

Maintenance of equatorial superrotation in the atmospheres of Venus and Titan

Xun Zhu*

The Johns Hopkins University Applied Physics Laboratory, 11100 Johns Hopkins Road, Laurel, MD 20723-6099, USA

Received 16 December 2005; received in revised form 10 May 2006; accepted 11 May 2006

Abstract

This paper extends Leovy's theory on Venus' equatorial superrotation by analytically examining additional terms in the mean zonal momentum equation that stably balances the momentum source of pumping by thermal tides. The general analytical solution is applied to the atmospheres of both Venus and Saturn's moon Titan. The main results are: (i) Venus' equatorial superrotation of 118 m s^{-1} results primarily from a balance between the momentum source of pumping by thermal tides and the momentum sink of meridional advection of wind shear by horizontal branches of the Hadley circulation; (ii) no solution is found for Titan's stratospheric equatorial superrotation centered at the 1-hPa level; (iii) however, if the main solar radiation absorption layer in Titan's stratosphere is lifted from 1 hPa ($\sim 185 \text{ km}$) to 0.1 hPa ($\sim 288 \text{ km}$), an equatorial superrotation of $\sim 110 \text{ m s}^{-1}$ centered at 0.1-hPa could be maintained. Titan's equatorial superrotation results mainly from a balance between the momentum source of tidal pumping and the momentum sink of frictional drag. © 2006 Elsevier Ltd. All rights reserved.

Keywords: Venus; Titan; Atmosphere superrotation

1. Introduction

A long-standing problem in planetary atmospheric dynamics (including planetary bodies such as Titan) is the maintenance of equatorial superrotation in the atmospheres of slowly rotating planets. Strong equatorial superrotational zonal winds of $\sim 100 \text{ m s}^{-1}$ were first observed at Venus' cloud-top level (e.g., Schubert, 1983), which yields an atmospheric angular velocity ~ 55 times greater than the Venus' solid body rotation. Several mechanisms have been proposed to explain the generation or maintenance of Venus' cloud-top level superrotation (e.g., Schubert and Whitehead, 1969; Fels and Lindzen, 1974; Gierasch, 1975; Rossow and Williams, 1979; Walterscheid et al., 1985; Leovy, 1987; Hou et al., 1990). Most theories have described the possible mechanisms from one (above first four references) or two (above last three references) particular zonal momentum sources or

sinks, either qualitatively or quantitatively. Also, most theories have studied equatorial superrotation based on a set of atmospheric parameters specifically adapted to Venus' atmosphere. Two well-known theories that explain the maintenance of the Venus equatorial superrotation are: (i) meridional momentum transport from mid-latitudes by eddy mixing (Gierasch, 1975; Rossow and Williams, 1979); and (ii) momentum pumping by thermal tides (Fels and Lindzen, 1974).

Several atmospheric general circulation models (GCMs) have produced near-global superrotations for Venus or/and Titan's atmosphere without a tidal pumping mechanism (e.g., Hourdin et al., 1995; Del Genio and Zhou, 1996; Yamamoto and Takahashi, 2003) by adopting a zonally averaged solar heating. The maintenance of equatorial superrotation in these models has been explained by horizontal eddy mixing that corresponds to an inverse cascade of energy from small-scale motions to large-scale ones in two-dimensional (2D) turbulence. Such an inverse cascade that transports the angular momentum equatorward by barotropic waves can also be simulated and

*Corresponding author. Tel.: +1 443 778 8764; fax: +1 443 778 1641.
E-mail address: xun.zhu@jhuapl.edu.

analyzed in a slowly rotating atmosphere by a shallow-water model (e.g., Luz and Hourdin, 2003; Iga and Matsuda, 2005) that carries the a priori assumption of a 2D flow.

The maintenance of stable equatorial superrotational winds requires a balance of two terms in the averaged momentum equation. Leovy (1987) examined the balance between the following two terms at Venus' cloud-top level: (i) wave pumping by semi-diurnal tides proposed by Fels and Lindzen (1974) as a momentum source, versus (ii) vertical advection of wind shear by the upward branch of the Hadley circulation as a momentum sink. The exact physical mechanism that maintains equatorial superrotation on Venus remains a mystery, since the complete mean zonal momentum equation contains at least 10 terms representing the physical processes contributing to the momentum sources and sinks. These terms have never been systematically and self-consistently evaluated in the same dynamical frame. This lack of resolution of the superrotation issue on Venus has led to further confusion and debate concerning equatorial superrotation in Titan's

equatorial superrotation. Furthermore, since the Earth's atmospheric circulation is best understood and does not have a strong equatorial superrotation in the stratosphere, we also briefly apply the analytic formula to the Earth atmosphere to demonstrate the non-existence of the equatorial superrotation with the extended theory.

In Section 2, we first perform a scale analysis of the primitive mean zonal momentum equation. Section 3 reviews and extends Leovy's analytic theory for Venus' equatorial superrotation. Section 4 derives all the analytic expressions for the zonal momentum budget and applies those expressions to Venus and Titan to solve for the equatorial superrotating winds. Section 5 discusses the physical reasoning for the existence or non-existence of an equatorial superrotation of a rotating planetary atmosphere. Section 6 gives concluding remarks.

