Report submitted as a part of the PhD Qualifying Research Preparation Criterion for the year 2013

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26th April, 2013
Optical Nonlinearities in Fibers: Review, Recent Examples, and Systems Applications

Abstract - Optical fibers offer a number of optical nonlinear effects, which come with their associated utilities and drawbacks. These nonlinearities are essential in order to implement the all-optical functionalities, which solely require tradeoff between the excellences and the adversities they offer, in order to utilize their potential to the fullest. In this condensed report, a concise and comprehensive study of the fiber nonlinearities and their consequences has been demonstrated. Along with the fundamentals, a special effort has been made to point out the impacts of different fiber parameters related to both the material and the fiber geometry and the interplay between the two. The report also refers to numerous recent applications that exploited these nonlinearities.

I. INTRODUCTION

The relatively low threshold of optical fibers for nonlinear effects can be a serious disadvantage in optical communications, especially in wavelength-division multiplexing (WDM) systems. On the other hand, nonlinearities in fibers can be very beneficial for a number of applications, starting with distributed in-fiber amplification and extending to wavelength conversion, multiplexing and demultiplexing, pulse regeneration, optical monitoring, and switching etc. [1]. Potentially, the development of the next generation of optical communication networks is going to rely strongly on fiber nonlinearities in order to implement all-optical functionalities; which solely requires the tradeoff between advantages and disadvantages of the nonlinear effects in order to utilize their potential to the fullest [2].

Interest in nonlinear fiber optics developed with the rapid growth of optical-fiber communications in the early 1980s and has been strong for the past 30 years. In this condensed report, summarized from the paper authored by J. Toulouse, a concise and comprehensive study of fiber nonlinearities has been demonstrated. Besides presenting a discussion on the fundamental nonlinear effects themselves, a special effort has been made to point out the impact of different fiber parameters related to both the material composition and the fiber geometry and the interplay between the two.

Although silica has a very small nonlinear index \( n_2 = 2.6 \times 10^{-16} \text{ cm}^2/\text{W} \), two of the fiber parameters can strongly enhance optical nonlinearities: the core size and the length of the fiber. The nonlinearities in bulk and silica fibers, respectively, are in the ratio [3]:

\[
\frac{I_{\text{f eff}}}{I_{\text{b eff}}} = \frac{\lambda}{\pi \omega^2 \alpha}
\]

where \( I_{\text{f b}} \) is the intensity in the fiber and the bulk, respectively, \( L_{\text{eff}} \) is the effective length, which for a long fiber is approximately equal to the inverse of the loss \((1/\alpha)\), \( \lambda \) is the wavelength, and \( r \) is radius of the fiber core. As seen from (1), a small core radius and low loss can greatly enhance the efficiency of optical nonlinearities.

Important fiber parameters for understanding fiber nonlinearity are the effective core area \( A_{\text{eff}} \) and effective fiber length \( L_{\text{eff}} \). \( A_{\text{eff}} \) is the area, the core would have if the optical intensity was uniformly distributed over it and zero outside. Assuming the fundamental optical beam launched is Gaussian, it is given by \( A_{\text{eff}} \sim \pi r_0^2 \). \( L_{\text{eff}} \), which is smaller than actual fiber length \( L \) due to fiber losses, is the length over which a signal would propagate through the fiber if it had a constant amplitude over that length and zero amplitude beyond. The third parameter is the group velocity dispersion (GVD) \( \beta_2 = [-\lambda^2/(2\pi c)](dn_2/d\lambda) \), where \( n_2 = [n - \lambda (dn/d\lambda)] \) is the group refractive index and \( n \), the normal index of refraction. The \( \beta_2 > 0 \) corresponds to normal dispersion regime where longer wavelengths travel faster, whereas \( \beta_2 < 0 \) corresponds to anomalous dispersion regime where the shorter wavelengths travel faster. In pure silica, \( \beta_2 = 0 \) at wavelength \( \sim 1310 \) nm, and that is why called the zero-dispersion wavelength (ZDW) \( \lambda_{ZDW} \). Fibers are often fabricated with \( \lambda_{ZDW} \) near \( 1550 \) nm in order to operate at the wavelength of minimum loss in silica, as well as to satisfy phase-matching conditions for nonlinear effects. This wavelength is also close to the maximum gain of erbium-doped fiber amplifiers (EDFA) at \( 1530 \) nm.
II. OPTICAL NONLINEARITIES

A. General

The optical nonlinearities considered in this report are those that can give rise to gain or amplification, conversion between wavelengths, generation of new wavelengths or frequencies, control of temporal and spectral shape of pulses, and switching. These effects result from the interaction between several optical fields simultaneously present in the fiber and may also involve acoustic waves or molecular vibrations. The two different types of nonlinearities can be distinguished as [4]:

