1-\phi(w,c)][U(w)+1]\} f(w)g(c) \, \mathrm{d}w \, \mathrm{d}c. \tag{18}$$

Corollary 2 *Suppose the private market is competitive so that prices there are equal to marginal costs. The public sector rations all consumers with cost above a threshold. For those consumers with costs below the threshold, the public sector supplies a consumer if and only if his wealth is below a value that is determined by his cost. That is,* $\phi(w,c) = 1$ *for* $c > \widetilde{c}^p$*, some* $\widetilde{c}^p > \underline{c}$ *and any* w*;* $\phi(w,c) = 1$ *for* $c < \widetilde{c}^p$ *and* $w > \mu(c)$ *where the function* μ *is implicitly defined by* $U(\mu - c) - U(\mu) + \lambda c = 0$ *for a constant* $\lambda > 0$*; otherwise,* $\phi(w,c) = 0$.

Corollary 2 is illustrated in Figure 6. The upward sloping line $\widehat{w}(c)$ is the marginal consumer given marginal cost pricing. The line above $\widehat{w}(c)$ is the function μ defined in the Corollary.[3] Consumers with cost $c < \widetilde{c}^p$ and wealth between $\widehat{w}(c)$ and $\mu(c)$ strictly prefer to purchase the good at cost, yet the supplier will assign the good to them for free. Very wealthy consumers are rationed even when their costs are low.

For consumers with costs higher than $\widetilde{c}^p$, it is not cost effective to supply them. The logic in Lemma 6 applies. For those with costs lower than $\widetilde{c}^p$, the logic in Lemma 7 applies with one modification. Cost consideration alone warrants the allocation. Nevertheless, wealthy consumers obtain higher incremental surplus from the private market. The condition in Corollary 2 can be rewritten as

$$U(\mu - c) + 1 = U(\mu) + 1 - \lambda c.$$

The left-hand side expression is the utility of a consumer with wealth μ buying from the private market; the right-hand side expression is the social net benefit, where λ is the multiplier of the budget constraint and λc measures the utility equivalent of cost c. If $w > \mu(c)$, the consumer gets more utility from the private market than the social net benefit from the good, so will be rationed. In contrast to Lemma 7, there is no unravelling of equilibrium price best responses in the private market.

[3] By definition, $U(\widehat{w}(c) - c) + 1 = U(\widehat{w}(c))$. Now because $c < \widetilde{c}^p$, we have $1 > \lambda c$. It follows that the value of μ that satisfies $U(\mu - c) + 1 = U(\mu) + (1 - \lambda c)$ must be greater than $\widehat{w}(c)$. We have drawn μ to be negatively sloped, but generally it does not have to be.

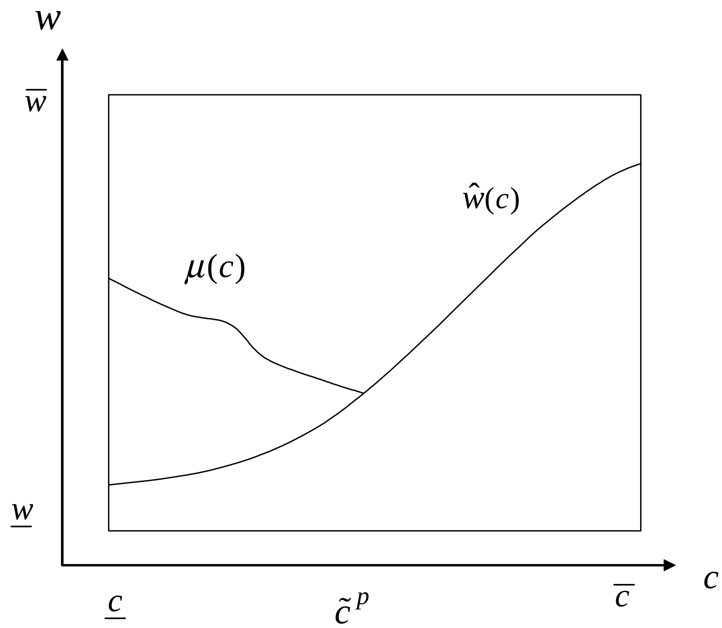

Figure 6: Optimal rationing in a competitive private market.