2. Mean zonal momentum equation and scale analysis

We start from the following mean zonal momentum equation (Holton, 1975):

$$\begin{aligned} \rho \bar{u}_t = & \underbrace{-\frac{1}{a \cos^2 \phi} (\rho \bar{v}' u' \cos^2 \phi)_\phi}_{\text{I}} - \underbrace{(\rho \bar{w}' u')_z}_{\text{II}} - \underbrace{\rho \bar{w} \bar{u}_z}_{\text{III}} - \underbrace{2\Omega \cos \phi (\rho \bar{v})}_{\text{IV}} \\ & - \underbrace{\bar{F}_{rx}}_{\text{VI}} + \underbrace{2\Omega \sin \phi (\rho \bar{v})}_{\text{VII}} - \underbrace{\frac{(\bar{u} \cos \phi)_\phi}{a \cos \phi} \rho \bar{v}}_{\text{VIII}} - \underbrace{\frac{\rho \bar{w}' u'}{a}}_{\text{IX}} - \underbrace{\frac{\rho \bar{w} \bar{u}}{a}}_{\text{X}}, \end{aligned} \quad (1)$$

stratosphere (e.g., Hourdin et al., 1995; Tokano et al., 1999). Titan is also a slowly rotating planetary body, which, similar to Venus' cloud layer, has a haze layer that absorbs a significant portion of its incident solar radiation.

This paper follows Leovy's (1987) analytic approach and examines additional terms in the mean zonal momentum equation that could be responsible for the maintenance of a stable equatorial superrotation in a slowly rotating planetary atmosphere. Because of the decoupling nature of a few fields in a slowly rotating fluid together with the sensitivity experiments based on a 2D numerical model (Zhu and Strobel, 2005), all of the possibly important terms can be evaluated analytically. The main advantage of an analytic approach is that the dependence of astronomical and atmospheric parameters can be expressed explicitly. As a result, the model parameters arising from the analytic formulation can also be easily adjusted to be consistent with the available measurements. The extended analytic model is tested by applying the formulations to the atmospheric circulation of both Venus and Titan because both planets slowly rotate and have a cloud or haze layer that strongly absorbs the solar radiation. A preferred theory should be able to consistently explain both the existence and non-existence of a strong and steady

where $(\bar{\quad})$ and $(\quad)'$ represent zonal mean and eddy components. The subscripts t , ϕ , and z denote time, latitude, and altitude derivatives, respectively. Other symbols are defined as follows:

- a = planetary radius
- Ω = angular velocity of the planet
- ρ = background air density
- u = eastward velocity (= prograde wind)
- v = northward velocity
- w = vertical velocity in log-pressure coordinates
- F_{rx} = eastward frictional force (= $-\alpha_R \rho \bar{u}$)

The "primitive Eq." (1) for mean zonal momentum contains 10 terms. The common approach of scale analysis is to simplify the equations of motion by eliminating the smaller terms based on a particular set of characteristic scales of the field variables (e.g., Holton, 2004). Frictional force F_{rx} represents the momentum source of the unresolved eddies. Since we will be focusing on an analytic formulation in this paper this term will be parameterized by a Rayleigh friction expression, which implies that eddies will always decelerate the mean flow. Therefore, its effect on the momentum

budget as measured by its magnitude needs to be treated cautiously.

We shall now attempt to simplify Eq. (1) by scale analysis before applying our analytic approach. Although our main objective is to analytically evaluate the momentum budget for a given equatorial superrotational zonal wind in the stratospheres of Venus and Titan, we will also apply the general formula to Earth's atmosphere to illustrate its non-existence of the equatorial superrotation. Note that the planetary parameters, such as a , Ω , etc., vary to a great extent for these three planets, and atmospheric motions in different regimes are thus expected. Furthermore, we are not specifying the magnitude of \bar{u} while making the simplifications, as long as it is not vanishingly small. As a result, only a few limited simplifications that fit all these planets can be made at this stage. First, a comparison between X and IV suggests that X can be neglected if the characteristic vertical scale of \bar{u} is at most of a few scale heights, which is much smaller than the planetary radius a . Second, III represents the divergence of the vertical eddy momentum flux that *drives* or *maintains* \bar{u} . Its characteristic vertical scale should be on the same order of magnitude as \bar{u} . When IX is compared to III, it is much smaller and therefore negligible. Third, for zonal mean atmospheric circulation, the seasonal timescale of a planet can be considered the timescale for \bar{u}_t . Although Venus' sidereal period is 224.7 Earth-days, both its orbital inclination (-2.7°) and orbital eccentricity (0.0068) are very small. Therefore, 90 Earth-days ($\approx \frac{1}{4}$ Earth-year) can be considered the shortest timescale of \bar{u}_t for all of the planets we are investigating. The typical Rayleigh friction coefficient for jet streams in the Earth's atmosphere has a relaxation time ($1/\alpha_R$) of ~ 15 Earth-days (e.g., Andrews et al., 1987, p. 304). It is generally believed that the momentum drag on zonal flows in the Earth's middle atmosphere is caused by internal gravity waves (e.g., Lindzen, 1981; Holton, 1982; Fritts, 1984). For Titan's atmosphere, its seasonal timescale is much longer ($\approx \frac{1}{4}$ Saturn-year = 169 Titan-days = 2695 Earth-days). Therefore, the timescales characterized by seasonal variation are

Charney's geostrophic turbulence theory that large-scale quasi-geostrophic flow with $Ro < 1$ behaves as 2D turbulence (Charney, 1971). However, on slowly rotating planetary bodies such as Venus and Titan, even planetary-scale motions could have $Ro > 1$ and thus are expected to behave as 3D turbulence. This leads to the decaying of large-scale eddies or zonal winds caused by nonlinear energy cascade to smaller scale eddies, which could contribute to α_R and its uncertainty. Even though α_R has large uncertainty, we expect it to be an internal property of the motion that parameterizes the effect of eddies not completely included in II and III. Thus, it should not be very sensitive to the external parameters of a planetary atmosphere of a slowly rotating planet. Consequently, I is negligible when compared to VI, so this paper investigates the momentum budget of the resulting steady state.