I) Nonlinearities that arise from scattering:
   a) Stimulated Brillouin Scattering (SBS)
   b) Stimulated Raman Scattering (SRS)

II) Nonlinearities that arise from optically induced changes in the refractive index and result in:
   Phase Modulation:
      a) Self-Phase Modulation (SPM)
      b) Cross-Phase Modulation (XPM)

Mixing of several waves and the generation of new frequencies:
   a) Modulation Instability (MI)
   b) Parametric processes, such as four-wave mixing (FWM)

A large optical field modifies the optical response of the material in both types of nonlinearities and this material response to the optical field can be represented by an expansion of the polarization:

\[ P = \chi^{(1)}E + \chi^{(2)}EE + \chi^{(3)}EEE \]  

where \( \chi^{(n)} \) is the \( n \)-th order susceptibility. The various types of nonlinearities considered here can be expressed in terms of the real and imaginary parts of one of the nonlinear susceptibilities \( \chi^{(n)} \) appearing in (3).

Nonlinearities in fiber are essentially distributed in a sense that they cumulate and further develop with distance along the length of the fiber. Different lengths characterize different contributions to the development of particular types of nonlinearities. GVD directly affects the propagation of pulses and, therefore, affects the nonlinearities they can experience. For a pulse of initial input width \( \tau_0 \), dispersion length is defined as \( L_D = \tau_0^2/|\beta_2| \). If the pulse is initially Gaussian, its temporal width \( \tau(z) \) at a distance \( z \) increases as \( \tau(z) = \tau_0[1 + (z/L_D)^2]^{1/2} \).

The nonlinear length, which is defined as the length over which the co-propagating pump is effective in providing energy or gain, is given by \( L_{NL} = (GP)^{-1} \), where \( G \) is the gain and \( P \) is the pump power. In case of phase-matched nonlinearities, coherence length is defined as the length over which several co-propagating light waves lose their mutual phase coherence and is given by \( L_C = 2\pi/|k| \), where \( k \) is the phase mismatch. Finally, when considering polarization effects, the polarization beat length is defined as the length over which a phase difference of \( 2\pi \) develops between the x and y field components of light and this length is given by \( L_B = 2\pi/n_x - n_y \), where \( n_x \) and \( n_y \) are the refractive indices along x and y directions, respectively.

The dominant contributions to the development of nonlinearity, among these length parameters, come from those for which the corresponding characteristic length is the shortest. Different nonlinear contributions are only important when the corresponding characteristic lengths are comparable or shorter than \( L_{eff} \) of the fiber. If these lengths are much longer than \( L_{eff} \), then corresponding nonlinear contributions can essentially be ignored.

B. Scattering Nonlinearities

Type I) nonlinearities involve the lattice or vibrational dynamics of the fiber material and therefore, must satisfy the laws of conservation of both energy and momentum of the light and lattice taken together:

\[ \Omega = \omega_L - \omega_S \quad \& \quad \vec{q} = \vec{k}_L - \vec{k}_S \]  

where \( L \) and \( S \) stand for laser and Stokes, respectively, \( \omega \) and \( k \) are the frequency and wave vector of the light, and \( \Omega \) and \( q \), those of a lattice phonon. In SBS, the scattered or Stokes light is downshifted by the frequency of an acoustic phonon (~ 10 GHz), and in SRS, by the frequency of an optical phonon or molecular vibration. In silica, the SRS gain is maximum at \( \Omega = 13.2 \) THz from the laser line [5], which corresponds to the dominant Raman band, called the ‘broadband.’ SBS occurs at relatively low powers and is maximum in the backward direction and zero in the forward direction. In case of SRS, the scattering cross section exhibits a much smaller angular dependence, and Raman scattering can be observed both in the forward and backward directions, essentially slightly more efficiently in the forward direction [6].
It is worth noting that SBS and SRS processes are not simply spontaneous, rather stimulated, which means that the light itself creates these phonons and subsequently scatters from them. The single most important difference between them is that, the thermal phonons in spontaneous process are incoherent, while the light coherently creates phonons, thereby making the stimulated process much more efficient. The input power, for which phonons are created at a higher rate than the rate at which they are annihilated, is defined as the threshold for stimulated processes and can be expressed as [7]:

\[ P_{th} = \frac{C A_{\text{eff}}}{g L_{\text{eff}}} \]  

where \( g \), and \( C \) are, respectively, the gain coefficient, and a constant that depends on the particular scattering process. The threshold power for SBS and SRS are found to be approximately 2 dBm (1.6 mW) and 28 dBm (700 mW), respectively. In practice, variations in core size along the fiber and other inhomogeneities tend to raise the threshold to higher powers, between 5 and 10 dBm (3–10 mW) for SBS and between 28 and 32 dBm (0.7–1.17 W) for SRS. However, these powers are commonly reached in fibers so that SBS and SRS nonlinearities are often encountered in optical communications and can be exploited for practical purposes.