6 Alternative assumptions on preferences, benefits, and correlation between wealth and costs

In this section, we discuss the general, nonseparable utility function, the situation where benefits may depend on costs, and the possible correlation between wealth and costs.

6.1 General utility function

If we have used the general utility specification, the willingness-to-pay function is implicitly defined by $U(w-\tau,1)=U(w,0)$. From total differentiation, we have

$$\frac{\mathrm{d}\tau}{\mathrm{d}w}=1-\frac{\dfrac{\partial U}{\partial w}(w,0)}{\dfrac{\partial U}{\partial w}(w-\tau,1)}.$$

The willingness-to-pay function is increasing if the goods are complements ($\frac{\partial U}{\partial w}(w,1)>\frac{\partial U}{\partial w}(w,0)$), but may still be increasing even when they are substitutes ($\frac{\partial U}{\partial w}(w,1)<\frac{\partial U}{\partial w}(w,0)$).

Consider first optimal rationing based on wealth. The welfare index is $\int_{\underline{w}}^{\overline{w}}[1-\theta(w)]U(w,1)\ \mathrm{d}F+\int_{\underline{w}}^{\overline{w}}\theta(w)U(w,0)\ \mathrm{d}F$. When the private market is inactive, the optimal rationing rule maximizes the welfare index subject to the budget constraint. The Lagrangean is $[1-\theta]U(w,1)+ \theta U(w,0)+\lambda[B-(1-\theta)\gamma]$, and the first-order derivative with respect to θ is $-U(w,1)+U(w,0)+\lambda\gamma$. This first-order derivative is increasing in w if and only if wealth and the good are substitutes. In this case, the public sector supplies the good to consumers with low wealth, so that $\theta(w)=0$ if w is below a threshold, and $\theta(w)=1$ otherwise.

When the willigness-to-pay function is increasing and when the optimal rationing rule favors consumers with low wealth levels, our arguments in Section 4 apply. Results in Section 4 will have to be modified if the two goods are strong substitutes because then the public supplier may ration poor consumers. Nevertheless rationing of poor consumers seems so rare that the case of strong substitutes is unrealistic.

Next, consider rationing based on wealth and cost. We can again derive the optimal rationing scheme when the private sector is inactive. The first-order derivative with respect to the rationing probability ϕ is $-U(w,1)+U(w,0)+\lambda c$, which is strictly increasing in c, and strictly increasing in w if and only if wealth and the good are substitutes.

Define (w^*,c^*) by $-U(w^*,1)+U(w^*,0)+\lambda c^*=0$. If wealth and the good are substitutes, the optimal rationing rule sets $\phi(w,c)=0$ if and only if $w\le w^*$ and $c\le c^*$. If they are complements, the optimal rationing rule sets $\phi(w,c)=0$ if and only if $w\ge w^*$ and $c\le c^*$. In either case, the optimal cost-effectiveness rule assigns the good to consumers with low costs. It is to be modified due to the change in the marginal utility of wealth from the consumption of the good.

In sum, the separable utility function is convenient. We have abstracted from secondary wealth effects due to the consumption of the good. In any case, when the consumption of the good results in small changes in marginal utilities of income, our results may continue to hold.

6.2 Benefits dependent on costs

Now we return to the separable utility function, but allow the consumer's benefit to vary with c. Let $\beta(c)$ be the benefit from the consumption of the good for consumer (w,c). When the public supplier does not observe cost, the expected value of $\beta(c)$ is used in computing optimal rationing policies. Our results regarding the public supplier's response against a private market quantity function remains valid.

Now suppose that the supplier has both wealth and cost information. The standard cost-effectiveness comparison will be modified as follows. If the private market is inactive, the comparison is between $\beta(c)$ and the shadow cost λc where λ is the multiplier of the budget constraint. The optimal rationing rule assigns the good to consumers where $\beta(c) - \lambda c > 0$. When β is concave, this means that low-cost consumers will be supplied by the public.

The private firm's profit-maximizing strategy is more complicated. The willingness to pay τ for consumer (w, c) is given by $U(w - \tau) + \beta(c) = U(w)$. Now τ depends on w and c, and is increasing in both arguments. At cost c, if the firm sets a price p, the demand is $\int_{p<\tau(w,c)} \mathrm{d}F(w)$ and the profit is $[p - c] \int_{p<\tau(w,c)} \mathrm{d}F(w)$. Profits may not be monotone, and may well be increasing in c; prices may not be monotone in cost either. Nevertheless, it seems unusual for firms to "prefer" costly patients. Our results in Sections 4 and 5 should continue to hold when the benefit function $\beta(c)$ is approximately a constant.