Terms II and III represent momentum sources due to two kinds of waves: (i) waves propagating horizontally generated either by horizontal shear (e.g., Drazin and Reid, 1981; Chapter 4) or by nonlinear momentum stirring (e.g., Lighthill, 1978, p. 60) and (ii) waves propagating vertically generated either by vertical shear or by radiative heat pumping. Based on the energy equations of disturbances and mean flow, waves generated by barotropic instability associated with the horizontal shear of zonal wind draw wave energy from the basic flow and thus only reduce the shear and the strength of the zonal jet (e.g., Kuo, 1949). Furthermore, this paper focuses on the analytic evaluation of all momentum source/sink terms. We know little about parameterization of the eddy momentum stirring in general, whereas the pumping by the thermally excited tides in a planetary atmosphere can be well characterized (e.g., Leovy, 1987; Zhu et al., 1999). Therefore, II is not included in this paper for a quantitative evaluation. We will focus on the acceleration or the maintenance of an equatorial zonal jet by thermally excited tides.

In summary, four out of 10 terms in Eq. (1) have been eliminated before making analytical evaluations of the remaining terms. Eq. (1) is now simplified as

$$0 = \underbrace{-(\overline{\rho w' u'})_z}_{\text{III}} - \underbrace{\rho \bar{w} \bar{u}_z}_{\text{IV}} - \underbrace{2\Omega \cos \phi (\rho \bar{w})}_{\text{V}} - \underbrace{\alpha_R \rho \bar{u}}_{\text{VI}} + \underbrace{2\Omega \sin \phi (\rho \bar{v})}_{\text{VII}} - \underbrace{\frac{\rho \bar{v}}{a \cos \phi} (\bar{u} \cos \phi)_\phi}_{\text{VIII}} + \underbrace{A^2 G(\bar{u}_0)}_{\text{I}} - \underbrace{A \bar{u}_0}_{\text{II}} - \underbrace{A}_{\text{X}} - \underbrace{\bar{u}_0}_{\text{VI}} + \underbrace{A}_{\text{VII}} - \underbrace{A \bar{u}_0}_{\text{VIII}}. \quad (2)$$

significantly different on the three planets, with the Earth's atmosphere having the shortest one. Observations show that, for the Earth's atmosphere, motions with horizontal scales greater ~ 200 km behave as 2D turbulence, whereas smaller scale motions behave as three-dimensional (3D) turbulence (Gage and Nastrom, 1986). The critical scale of ~ 200 km between 2D and 3D turbulent motions corresponds to a typical value of the Rossby number (Ro) of 1 for the Earth's atmosphere. This is consistent with

The physical meanings of each term in Eq. (2) are as follows: III = $-(\overline{\rho w' u'})_z$ is the momentum source due to wave pumping; IV = $-\rho \bar{w} \bar{u}_z$ is the momentum sink due to the vertical advection of the vertical wind shear by the upward branch of the Hadley circulation; V = $-2\Omega \cos \phi (\rho \bar{w})$ is the momentum sink due to the vertical advection of the horizontal component of planetary vorticity by the upward branch of the Hadley circulation; VI = $-\alpha_R \rho \bar{u}$ is the momentum sink due to parameterized

Rayleigh friction; VII = $2\Omega \sin \phi (\rho \bar{v})$ is the momentum source due to the advection of the vertical component of planetary vorticity by the horizontal branches of the Hadley circulation; and VIII = $-\rho \bar{v} (a \cos \phi)^{-1} (\bar{u} \cos \phi)_\phi$ is the momentum sink due to the advection of the horizontal wind shear by the horizontal branches of the Hadley circulation. The vertical and horizontal wind shears in IV and VIII are also the horizontal and vertical components of relative vorticity for the mean flow, respectively. When the conversion of the planetary vorticity and relative vorticity becomes dominant in one direction, conservation of Ertel's potential vorticity is a better framework for understanding the physical mechanism of motion. The last row in Eq. (2) shows the parameter-dependence of each term on two important quantities: the strength of radiative heating A and the mean zonal velocity at the jet center \bar{u}_0 . In this paper, waves represent the thermally driven tides whose amplitudes are linearly proportional to A , which leads to an A^2 -dependence in the vertical momentum transport by the correlation terms. The strength of the thermally forced Hadley circulation (\bar{v} , \bar{w}) is also linearly proportional to A . This leads to a linear dependence of those terms that are linearly dependent on \bar{v} or \bar{w} . For a specified form of \bar{u} distribution, its strength depends on \bar{u}_0 , which leads to a \bar{u}_0 -dependence in those terms containing \bar{u} or its derivatives. The strength of the wave pumping depends nonlinearly on \bar{u}_0 (Section 3) and is denoted by $G(\bar{u}_0)$ in Eq. (2). The function $G(\bar{u}_0)$ becomes vanishingly small as \bar{u}_0 approaches 0 or ∞ (Leovy, 1987). The “+” or “-” signs represent acceleration or deceleration of \bar{u} under nominal specifications of the atmospheric states, which will be shown in the next two sections.

3. Review of Leovy's theory on venus superrotation and its extension

There will always be viscous friction in a moving fluid that decelerates a flow (Batchelor, 1967, Chapter 3). Therefore, the essential difficulty in explaining the observed equatorial superrotation in Venus' stratosphere is defining a mechanism that accelerates the equatorial jet. The conservation of total atmospheric angular momentum suggests that large equatorial superrotation can only be produced by eddy momentum fluxes, which redistribute angular momentum within the atmosphere (e.g., Zhu and Strobel, 2005) and correspond to II and III in Eq. (1). It was first proposed by Fels and Lindzen (1974) that thermally excited tides were responsible for the maintenance of the equatorial superrotation of Venus' cloud-top level wind. Leovy (1987) developed an analytic model of Venus' equatorial superrotation and investigated the strength and stability of the superrotation by balancing III and IV:

$$0 = \int_{-\infty}^{\infty} \rho \overline{w' \zeta'} dz - \int_{-\infty}^{\infty} \rho \bar{w} \bar{u}_z dz. \quad (3)$$