1) Stimulated Brillouin Scattering (SBS)

In SBS, the photo generated acoustic wave propagates collinearly with the incident pump beam, while the Stokes wave is backscattered, resulting in rapid increase in the reflected light with incident optical power and eventual saturation in the transmitted light. SBS threshold is particularly low due to the gain coefficient being relatively high. The effective Brillouin gain coefficient is given by:

\[ g_B = \frac{\Delta v_B}{\Delta v_B + \Delta v_S} g_B(v_B) \]

where \( g_B(v_B) \) (~5 × 10\(^{-11}\) m/W) is the maximum Brillouin gain obtained for a perfectly monochromatic signal, \( \Delta v_B \) and \( \Delta v_S \) are the spectral widths of the Brillouin and signal beams respectively. The Brillouin gain can thus be reduced, accordingly the threshold can be raised by increasing \( \Delta v_S \) through dithering. New fiber structures are being investigated with the development of new schemes to maximize SBS threshold. Promisingly, photonic crystal fibers (PCFs) showed SBS thresholds as high as 18 or 20 dBm [8]. The fundamental reason for the higher SBS thresholds of these new fibers may be a lower degree of overlap between the acoustic and optic modes.

Other common applications of SBS are in narrow line-width amplifiers [9] and lasers [10]. An original implementation of a Brillouin laser in an erbium-doped fiber was proposed and demonstrated [11], where the combination of SBS and erbium gain leads to the appearance of strong higher order Stokes waves [12] or a comb of frequencies with ~10-GHz line spacing [13]. Polarization plays an important role in SBS by minimizing the interaction of two optical modes having orthogonal states of polarization (SOP). Thus, the scheme of multiplexing two equivalent signals with orthogonal SOP, each with half the total launched power desired, is suggested to be a potential way to mitigate SBS is communication system.

SBS can also be used for remote time-domain reflectometry by exploiting the backscattered Stokes wave. From the corresponding change noted in the Stokes signal it is possible to detect any mechanical change in the fiber, and the location of this change can be determined remotely by its arrival time. Because of the polarization dependence of the SBS gain, this technique can also be used to determine the SOP of the light at any point along the fiber [14].

2) Stimulated Raman Scattering (SRS)

SRS differs from SBS in three ways. First, due to the lower Raman-gain coefficient \( g_R \) (~1 × 10\(^{-13}\) m/W, SRS occurs at much higher powers than SBS, which are typically greater than ~1 W. Second, the Raman shift, ~13.2 THz in silica, which is much greater than the Brillouin shift (~ 10 GHz). Thirdly, SRS generates a Stokes beam both in forwards and backwards directions, although more efficiently in the forward direction [6].

Raman fiber amplifiers offer two significant advantages over EDFAs. The first one is that, rather than giving maximum gain at a fixed absolute frequency, SRS provides maximum gain at a frequency shift that is relative to the pump wavelength and not at a fixed absolute frequency [15], which allows to choose maximum gain in any desired wavelength range, S-, C-, or L-band. Second, the gain bandwidth is much greater than that provided by EDFAs (> 100 nm versus 35 nm). Although the SRS gain curve is not really flat over the 100nm or so, it can
be made flat within 1–2 dB either by introducing wavelength-selective loss mechanism or by using several pumps at staggered wavelengths.

SRS can be used for lumped as well as distributed amplification in communication systems. The characteristic Raman-gain length \( L_G = \left( \frac{\phi_R P/A_{\text{eff}}}{P_0} \right)^{-1} \) is defined as the length at which pump has transferred most of its power to the signal, which then progressively attenuates itself along the remainder of the fiber that may result in low SNR. Therefore, a combination of pumps is often used, some co-propagating and the others counter-propagating with the signals, to optimize the optical-signal-to-noise (OSNR) ratio at the receiver.

SRS gain is also being extensively used in a cavity configuration for laser applications. A Raman active fiber (usually a small-core HNLF) placed between two sets of cascaded Bragg gratings makes a Raman fiber laser. Each grating pair defines a cavity that lases at a particular Stokes wavelength, and the successive gratings reflect light from increasingly higher order Stokes. A pair of gratings ensures sufficient gain at each successive Stokes wavelength. Although the intrinsic threshold for SRS is relatively high for a single pass, as in an amplifier, it can be much lower in a laser-cavity configuration because of the multiple passes of the beam through the Raman-gain medium [16].

Raman-mediated power transfer is a function of the signal power; this mechanism of SRS is exploited to transfer negative or complementary of data stream from Stokes to pump, also allows for switching and modulation of the pump by the Stokes.