6.3 Correlation between wealth and costs

The case of correlated wealth and costs shares the same issues as the case of benefits dependent on costs. First, for the public supplier observing only wealth, the relevant distribution of cost is now a conditional distribution $G(c|w)$, and the conditional expected costs $\gamma(w) \equiv \int \gamma \ \mathrm{d}G(c|w)$. The budget constraint (6) becomes $\int_{\underline{w}}^{\overline{w}} \gamma(w)(1-\theta(w))f(w) \ \mathrm{d}w \leq B$. The first-order derivative with respect to θf becomes $-1+\lambda\gamma(w)$. Hence, if the conditional expected cost $\gamma(w)$ is increasing in w, the public supplier rations the rich consumers, and the converse is true if $\gamma(w)$ is decreasing in w. Second, the case of rationing by the public supplier observing both wealth and cost is unaffected by any correlation. The public supplier already observes both benefit and cost information, and applies the cost-effectiveness principle.

The private firm's profit function is now $[1-F(w|c)][\tau(w)-c]$, where $F(w|c)$ is the conditional distribution of w given c. The profit function may not be decreasing in c and the profit-maximizing price may not be increasing in c. For example, suppose that w and c are negatively correlated. Then if the firm observes consumers with low costs, then these must be relatively wealthy consumers and the firm sets a high price. Nevertheless, prices decreasing with costs seems unrealistic. When prices are increasing in costs, our results in Sections 4 and 5 should be valid.

7 Concluding remarks

We have introduced a framework for studying strategic interactions between public and private sectors. In our model, the public sector uses nonprice rationing and the private sector uses a pricing rule. We derive equilibria when rationing is based on consumers' wealth or both wealth and cost information. Equilibria do look like common rationing schemes: rich consumers are rationed, public supply is based on a cost-effectiveness measure, while prices in the private sector are high because rich consumers seek services there.

While quantity policies in the public sector are common, our model can be adapted for studying monetary subsidies and their effects on pricing in the private sector. It can also be used for studying quality differences between the public and private sectors. The previous section reviews some of the possible issues for more general model specifications. While we believe that our results are robust against small changes in model specifications, extending the model to include the general specifications may be worthwhile.

Finally, we have assumed a fixed budget for the public supplier. Extending the model for to determine the budget endogenously seems interesting. Given a budget, our model yields equilibrium outcomes in Propositions 1 and 3. Moreover, the equilibrium values for the consumer welfare index can be obtained. Once the social cost for the budget is specified, the optimal level for the budget can be studied.

Appendix

Proof of Lemma 1: The proof of Lemma 2 below shows that the quantity function $\widehat{w}^m(c)$ must be increasing; simply set $\theta(w)$ to 1 for all w in Lemma 2. Here we show that it is strictly increasing. The profit function (3) is differentiable in w. The first-order derivative of $\pi(w, c)$ with respect to w is $-f(w)[\tau(w) - c] + \tau'(w)[1 - F(w)]$, and this vanishes at $w = \widehat{w}^m(c)$ since $f(w) > 0$ by assumption. Hence, for $\epsilon > 0$, $-f(w)[\tau(w) - (c - \epsilon)] + \tau'(w)[1 - F(w)]$ is strictly negative at $w = \widehat{w}^m(c)$. Again because $f(w) > 0$, lowering w from $\widehat{w}^m(c)$ must strictly increase profit at cost $c - \epsilon$. In other words, $\widehat{w}^m(c)$ does not maximize $\pi(w, c - \epsilon)$. We conclude that $\widehat{w}^m(c)$ must be strictly increasing. ■

Proof of Lemma 2: For $c_1 < c_2$, let $w_1 \in \widehat{w}(c_1)$ and $w_2 \in \widehat{w}(c_2)$. Because the profit function $\pi(w; c, \theta)$ in (9) is strictly decreasing in c, we have $\widehat{\pi}(c_1) \geq \pi(w_2; c_1, \theta) > \pi(w_2; c_2, \theta) = \widehat{\pi}(c_2)$. Hence, the maximum profit function $\widehat{\pi}(c)$ is strictly decreasing in c.