In Eq (3), III has been replaced by the vorticity flux (Leovy, 1987)

$$-(\rho \overline{w' u'})_z = \overline{\rho w' \zeta'}, \quad (4)$$

where the horizontal vorticity (along the y -direction) of the perturbation field is given by

$$\zeta' = -u'_z + w'_x. \quad (5)$$

In Fig. 1, we show schematic representations of the terms in Eqs. (3)–(4) and illustrate how the two terms in the mean zonal momentum equation are balanced. Assuming an oscillatory heating source moving in a direction opposite to an existing zonal wind, panel (a) shows the phase surfaces and air parcel trajectories of the excited tidal waves, adapted from Leovy (1985). The figure shows that the phase surfaces and air parcel trajectories of the excited tidal waves are structured in such a way that a vertical momentum flux convergence arises across the heating layer, which produces an acceleration of the mean zonal flow. Eq. (4) shows that the momentum flux convergence within the heating layer can also be represented as a vorticity flux. Panel (b) shows how a positive correlation between w' and ζ' leads to an acceleration of the mean zonal flow at the jet center as the vortex parcels move in an organized way. Panel (c) shows the deceleration effect of vertical advection of the mean zonal wind shear. Vertical advection leads to a decrease in \bar{u} below the jet center and an increase in \bar{u} above the center. This will lead to a major cancellation between two terms below and above the jet center, which is also shown in numerical simulations by Baker and Leovy (1987). Because the exponential decrease of air density with altitude ($\rho \sim e^{-z/H}$) is faster than the increase of the thermally forced vertical velocity $\bar{w} \sim e^{z/2H}$, the net effect of the vertical average is more heavily weighted by the decrease in \bar{u} below the jet center, which causes the deceleration of the vertically integrated zonal wind.

Assuming a parabolic wind profile that is vertically symmetric with respect to the jet center $z = 0$, Leovy (1987) derived analytic expressions for the two terms on the rhs of Eq. (3):

$$F_{\text{III}-0} \equiv \int_{-\infty}^{\infty} \overline{\rho w' \zeta'} dz = \frac{\pi \rho_0 A^2 \hat{u}^{-3}}{4kN^4 \beta^2 h} \exp\left(\frac{-1}{2\hat{u}^2}\right) \quad (6)$$

and

$$F_{\text{IV}-0} \equiv \int_{-\infty}^{\infty} \rho \bar{w} \bar{u}_z dz = \frac{\sqrt{\pi} \rho_0 \beta \lambda^2 h^3}{\gamma N} \epsilon e^{\epsilon^2} A \hat{u}, \quad (7)$$

where the newly introduced symbols in Eqs. (6)–(7) are defined as follows:

$A = (\gamma g / \bar{T}) [d\bar{T}/dt]_{\text{radiative}}$, amplitude of the semidiurnal tidal heating per unit volume

γ = ratio of A to the mean radiative heating

N = buoyancy frequency

$k = 2/a$, zonal wavenumber of the semidiurnal tide

$\beta \approx 1.3$, scaling factor that converts gravity wave formulas into the semidiurnal tide

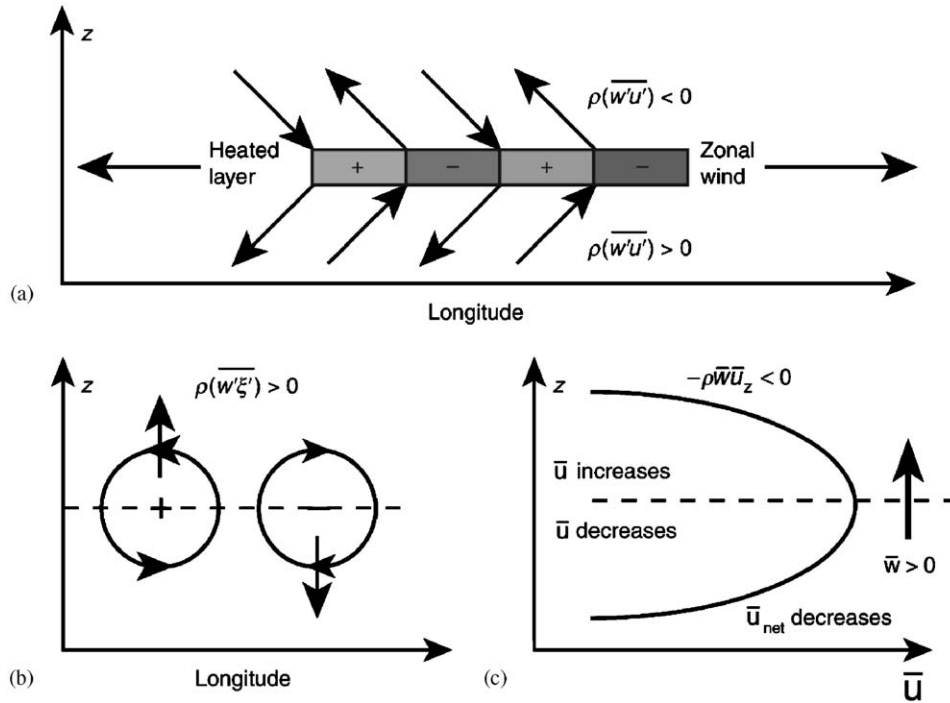


Fig. 1. (a) Oscillating heating (+) and cooling (–) centers moving in an oppose direction as the zonal wind. The phase surfaces (tilt lines) and air parcels trajectories (arrows) of the excited tidal waves leads to a negative mean momentum flux above the heating layer and a positive one below the heating layer. The vertical momentum flux convergence across the heating layer produces an acceleration of the mean zonal flow. (b) The momentum flux convergence within the heating layer can also be represented as the vorticity flux. A positive correlation between w' and ζ' also leads to the acceleration of the mean zonal flow. (c) Vertical advection of the mean zonal wind shear leads to decrease in \bar{u} below the jet center and increase in \bar{u} above the center. Because the exponential decrease of air density with altitude is faster than the increase of the thermally forced vertical velocity, the net effect of the vertical average is to cause deceleration of the zonal wind.

h = vertical scale characterizing the radiative heating
 λ = inverse depth scale for a symmetric jet of
 $\bar{u}(z) = \bar{u}_0(1 - \lambda^2 z^2/2)$
 $\hat{u} = \bar{u}_0/(\beta N h)$, dimensionless zonal velocity at the jet center ($z = 0$)
 $\varepsilon = h/(4H)$, dimensionless scale characterizing the heating depth.