In case of short pulse pumps rather than CW pump, walk-off length can be significantly shorter than the fiber length, which limits the Raman process to a duration, only during which pump and Stokes overlap, thereby limiting efficiency of the process. Also combination of SRS with other higher order nonlinearities becomes prominent when using short pump pulse. One of these effects is intrapulse Raman scattering, a combination of SRS and SPM or XPM that can lead to the formation of very short pulses or Raman solitons in the anomalous dispersion regime [17].

This higher order effect can be used to generate subpicosecond Stokes soliton pulses that are both tunable in frequency and in duration [18]. SRS and parametric FWM between pump and Stokes can generate anti-Stokes or, equivalently, two pump photons can simultaneously generate a Stokes and an anti-Stokes photon. With high-power picoseconds pulses, SRS & FWM together lead to supercontinuum generation.

C. \( \chi^{(3)} \) Nonlinearities

Type II) nonlinearities are often referred to as \( \chi^{(3)} \) nonlinearities which arise from light-induced nonlinear electronic polarization of the medium and can result in nonlinear refraction, a phenomenon referring to the intensity dependence of the refractive index (Kerr effect), or the mixing of optical waves (parametric interactions). The refractive index can be expressed as (7), where \( n_0 \) is linear index, \( n_2 \) is nonlinear coefficient, and I is the optical intensity. In silica, \( n_2 \approx 2.6 \times 10^{-16} \text{ cm}^2/\text{W} \). The relation between \( n_2 \) and \( \chi^{(3)} \) is given by (8) [19]. Practically, the coefficient that determines \( n_2 \) is the nonlinear parameter \( \gamma \), which can be expressed by (9).

\[
n = n_0 + n_2 I \\
n_2 = \frac{3}{8n_0} \text{Re} (\chi^{(3)}) \\
\gamma = \frac{2\pi n_2}{\lambda A_{\text{eff}}} \\
\]  

(7)  

(8)  

(9)

For a typical single-mode silica fiber, \( \gamma \sim 20 \text{ W}^{-1}\text{km}^{-1} \). The result of these nonlinearities introduce a nonlinear shift \( \phi_{\text{NL}} \) in the phase of the propagating light, given by (10), where \( P_0 \) is the peak input power. \( \phi_{\text{NL}} \) can also be rewritten in terms of a nonlinear length \( L_{\text{NL}} = \left( \frac{\gamma P_0}{\lambda A_{\text{eff}}} \right)^{-1} \) as in (11):

\[
\phi_{\text{NL}}(z) = \gamma P_0 z \\
\phi_{\text{NL}} = \frac{z}{L_{\text{NL}}} \\
\]

(10)  

(11)

For a 1mW input power at \( \lambda = 1.55\mu\text{m} \) in a single-mode fiber with \( A_{\text{eff}} = 50\mu\text{m}^2 \), \( L_{\text{NL}} \approx 500\text{m} \). This illustrates the importance of these nonlinearities in optical communication systems. \( \chi^{(3)} \) nonlinearities can be categorized into two groups: (a) SPM and XPM, (b) parametric processes, such as FWM and third harmonic generation (THG). SPM, XPM, and FWM, have a common origin, which can be shown mathematically by considering the interaction of two beams. The total electric field can then be written as in (12):
\[ E(r, t) = \frac{1}{2} [E_1 \exp(-i\omega_1 t) + E_2 \exp(-i\omega_2 t)] + c. c. \] (12)

Substitution of \( E(r, t) \) in (3) results in a number of different \( P_{NL} \) terms:

1) \( P_{NL}(\omega_1) \propto (|E_1|^2 + 2|E_1|^2)E_1 \) and \( P_{NL}(\omega_2) \propto (|E_2|^2 + 2|E_1|^2)E_2 \) contain both SPM (first term in each) and XPM (second term in each) terms

2) \( P_{NL}(2\omega_1 - \omega_2) \propto E_1^2E_2^* \) and \( P_{NL}(2\omega_2 - \omega_1) \propto E_2^2E_1^* \) represent FWM terms

Specific aspects and applications of each of these fiber nonlinearities have been introduced here successively.

1) Self-Phase Modulation (SPM)

In SPM, the intensity modulation of an optical beam results in the modulation of its own phase via refractive index modulation of the medium, resulting in time-dependent change or modulation of the phase itself, leading to spectral broadening or frequency chirping, which can be given by (13) [20]:

\[ \Delta \omega(z, t) = -\frac{\partial \phi_{NL}}{\partial t} = -\frac{2\pi n_2}{\lambda A_{eff}} \frac{dP(t)}{dt} z = -n_2 \frac{dI(t)}{dt} kz \] (13)

where \( I \) is optical intensity and \( P \) is the optical power. The time derivative in (13), says that SPM is essentially a pulse effect, with the leading edge of the pulse being red-shifted and the trailing edge blue-shifted. Interestingly, this nonlinear spectral broadening can be either compensated or magnified by the chromatic dispersion of the fiber. In the normal chromatic dispersion regime (\( \beta_2 > 0, \lambda < \lambda_{ZDW} \)), the nonlinear dispersion is magnified by the chromatic dispersion, resulting in enhanced broadening. In the anomalous dispersion regime (\( \beta_2 < 0, \lambda > \lambda_{ZDW} \)), the nonlinear dispersion is compensated, leading to pulse compression or, to the formation of solitons when exactly balanced. SPM can be used for the spectral and temporal compression of pulses, soliton generation and pulse regeneration with appropriate dispersion and pulse characteristics.