Next, by the definitions of w_1 and w_2, we have

$$\int_{w_1}^{\overline{w}} \theta(w) f(w) \, \mathrm{d}w \;\; [\tau(w_1) - c_1] \;\; \geq \;\; \int_{w_2}^{\overline{w}} \theta(w) f(w) \, \mathrm{d}w \;\; [\tau(w_2) - c_1]$$

$$\int_{w_2}^{\overline{w}} \theta(w) f(w) \, \mathrm{d}w \;\; [\tau(w_2) - c_2] \;\; \geq \;\; \int_{w_1}^{\overline{w}} \theta(w) f(w) \, \mathrm{d}w \;\; [\tau(w_1) - c_2].$$

Adding these two inequalities yields

$$\int_{w_1}^{w_2} \theta(w) f(w) \, \mathrm{d}w \, [c_2 - c_1] \geq 0,$$

which says that w_2 must be at least w_1 since $\theta(w) \geq 0$. ■

Proof of Lemma 3: Consider w_1 and w_2 with $w_1 < w_2$. Evaluating (14) at w_1 and w_2 and then taking the difference, we have

$$\begin{aligned} \frac{\partial V}{\partial \theta f}_{w=w_2} &- \frac{\partial V}{\partial \theta f}_{w=w_1} \\ &= \int_{\underline{c}}^{\widehat{c}(w_1)} \{[U(w_2 - \tau(\widehat{w}(c))) - U(w_1 - \tau(\widehat{w}(c)))] - [U(w_2) - U(w_1)]\} \, g(c) \, \mathrm{d}c \\ &\qquad + \int_{\widehat{c}(w_1)}^{\widehat{c}(w_2)} \{U(w_2 - \tau(\widehat{w}(c))) + 1 - U(w_2)\} \, g(c) \, \mathrm{d}c \geq 0 \end{aligned} \tag{19}$$

The inequality in (19) follows from the concavity of U, and $\widehat{c}$ being increasing whenever $\widehat{c}(w) \neq \underline{c}$. Finally, (19) is zero if and only if $\widehat{c}(w_1) = \widehat{c}(w_2) = \underline{c}$. ■

Proof of Lemma 4: Because of the limited budget, the public supplier must leave some consumers to the private sector, so that $\theta(w) > 0$ for some w. Because the rationing rule depends only on wealth, some consumers must purchase from the private market. Let $\widetilde{w} = \inf\{w : \theta(w) > 0\}$, and for some $w > \widetilde{w}$, $\widehat{c}(w)$ must be higher than $\underline{c}$. By Lemma 3, the first-order derivative of the Lagrangean with respect to θf must be strictly increasing in w for $w > \widetilde{w}$. If $\theta(w) > 0$, then the first-order derivative (15) must be nonnegative, and for any $w' > w$, the value of (15) must be strictly positive, and $\theta(w') = 1$. ■

Proof of Lemma 5: If $\widehat{w}(c)$ is an equilibrium quantity function, it is increasing. Suppose that the Lemma is false. That is, suppose that for some $\widetilde{c} > \underline{c}$, $\widehat{w}(c)$ is strictly increasing for all $c < \widetilde{c}$. Then $\underline{w} < \widehat{w}(c) < \overline{w}$, for $c < \widetilde{c}$. which implies that $\widehat{c}(w) > \underline{c}$ for all w. By Lemma 3, the first-order derivative (15) must be strictly increasing at any w. This means that the set of w at which the first-order derivative (15) vanishes is a single point. Hence, in this equilibrium, the public supplier assigns the good to consumers with wealth below a certain threshold, and $\theta(w) = 0$ for $w < \widetilde{w}$. Against such a rationing scheme, the quantity function $\widehat{w}(c)$ that is strictly increasing for $c < \widetilde{c}$ cannot be optimal, because switching $\widehat{w}(c) < \widetilde{w}$ to $\widehat{w}(c) = \widetilde{w}$ yields strictly higher profit. ■