The notation in Eqs. (6)–(7) mostly follows Leovy (1987), to which readers can refer for more detailed definitions. Following Leovy (1985), we have introduced $\hat{u} = \bar{u}_0/(\beta N h)$, rather than its inverse, to represent the dependence of various terms on the zonal wind in calculations and analyses. Substituting Eqs. (6)–(7) into Eq. (3) and setting $\gamma \equiv \gamma_L = 3.1$, Leovy found a stable solution of Venus superrotation of $\sim 100 \text{ m s}^{-1}$, which corresponds to an exact balance between the two terms as described schematically by Panels (b) and (c) in Fig. 1.

There is evidence that the zonal wind is asymmetric with respect to the jet center (Newman et al., 1984). As a result, the net effect on the deceleration due to the vertical advection of the wind shear is sensitive to the vertical asymmetry of the wind profile (Zhu, 2005). However, we will see later that Eq. (7) only makes a very minor contribution to the overall momentum budget in Eq. (2).

Therefore, Leovy's nominal wind profile of a symmetric parabolic function is adopted in this paper.

Generally speaking, a thermal force applied to a mechanically closed system only redistributes the angular momentum within the system (Zhu and Strobel, 2005). The production of angular momentum within a closed system requires an external torque generated by a mechanical force. Therefore, strictly speaking, thermal pumping by vertically propagating tidal waves, when integrated over the whole domain from $-\infty$ to ∞ , should be zero. Physically, this means that tidal waves thermally generated from the pumping layer will be damped in regions away from the pumping layer. The acceleration in the pumping region caused by the excitation of the tidal waves will be exactly compensated by deceleration in regions outside the heating layer of the significant wave damping. This can also be seen from Fels and Lindzen's (1974) numerical calculations where retrograde winds are produced when the wave model also includes the damping effect. In Baker and Leovy (1987), the significant retrograde acceleration is induced above the heating region. Therefore, strictly speaking, the evaluations of the integrated momentum sources should only be limited to the heating region when examining the momentum budget of the jet stream. Using the Green's function method, Leovy (1987) developed an analytic model that allowed him to focus on the contribution to

the prograde acceleration. Though the integration of the vorticity flux is over the entire atmosphere, it actually represents the wave pumping within the region of solar heating where the tidal waves are generated and radiated.

However, the above strategy may not be applicable to other terms in Eq. (2). To make evaluations of all terms consistent, we first extend Leovy's analytic theory by replacing the bounds of the integrals with a finite domain of $[-D, D]$:

$$0 = \int_{-D}^D \overline{\rho w' \zeta'} dz - \int_{-D}^D \rho \bar{w} \bar{u}_z dz. \quad (8)$$

Note that the vertical scale h that characterizes the radiative heating in Leovy (1987) as shown in Eqs. (6) and (7) is based on the specification of a heating rate profile proportional to $e^{-(z/h)^2}$. The setting of $h = H$ in Leovy (1987) provides a reasonably good fit to the heating rate profile derived from the detailed radiative calculations. Since $e^{-2.25} = 0.1$, we consider $h = H$ and $D = 1.5h$ as reasonable standard settings for h and D , respectively, in this paper. The analytic expressions of the two terms in Eq. (8) are

$$F_{\text{III}} \equiv \int_{-D}^D \overline{\rho w' \zeta'} dz = F_{\text{III}-0} R_{\text{III}}(\hat{u}), \quad (9)$$

and

$$F_{\text{IV}} \equiv \int_{-D}^D \rho \bar{w} \bar{u}_z dz = F_{\text{IV}-0} R_{\text{IV}}(\varepsilon), \quad (10)$$

where R_{III} and R_{IV} are the ratios of two momentum fluxes that represent the effects of the finite domains of the integrations and are given by

$$R_{\text{III}}(\hat{u}) = \frac{4}{\pi} \exp\left(\frac{1}{2\hat{u}^2}\right) \left[\int_0^\delta \cos(x/\hat{u}) \exp(-x^2) dx \right]^2 \quad (11)$$

and

$$R_{\text{IV}}(\varepsilon) = \frac{1}{2} [\text{erf}(\delta + \varepsilon) + \text{erf}(\delta - \varepsilon)] - \frac{e^{-\delta - \varepsilon^2}}{\sqrt{\pi\varepsilon}} \sinh(2\delta\varepsilon) \quad (12)$$

with $\delta = D/h$. In Fig. 2, we show plots of R_{III} and R_{IV} as functions of \hat{u} and h/H ($= 4\varepsilon$), respectively, for a given value of $\delta = 1.5$. Setting $N^2 = 2 \times 10^{-5} \text{ s}^{-2}$, $h = H$, and $\bar{u}_0 = 115 \text{ m s}^{-1}$ for the Venus cloud layer, we find $\hat{u} = 4.0$ and $\varepsilon = 1/4$, which gives $R_{\text{III}} = 0.939$ and $R_{\text{IV}} = 0.771$. These two characteristic values are indicated in the figure by two circles. Since the determination of \hat{u} and thus \bar{u}_0 relies on the balance between the two terms in either Eq. (3) or Eq. (8), the fact that both terms remain close to unity for $\hat{u} = 4.0$ and $h = H$ suggests that Leovy's analytic theory with an infinite domain of integration is a good approximation for a finite domain solution.