SPM results in spectral compression of a pulse, provided the pulse is negatively chirped initially. In that case, the higher frequency components at the leading edge of the pulse are being red-shifted by SPM, while the lower frequency components at the trailing edge of the pulse are simultaneously being blue-shifted, thus canceling the initial chirp of the pulse.

Temporal pulse compression is usually achieved in two stages, first by broadening the pulse spectrally through SPM in the presence of normal dispersion and then compressing the frequency components of the chirped pulse temporally in a section of fiber with anomalous dispersion. Pulse compression is an integral part of the pulse regeneration process in optical communication networks including retiming, recompression, and reamplification (3R). 3R regeneration can be achieved by synchronous modulation, SPM in an HLNF section, narrowband filtering or slicing and finally Raman or parametric reamplification [21].

Modulation instability (MI) is another nonlinear effect resulting from SPM in the anomalous dispersion regime, which leads to the breakup of a CW wave into a train of very narrow pulses [22], when the CW beam is subjected to a small periodic perturbation with frequency \( \Omega \) and wavevector \( k \). It can be shown that \( k \) becomes imaginary for perturbation frequencies \( \Omega < \Omega_\text{c} \), where \( \Omega_\text{c} \) can be expressed as [4], [23]:

\[ \Omega_\text{c}^2 = \frac{4\gamma P_0}{|\beta_2|} = \frac{4}{|\beta_2| L_{NL}} \] (14)

The perturbation introduces a dynamical modulation of the \( \phi_{NL} \) and a periodic chirp of the CW beam. Under the influence of the GVD (\( \beta_2 \)), this periodic chirp leads to the breakup of the CW beam into a train of ultrashort pulses with a repetition rate of \( \Omega \). The fastest growth of these pulses or the maximum MI gain occurs for \( \Omega_{\text{max}} = \Omega_c / \sqrt{2} \). In case of vectorial or polarization modulation instability (PMI), where the CW beam excites both of its polarization components concurrently, PMI occurs due to the exponential growth of a periodic perturbation and manifests itself by the breakup of the CW beam into a train of pulses. PMI can, in fact, be described as the result of XPM between these two polarized components in the presence of a periodic perturbation on the amplitude of the CW beam. As it involves XPM between two polarization components, PMI can occur in both the anomalous and the normal dispersion regime.

2) Cross-Phase Modulation (XPM)

XPM is a similar effect to SPM, but it involves two optical beams instead of one. In XPM, the intensity modulation of one of the beams results in a phase modulation of the other and the phase modulation translates into a frequency modulation through the pulse, which accordingly broadens the spectrum. However, due to the
fact that the total intensity is equivalent to the square of the sum of the two electric-field amplitudes, spectral broadening in XPM is twice as large as in SPM.

\[ n = n_0 + n_2 I = n_0 + n_2 |E_1 + E_2|^2 \quad \Rightarrow \quad \phi_{NL}(z) = \frac{2\pi}{\lambda} n_2 z \left(|E_1|^2 + 2|E_2|^2\right) \]  (15)

The expression for \( \phi_{NL} \) in (15) manifests the fact that XPM (second term) is always accompanied by SPM (first term). A similar expression can be written for the second beam, \( \phi_{NL}^{(2)} \). If one of the two beams (the pump) is much stronger than the other (the probe or signal), XPM primarily acts from that pump beam to the weaker signal beam. XPM requires that the two beams overlap in time and in space, that means in case of pulses that, they should have similar GVDs, so that the two modes do not walk off each other.

Similar to SPM, XPM introduces a nonlinear phase shift to the pulse which, according to (13), translates into a spectral broadening. The combined effect of the XPM induced chirp and the GVD, leads to the development of a multipeak temporal structure of the pulses or optical wavebreaking. As for SPM, the XPM induced MI, resulting from a periodic perturbation, can lead to breakup of a CW pulse into a train of pulses.

Unlike SPM, XPM can occur when either one or both of the beams are in normal as well as in anomalous regime, although there will be difference in the stability range for the two cases since the nonlinear dispersion can either be compensated or magnified by the intrinsic chromatic dispersions of the beams. The nonlinear phase shift and its relative weight with respect to intrinsic dispersion can change as the two beams with their different GVDs propagate along the fiber, unlike SPM.