Proof of Proposition 1: We verify that the strategies in the proposition form an equilibrium. Given the public supplier's rationing scheme, it is optimal for the firm to set prices at $\tau(\widehat{w}^m(c))$ when $c > \widetilde{c}^s$, and at $\tau(\widetilde{w}^s)$ when $c < \widetilde{c}^s$. Given this quantity function $\widehat{w}$, we have the equivalent function $\widehat{c}$ where $\widehat{c}(w) = \underline{c}$ for $w < \widetilde{w}^s$, and $\widehat{c}(w) > \underline{c}$ for $w > \widetilde{w}^s$. For (15) we set the multiplier λ to $1/\gamma$. Then the first-order derivative (15) is zero for $w < \widetilde{w}^s$ and strictly positive for $w > \widetilde{w}^s$. Moreover, the budget constraint holds as an equality. Hence the rationing scheme defined in the proposition is optimal. The strategies form an equilibrium. ■

Proof of Proposition 2: In any equilibrium, the budget constraint $\gamma \int_{\underline{w}}^{\overline{w}} (1 - \theta(x)) f(x) dx \leq B$ must hold as an equality. The equilibrium in Proposition 1 supplies those with wealth between $\underline{w}$ and $\widetilde{w}^s$, where $F(\widetilde{w}^s)\gamma = B$, so $\theta(w) = 0$ for $w < \widetilde{w}^s$ and $\theta(w) = 1$ for $w > \widetilde{w}^s$. Consider any other equilibrium. Here, the supplier must ration some consumers with wealth below $\widetilde{w}^s$ because the budget constraint must hold.

Hence, for this equilibrium the threshold $\widetilde{w}^e$ at which $\theta(w) = 1$ for $w > \widetilde{w}^e$ must be strictly higher than $\widetilde{w}^s$.

Let $\widetilde{c}^e$ be defined by $\widehat{w}^m(\widetilde{c}^e) = \widetilde{w}^e$. Clearly, $\widetilde{c}^e > \widetilde{c}^s$ because $\widehat{w}^m$ is strictly increasing and $\widetilde{w}^e > \widetilde{w}^s$. We now show that the firm's equilibrium price is $\tau(\widehat{w}^m(\widetilde{c}^e))$ for $c < \widetilde{c}^e$. At $w < \widetilde{w}^e$, the first-order derivative of the Lagrangean (15) must be nonnegative; if that derivative was negative, then $\theta(w) = 0$, which would violate the budget constraint. Because $\theta(w) = 1$ for $w > \widetilde{w}^e$, the value of (15) is positive for all $w > \widetilde{w}^e$. Therefore, by Lemma 3, for $w < \widetilde{w}^e$ the value of (15) must be exactly zero and $\widehat{c}(w) = \underline{c}$ for $w < \widetilde{w}^e$. Because $\theta(w) = 1$ for $w > \widetilde{w}^e$ the equilibrium quantity must be $\widehat{w}^m(c)$ for $c > \widetilde{c}^e$, and remains constant at $\widehat{w}^m(\widetilde{c}^e)$ for $c < \widetilde{c}^e$.

From $\widetilde{c}^s < \widetilde{c}^e$ and $\widetilde{w}^e > \widetilde{w}^s$, by comparing the values of (12) across the equilibrium in Proposition 1 and the alternative, we conclude that the supplier's payoff is higher in the equilibrium in Proposition 1.

Finally, consider the equilibrium price function in Proposition 1 and any other equilibrium price function. The cost threshold is $\widetilde{c}^s$ in Proposition 1, and $\widetilde{c}^e$ in the other, where the prices remain constant at $\tau(\widehat{w}^m(\widetilde{c}^s))$ and $\tau(\widehat{w}^m(\widetilde{c}^e))$ respectively when cost falls below these thresholds. The private firm sets the same price and sells to the same set of consumers when $c > \widetilde{c}^e$, making the same profit in the two equilibria. Now for $c < \widetilde{c}^e$, in the equilibrium in Proposition 1, the firm could have set a price at $\tau(\widehat{w}^m(\widetilde{c}^e))$ and sell to less consumers but has chosen to set a lower price at $\tau(\widehat{w}^m(c))$ or $\tau(\widehat{w}^m(\widetilde{c}^s))$. Therefore, the firm makes more profit at $c < \widetilde{c}^e$ in the equilibrium in Proposition 1. We conclude that the private firm makes the largest equilibrium expected profit in the equilibrium in Proposition 1. ■

Proof of Lemma 6: In an equilibrium, the public supplier chooses ϕ to maximize (17) subject to the budget constraint (8), given a quantity function $\widehat{w}(c)$. Consider pointwise maximization at each (w, c). The first-order derivative with respect to ϕ is

$$\max\left[U(w - \tau(\widehat{w}(c))) + 1, U(w)\right] - [U(w) + 1] + \lambda c, \tag{20}$$

where $\lambda > 0$ is the multiplier.