Substitution of Eqs. (9) and (10) into Eq. (8) yields an estimate of the zonal wind at the jet center for a given

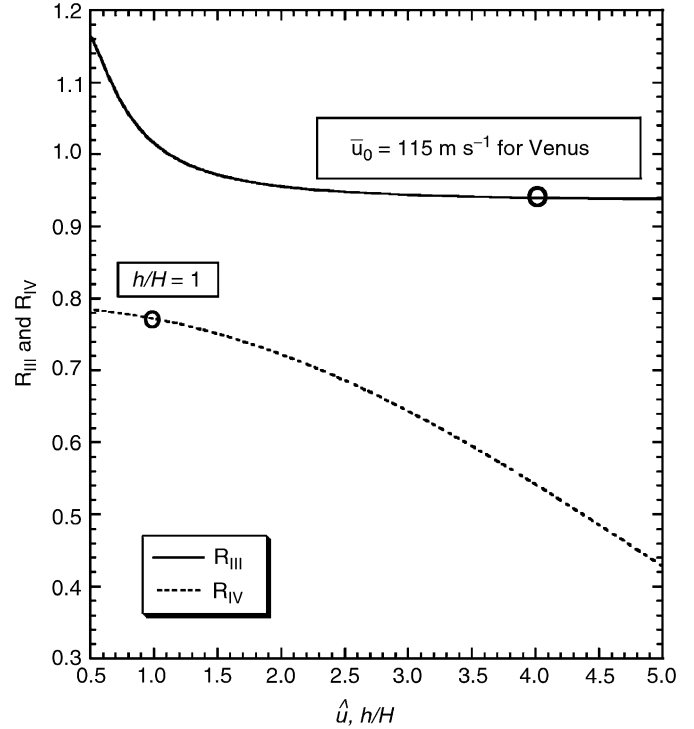


Fig. 2. Plots of R_{III} and R_{IV} defined in Eqs. (11)–(12) as functions of \hat{u} and h/H ($= 4\varepsilon$), respectively. Two circles indicate the corresponding values of R_{III} and R_{IV} for the conditions of Venus stratospheric superrotation with $\bar{u}_0 = 115 \text{ m s}^{-1}$ and $h = H$.

planetary atmosphere:

$$\hat{u} = A\Psi G(\hat{u}) \quad (13)$$

where

$$\Psi = \frac{\sqrt{\pi}\gamma e^{-\varepsilon^2}}{4kN^3\beta^3(\lambda h)^2 h^2 \varepsilon R_{\text{IV}}(\varepsilon)}, \quad G(\hat{u}) = \hat{u}^{-3} e^{-1/(2\hat{u}^2)} R_{\text{III}}(\hat{u}). \quad (14a,b)$$

The lhs of Eq. (13) is a linear function of the dimensionless wind \hat{u} , whereas the rhs is a linear function of the tidal heating amplitude A and a nonlinear function of \hat{u} . A stable solution of Eq. (13) corresponds to the intersection of two curves (Leovy, 1985). In the example shown in Fig. 3, a circle denotes the stable intersection between Curves A (wave pumping momentum source) and A' (the vertical advection of the wind shear momentum sink). Fig. 3 also shows other examples of different choices of parameters in the two sets of curves $y_1 = C_1 G(\hat{u})$ and $y_2 = C_2 \hat{u} + \alpha_0$ with different values of C_1 , C_2 and α_0 , where $C_1 = A\Psi$. Note from the last row in Eq. (2) that the remaining momentum sources or sinks are either independent of or linearly dependent on \bar{u}_0 . Therefore, those terms can be included into the lhs of Eq. (13) by replacing \hat{u} with $C_2 \hat{u} + \alpha_0$, while the rhs of Eq. (13) remains unchanged. Different choices of Ψ , A , C_2 , and α_0 lead to different stable solutions of \hat{u} . Returning to Fig. 2, we note that $R_{\text{III}}(\hat{u})$, which measures the effect of the finite domain, varies slowly in the regime of superrotational winds. To simplify