XPM can be used for a number of all-optical applications in communication networks: wavelength conversion, demultiplexing, switching, and other optical-control applications. Since XPM occurs jointly with SPM, it can be used for simultaneous demultiplexing and regeneration [24]. It can be used advantageously for control applications because it can alter pulse shape and timing significantly without causing energy exchange between the beams. In particular, one can use a ‘shepherd’ pulse at a separate wavelength from the signal to manipulate and control the signal pulses [25].

On the reverse side, XPM can induce crosstalk between nearby channels causing potential problem in WDM networks. This can affect the pulse shapes and amplitudes in different channels and lead to the time-dependent depolarization of nearby channels [26]. This problem is handled by using dispersion managed fibers. XPM also results in nonlinear contributions to birefringence, given in (16). It appears clear that, the nonlinear birefringence and related effects must depend on the relative optical intensities in the x and y direction. These two components interact and modulate each other nonlinearly in a way analogous to XPM, resulting in a relative nonlinear phase shift between the two components, given in (17).

\[ \Delta n_x = n_2 \left(|E_x|^2 + \frac{2}{3}|E_y|^2\right) \quad \text{and} \quad \Delta n_y = n_2 \left(|E_y|^2 + \frac{2}{3}|E_x|^2\right) \]  (16)

\[ \Delta \phi_{NL} = \gamma L_{\text{eff}} (1 - B)(P_x - P_y) \]  (17)

where \( P_{x,y} \) are the powers in the x and y components, respectively, and B is the ellipticity of the fiber (B= 2/3 for a linearly birefringent fiber). Such a relative nonlinear phase shift can be introduced by co-propagating, along with a weak arbitrarily polarized signal, a strong pump polarized along the x-axis of the fiber.

3) Solitons

Under the combined effect of SPM or XPM and dispersion, short pulses can evolve towards a solitonic state, in which they preserve their shape as they propagate, and can travel undistorted over long distances. Solitons generate when the nonlinear dispersion is exactly compensated by the intrinsic chromatic dispersion across the entire pulse. Pulses, that satisfy the conditions for the solitonic state, are those whose normalized shape can be described by the sech function:

\[ u(\xi, \tau) = \eta \text{sech}(\eta \tau) \exp \left( \frac{i\eta^2 \xi}{2} \right) \]  (18)

where \( \eta \) designates soliton amplitude, \( \tau = t/t_0 \) is the time \( t \) normalized by the width of the incident pulse \( t_0 \), and \( \xi = z/L_0 \) is the distance traveled normalized by the dispersion length \( L_0 \). A fundamental characteristic of solitons is that, its width changes with its amplitude as \( t_0/\eta \), i.e., width is inversely proportional to amplitude.

In practice, very short pulses can spontaneously evolve towards a solitonic state. For example, the ultra short pulses generated through MI can evolve into solitons by shedding energy at their edges where SPM is not as strong as in the central part of the pulse. Alternatively, sufficiently narrow and energetic pulses directly launched into a fiber can also evolve into solitonic state in the same manner. However, they can be annihilated...
by perturbations, such as loss or noise along the fiber. The dispersion managed solitons, those created in dispersion managed fibers, are not exactly similar to the solitons mentioned above. Although they can retain their shape over long distances, their amplitude and width oscillate in a periodic manner, they are chirped, and their shape is closer to a Gaussian form than to a hyperbolic secant form.

Solitons are now extensively being used in long-haul optical communication systems. The solitons discussed above are temporal solitons. Another type of soliton has been extensively studied, the spatial soliton [27]. A soliton is a self-reinforcing linearly localized wave. Spatial solitons are now extensively being used in long-haul optical communication systems. The solitons discussed above are temporal solitons. Another type of soliton has been extensively studied, the spatial soliton [27].

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4) Parametric Processes – FWM

The interaction of two or more light waves can lead to a second kind of $\chi^{(3)}$ nonlinearities, referred to as parametric processes, which involves an energy transfer between waves and not simply a modulation of the index seen by one of them due to the other. It requires sufficiently high powers to induce optical nonlinearities ($n_2$ or $\gamma$), and also the two interacting light waves be phase-matched meaning that their phase velocities be the same. These make FWM much more stringent to occur than XPM, where only the group velocities needed to be similar so that pulses would overlap. The phase matching condition applies to the sum of the wavevectors of the different waves participating in the process and can be written as:

$$\kappa \equiv \Delta k_M + \Delta k_W + \Delta k_{NL} = 0$$  \hspace{1cm} (19)

where $\Delta k_M$ is material contribution, $\Delta k_W$ is waveguide contribution, and $\Delta k_{NL} = \sum \gamma P_i$ is nonlinear contribution. It is clear from (19) that $\kappa = 0$ can be satisfied when one of the dispersion terms is negative, for example, $\Delta k_M$ in the anomalous dispersion regime. Wavelength range near the ZDW can be used, where the waveguide and nonlinear dispersion can be adjusted to cancel the small material dispersion; the first one can be adjusted through proper fiber design, and the second, by proper choice of the optical powers.