Given a quantity function, if consumer (w, c) prefers to purchase from the private firm, the expression in (20) becomes

$$U(w - \tau(\widehat{w}(c))) - U(w) + \lambda c \tag{21}$$

Otherwise, the expression in (20) becomes

$$-1 + \lambda c. \tag{22}$$

When $U(w - \tau(\widehat{w}(c))) + 1 > U(w)$, the value in (21) is larger than (22).

Consider all consumers (w, c) who do not purchase from the private firm. Now the first-order derivative is given by (22), which is strictly increasing in c. Furthermore, if $-1 + \lambda c > 0$, then the expression in (21) is also strictly positive.

Now we claim that in an equilibrium expression (22) must not be always strictly positive. Suppose not, then $-1 + \lambda c > 0$ for all c, and therefore, the first-order derivative (20) is always strictly positive. The supplier rations all consumers so that $\phi(w, c) = 1$ for all w and c. This implies that the supplier does not use its budget, and this cannot be optimal. We conclude that there must exist $\widetilde{c} > \underline{c}$ such that (22) vanishes at $c = \widetilde{c}$.

Finally, whenever $c > \widetilde{c}$, $-1 + \lambda c > 0$ so that the first-order derivative (20) is strictly positive. We conclude that $\phi(w, c) = 1$, all $c > \widetilde{c}$ and any w. ■

Proof of Lemma 7: Consider $c < \widetilde{c}$. From Lemma 6, the value of (22) is strictly negative for $c < \widetilde{c}$. For $w < \widehat{w}(c)$, $U(w - \tau(\widehat{w}(c))) + 1 < U(w)$ so that (20) takes the value of (22) and it is negative. We conclude that $\phi(w, c) = 0$.

If $\widehat{w}(c) = \overline{w}$, the proof is complete, so now we prove that in an equilibrium, for $c < \widetilde{c}$, $\widehat{w}(c) = \overline{w}$. Suppose not; that is, suppose that $\widehat{w}(c) < \overline{w}$. Now at $w = \widehat{w}(c)$, $U(w - \tau(\widehat{w}(c))) + 1 = U(w)$, so that the derivative (20) is negative. For $\epsilon > 0$ and sufficiently small, $U(w + \epsilon - \tau(\widehat{w}(c)) - U(w + \epsilon) + \lambda c < 0$, so that the derivative (20) remains negative, and $\phi(w + \epsilon, c) = 0$. Now given that at c, consumers with wealth between $w = \widehat{w}(c)$ and $w = \widehat{w}(c) + \epsilon$ are supplied by the public sector, the private firm will raise the price from $\tau(\widehat{w}(c))$, or equivalently raise the equilibrium quantity function from $\widehat{w}(c)$. This contradicts the assumption that $\widehat{w}(c) < \overline{w}$ is an equilibrium quantity function. We conclude that for $c < \widetilde{c}$, $\widehat{w}(c) = \overline{w}$. ■

Proof of Corollary 2: First, set $\tau(\widehat{w}(c))$ to c in the proof of Lemma 6. It follows that $\phi(w, c) = 1$ for $c > \widetilde{c}^p$, some $\widetilde{c}^p > \underline{c}$. Now consider $c < \widetilde{c}^p$. For those consumer (w, c) who do not purchase from the private sector, the proof of Lemma 7 applies and $\phi(w, c) = 0$.

Consider consumer (w, c), $c < \widetilde{c}^p$, and $U(w-c)+1 > U(w)$, so that this consumer purchases from the private sector at cost c. The first-order derivative of the Lagrangean with respect to ϕ is

$$U(w-c) - U(w) + \lambda c, \tag{23}$$

where $\lambda > 0$ is the multiplier for the budget constraint. Expression (23) is negative at $c < \widetilde{c}^p$ and $w = \widehat{w}(c)$. Hence if there exists $w' > \widehat{w}(c)$ such that $U(w'-c) - U(w') + \lambda c > 0$, then $\phi(w', c) = 1$. The function $\mu(c)$ in the corollary is implicitly defined by setting the first-order derivative (23) to 0. ■

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