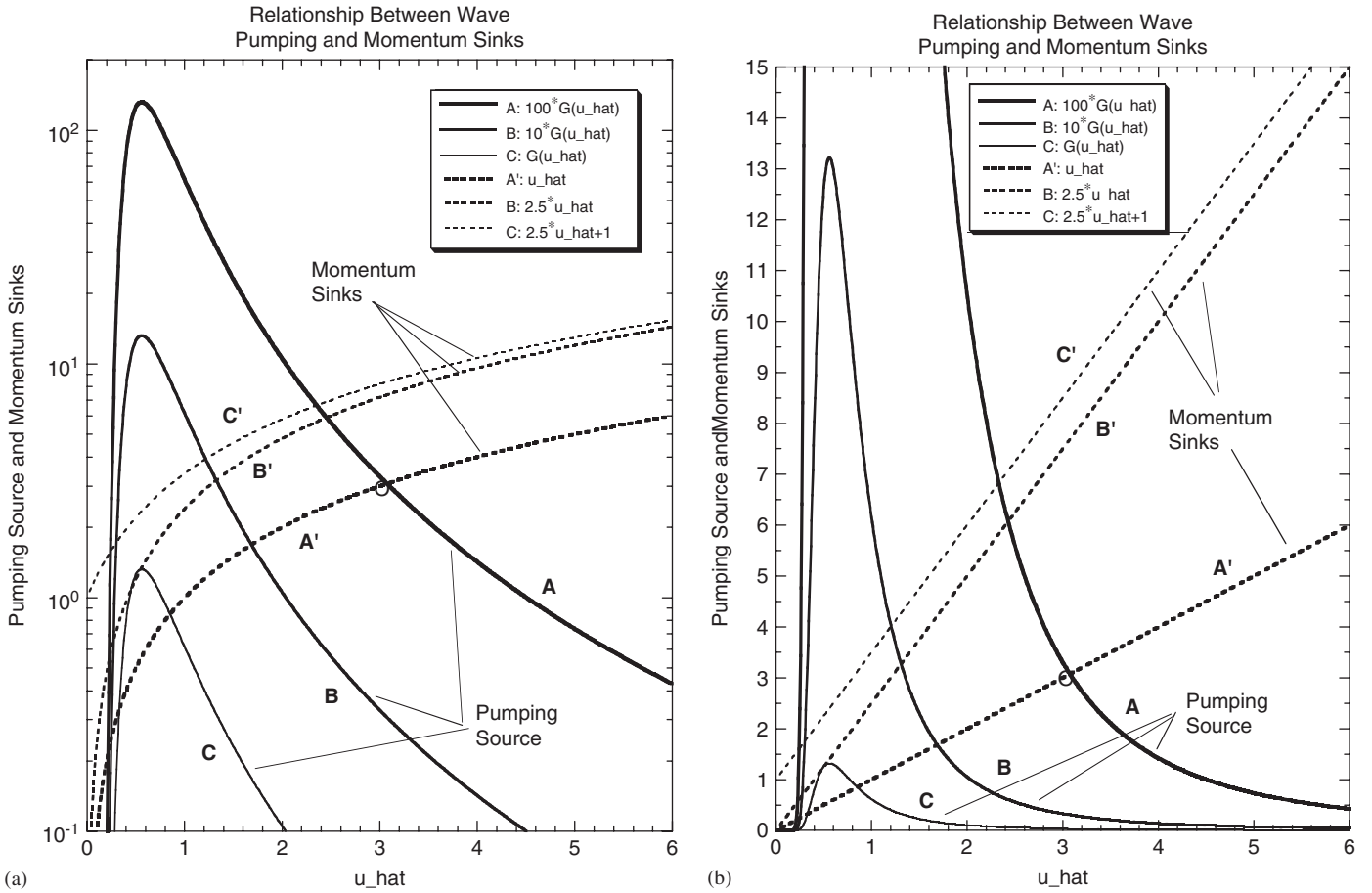


Fig. 3. Dimensionless relationship between the wave pumping and momentum sinks. Solid curves A, B, and C represent the rhs of Eq. (13) $y_1 = C_1 G(\hat{u})$ with $C_1 = 625, 25,$ and $1,$ respectively. Dashed curves A', B', and C' represent the advective and Rayleigh friction sink terms of $y_2 = \hat{u}, y_2 = 2\hat{u},$ and $y_2 = 2\hat{u} + 3,$ respectively. An intersection of any solid curve with a dashed curve at a larger value of \hat{u} (u_{hat} in plot) corresponds to a stable solution of superrotation. Panel (a) is plotted in a log-ordinate whereas Panel (b) is plotted in a linear ordinate.

our analyses and calculations, we will still use Leovy’s closed expression with a coefficient for the momentum source III. Thus, function $G(\hat{u})$ in Eq. (13) is given by

$$G(\hat{u}) = R_{\text{III}}^* \hat{u}^{-3} e^{-1/(2\hat{u}^2)} \tag{15}$$

with $R_{\text{III}}^* = 0.9387.$

4. Analytic evaluation of the equatorial zonal momentum budget for a planetary atmosphere

Once the form of the equatorial jet stream is prescribed, other terms in Eq. (2) can also be evaluated analytically. In Leovy (1987), only the vertical branch of the Hadley circulation is used. To evaluate VII and VIII, we also need the zonal mean meridional wind of the Hadley circulation. Classical models of Hadley circulation assume a geostrophic approximation, which leads to a thermal wind balance to relate the zonal wind and temperature fields. This in turn leads to a meridional circulation that depends on the planetary rotation rate (e.g. James, 1994, Chapter 4). Using an axially symmetric 2D model for Titan’s

stratosphere, Zhu and Strobel (2005) recently examined the relationships among thermal wind balance, meridional circulation, and zonal wind for various choices of planetary parameters. It is found that the strength of the meridional circulation is exclusively sensitive to the radiative drive characterized by the meridional gradient of the net diabatic heating rate, and insensitive to the planetary rotation rate.

Here, we use the continuity equation alone to derive the meridional velocity from the known vertical velocity derived from the radiative heating rate. This approach was first used by Murgatroyd and Singleton (1961) for deriving meridional circulation in Earth’s middle atmosphere. First, we add a meridional distribution of the heating rate to the corresponding \bar{w} expression used in Leovy (1987)

$$\bar{w} \approx \frac{\bar{q}}{N^2} = \frac{A}{\gamma N^2} e^{-\zeta^2 + 2\epsilon\zeta} \Theta(\phi), \tag{16}$$

where $\zeta = z/h.$ Also, $\Theta(\phi) = 1$ at the central latitude $\phi = \phi_c$ where peak radiative heating per unit volume occurs. ϕ_c characterizes the seasonal variation of $\bar{q}.$ When $\phi_c = 0,$ we

recover the expression used by Leovy (1987). Based on the zonal mean continuity equation (Holton, 1975)

$$\frac{\partial}{\partial \phi} (\cos \phi \rho \bar{v}) + \frac{\partial}{\partial z} (\cos \phi \rho \bar{w}) = 0, \quad (17)$$

the meridional distribution function for \bar{w} or the heating rate needs to satisfy the condition

$$\int_{-\pi/2}^{\pi/2} \Theta(\phi) \cos \phi \, d\phi = 0, \quad (18)$$

which indicates that the global mean net radiative heating rate vanishes when averaged over pressure surfaces. Combining Eqs. (17)–(18), we obtain the meridional velocity:

$$\bar{v} = \left(\frac{2aA}{\gamma N^2 h} \right) \frac{(\zeta + \varepsilon) e^{-\zeta^2 + 2a\zeta}}{\cos \phi} \int_{\phi_c}^{\phi} \Theta(x) \cos x \, dx. \quad (19)$$