The $\chi^{(3)}$ parametric processes generate FWM, THG, and parametric gain. Here, three waves interact to produce a fourth one. Being a coherent interaction (between fields and not just between intensities or powers), both frequencies and wavevectors must be conserved (energy conservation and wavevector conservation or phase matching) (20), which results in a variety of parametric processes depending on the particular product of the four fields.

However, mixing of four waves of different frequencies under the condition of phase matching is not very likely in general. Two common such processes are FWM (21) and THG (23). In degenerate FWM, a special case, two high-intensity waves, with respective frequencies $\omega_1 = \omega_2$ and $\omega_3 = \omega_4$, interact and generate two new waves at $\omega_1$ and $\omega_3$ and $\omega_2$, respectively designated as Stokes and anti-Stokes, by analogy with the SRS, or the signal and the idler.

$$\pm \omega_1 \pm \omega_2 = \pm \omega_3$$

- in FWM: $\omega_1 + \omega_2 = \omega_3 + \omega_4$ and $k_1 + k_2 = k_3 + k_4$  \hspace{1cm} (20)

- in Partially degenerate FWM: $\omega_1 = \omega_2$ \rightarrow $2\omega_1 = \omega_3 = \omega_4 \rightarrow \omega_1 - \omega_3 = \omega_4 - \omega_1$  \hspace{1cm} (22)

$\kappa$ is nonlinear parameter (9), $\kappa$, the phase mismatch introduced earlier (19), and $P_1$ is the pump power. As indicated above, $\kappa = 0$ is required for perfect phase matching. It turns out that, because of the generation of
an idler, with FWM both amplification of the signal and wavelength conversion can be done concurrently. Other dual- purpose applications are parametric amplification and demultiplexing in WDM systems, when the signal is composed of multiwavelengths and the desired wavelength can then be isolated through filtering [30].

FWM is also used for optical regeneration and reshaping of pulses. FWM-based dispersion monitoring method has also been demonstrated, where the signal pulse stream, used as a parametric pump, is mixed with a weak CW wave and the power of the idler generated is shown to depend on the pulse width, and therefore on the accumulated dispersion [31]. Due to the phase-matching condition, dispersion plays an even bigger role in FWM than in other nonlinear processes. In conventional fibers, dispersion fluctuations can translate into $\lambda_{ZDW}$ fluctuations and reduce FWM efficiency. Polarization mode dispersion (PMD) constitutes another source of dispersion, resulting in fluctuations in idler power causing fluctuations in $\lambda_{ZDW}$. Finally, parametric processes, in general, and FWM in particular, make a special contribution to the nonlinear generation of a supercontinuum.

### III. HIGHLY NONLINEAR FIBERS (HLNFs)

In order to take advantage of the nonlinearities described above and to further enhance them, novel optical fibers have been designed and fabricated. We know that $A_{\text{eff}}, L_{\text{eff}}$ and $\gamma$ are the parameters that determine strength of these nonlinearities. Clearly, these nonlinearities become important when $L_{\text{eff}}$ exceeds the relevant nonlinear length $L_{\text{NL}}$. For the development of distributed optical nonlinearities in fibers, chromatic dispersion length $L_D$ is again an important parameter. The shorter the $L_D$, the more important the role dispersion plays.

HNLFs can be obtained by reducing $A_{\text{eff}}$ and increasing the index contrast $\Delta n = n_{\text{core}} - n_{\text{clad}}$, in order to confine the mode more tightly in the core, or by increasing the gain coefficient $g$. Reduced size of the doped region increases the optical power density in the core. Simultaneously, increased doping level increases the $\Delta n$, thus pulling the mode further inward to the center of the core region. Rather than using Ge-doping, Pb-doped and Bi$_2$O$_3$-doped small core silica fibers showed much higher nonlinear coefficients but with greater loss.

An advantage in changing the index profile is the possibility to shift the ZDW from 1310 nm to the most common operating wavelength of 1550 nm, and thus enhance optical nonlinearities at that wavelength. This can be achieved with a high Ge-doping level in the core and fluoride doping of the cladding, which, in addition to shifting $\lambda_{ZDW}$ to 1550 nm, also increases the $\Delta n$ and therefore the confinement [32].

Fibers made of glasses other than silica also show much promise for nonlinear applications. The most investigated are tellurites, chalcohalides, chalcogenides etc. In designing fibers with particular dispersion characteristics, PCFs offer the greatest versatility yet. These are fibers in which the cladding is composed of air holes running the length of the fiber parallel to the core.

**Photonic Crystal Fibers (PCFs)**

The invention of the microstructured fibers (PCF and photonic bandgap fibers (PBGF)) has added an entirely new dimension to nonlinear optical effects and applications in fibers. PCFs are index guiding, i.e., they guide light through total internal reflection (TIR). Here the effective refractive index ($n_{\text{eff}}$) of holey cladding is a decreasing function of wavelength, which allows PCFs to operate as a single mode structure over a broad range of wavelengths. This phenomenon, called endlessly single mode guidance, can be understood from (25):

$$V = \frac{2\pi}{\lambda} a (n_{\text{core}}^2 - n_{\text{clad}}^2(\lambda))^{1/2}$$

(25)

where $\lambda$ is free-space wavelength, $a$ is core radius, and $n_{\text{clad}}$ is an effective index that depends on the particular geometry of the cladding (hole size $d$ and hole-to-hole distance or pitch $\Lambda$). In practice, PCFs with $d/\Lambda \leq 0.45$ are single mode at all wavelengths, $V$ being less than 2.405 which is the cutoff $V$ parameter for fundamental mode. For values greater than 0.45, PCFs still can behave as single-mode fibers for wavelengths that are longer than a certain cutoff wavelength. Besides, due to the large index contrast, $A_{\text{eff}}$ can be made very small compared to conventional fibers and the light can more tightly be confined within the solid core, leading to high optical power density and efficient nonlinearity. Consequently, PCFs are characterized by a greater critical angle $\theta_c$ and a much greater numerical aperture NA (26), which in turn increases coupling loss between PCF and the conventional fiber.

$$NA = \sin \theta_c \approx \left(1 + \frac{\pi A_{\text{eff}}}{\lambda^2}\right)^{-1/2}$$

(26)

Although a small-core PCF offers tight mode confinement, it can also be accompanied by significantly higher confinement losses, which are damaging for applications [33]. Proper design can minimize these losses and
yield optical nonlinearities in PCF more than 50 times higher than in conventional fibers. Importantly, shorter lengths of fibers are needed for this, so that loss becomes less of an issue. In addition to these special modal properties, PCFs also possess very special dispersion properties, which depend again on the geometry parameters, d and Λ. In particular, they often exhibit an inverted bell-shaped dispersion curve, with two ZDWs.

PCF is particularly suitable for the observation of one of most spectacular nonlinear effects, super continuum generation (SCG). Interestingly, it appears that this mechanism depends on the duration of the initial pulse. For pulses in the femtosecond range (100–200 fs), continuum is initially generated through the formation of a Raman soliton, followed by soliton fission. For pulses in the picosecond range (10–100 ps), SCG results from SRS for generation of the longer wavelengths, followed by FWM for the generation of the shorter ones. However, it is important to stress that, the shape and width of the generated spectrum are also very sensitive to the initial pulse energy [34]. Because some of the processes involved in SCG require phase matching, it is also sensitive to the dispersion characteristics of the fiber. When applied to the case of SCG, the dispersion of the fiber can be engineered so as to enhance the transfer of energy from the monochromatic pump beam preferentially to a particular wavelength range. PCFs have also fabricated with other glass materials, like tellurite and Bi$_2$O$_3$ providing much higher $\gamma$ parameters (~ 460W$^{-1}$km$^{-1}$).

A number of applications of PCFs have already been demonstrated in optical communication networks. For example, tunable wavelength converter, a retiming and reamplification (2R) regenerative all optical switch, broadband pulse source etc.

The other type of microstructured fiber is the hollow core fiber or PBGF, in which light is guided through a photonic bandgap mechanism. Although these fibers are not intrinsically nonlinear due to having air core, they can be loaded with gases or liquids that can themselves exhibit nonlinear effects. SRS has thus been reported in a hydrogen loaded PBGF [35].

**IV. CONCLUSION**

Optical nonlinearities in fibers give rise to numerous potential effects. The extensive number of new phenomena occur due to the facts that, these nonlinearities operate in a distributed manner in the fiber and, most importantly, several nonlinearities may act simultaneously, resulting in an even greater variety of manifestations; supercontinuum generation (SCG) and soliton formation, etc. Nonlinear effects can be detrimental for optical communications, especially in WDM, where they can result in back scattering (SBS), noise (spontaneous Raman), pulse distortion (SPM, XPM, MI), and crosstalk between channels (XPM, FWM). Conversely, they are extremely useful for a variety of applications, from fiber lasers and amplifiers to wavelength converters, demultiplexers, optical switches, etc. Nonlinear effects is particularly important in the formation of all optical networks, where the prior challenge is to control these nonlinearities and their interplay. This certainly requires new types of fibers, e.g., new glass compositions, fiber designs, dispersion maps, and birefringence characteristics, that can be precisely tuned to properly balance the desired nonlinearities. In this regard, PCFs hold greater promise.

**REFERENCES**