Note that the meridional velocity as defined by Eq. (19) is linearly proportional to the zonal mean radiative drive (A/γ) and planetary radius (a), but independent of the planetary rotation rate (Ω), which is consistent with the recent numerical experiments by Zhu and Strobel (2005). Also note from Eqs. (16) and (19) that the ratio of \bar{v} to \bar{w} is proportional to the aspect ratio of the Hadley circulation a/h , which is $\sim 10^3$ for Venus' stratosphere and ~ 61 for Titan's stratosphere, respectively. If we substitute the values of A , N^2 (below, Table 1), and $\gamma = \gamma_L (= 3.1)$ into Eq. (16) we get $\bar{w} = 0.40 \text{ m s}^{-1}$ at the jet center and $|\bar{v}| \sim 280 \text{ m s}^{-1}$ at the mid-latitude cloud-top level on Venus, compared with observations of $|\bar{v}| \sim 10 \text{ m s}^{-1}$ (Del Genio and Rossow, 1990). Therefore, in this paper we set $\gamma = 25\gamma_L = 77.5$, which yields values of \bar{w} to be

Table 1
Amplitude of the semidiurnal tidal heating A (m s^{-3}) and several parameters for atmospheres of Venus, Titan and Earth^a

($S = 1366 \text{ W s}^{-2}$)	Venus	Titan	Earth
κ ($= R/c_p$)	0.25	0.29	0.29
g (m s^{-2})	8.87	1.35	9.82
a_p	0.77	0.30	0.30
η	0.6	0.7	0.2
z (km)	65	180	5
p_r (Pa)	10^4	10^2	5×10^4
H (m)	5×10^3	42×10^3	7×10^3
d_{sp}	0.7233	9.5719	1
A (m s^{-3})	2.477×10^{-5}	1.056×10^{-6}	2.412×10^{-6}
N^2 (s^{-2})	2×10^{-5}	10^{-5}	1.5×10^{-4}
a (m)	6.052×10^6	2.575×10^6	6.371×10^6
Ω (s^{-1})	3.001×10^{-7}	4.573×10^{-6}	7.292×10^{-5}
\bar{u}_{0c} (m s^{-1})	14.5	86.3	55.7
\bar{u}_{0x} (m s^{-1})	16.8	99.7	64.3

^a η is the fraction of the solar radiation deposited within an absorption layer. \bar{u}_{0c} is the critical velocity of the jet center below which there exists no stable solution for equatorial superrotation. \bar{u}_{0x} is the maximum pumping velocity that yields the maximum tidal pumping. Definitions of the remaining symbols are given right below Eqs. (1), (7) and (34).

0.016 m s^{-1} at the jet center and $|\bar{v}|$ to be $\sim 11 \text{ m s}^{-1}$ at mid-latitudes, respectively.

The physical justification for a much greater γ is as follows: In Leovy (1987), the parameter γ is defined as the ratio of the equatorial semidiurnal heating amplitude to the zonal mean net radiative heating rate. As a first approximation, the semidiurnal heating amplitude is solely determined by solar heating (e.g., Leovy, 1987; Zhu et al., 1999). On the other hand, the zonal mean net radiative heating rate is the difference between the zonal mean solar heating and infrared radiative cooling (e.g., Goody and Yung, 1989; Zhu, 2004). In a special case of a strictly zonal mean radiative equilibrium, the zonal mean solar heating exactly balances the zonal mean infrared cooling, which yields a vanishing zonal mean net radiative heating and $\gamma \rightarrow \infty$. Leovy (1987) assumed a very long radiative relaxation time and used a global mean cooling rate as the equatorial zonal mean radiative cooling, which resulted in a minimum value of $\gamma_L (= 3.1)$ for a thermally driven atmosphere. The actual value of γ should lie between these two extremes (∞ and 3.1) for a thermally driven atmosphere. In the Earth's stratosphere, regions of higher zonal mean temperatures generally coincide with overhead Sun, suggesting that zonal mean radiative equilibrium is a good approximation (e.g., Andrews et al., 1987). Detailed radiative calculations suggest a relatively short radiative relaxation time of ~ 15 Earth-days near Venus' cloud-top level (Crisp, 1989), which is comparable to the dynamical time scale as measured by the parameterized Rayleigh friction coefficient $1/\alpha_R$. Increasing γ_L by a factor of 25 for γ_L , which significantly reduces \bar{w} at equator from 0.40 to 0.016 m s^{-1} , is equivalent to assuming a quasi-equilibrium between zonal mean solar heating and infrared cooling as a result of strong radiative relaxation. Moreover, choosing $\gamma = 77.5$ yields a meridional wind $|\bar{v}|$ ($\sim 11 \text{ m s}^{-1}$) for Venus that agrees with the measurements presented by Del Genio and Rossow (1990).

To explicitly evaluate Eq. (19), we assume the meridional distribution function $\Theta(\phi)$ to be the following simple form

$$\Theta(\phi) = \begin{cases} \cos[\mu_1(\phi - \phi_c)], & -\pi/2 < \phi \leq \phi_c, \\ \cos[\mu_2(\phi - \phi_c)], & \phi_c < \phi \leq \pi/2, \end{cases} \quad (20)$$

which has the property of $\Theta(\phi_c) = 1$, as required by Eq. (16). Two parameters μ_1 and μ_2 are determined by the following conditions, which are more restrictive than Eq. (18),

$$\int_{-\pi/2}^{\phi_c} \Theta(\phi) \cos \phi \, d\phi = \int_{\phi_c}^{\pi/2} \Theta(\phi) \cos \phi \, d\phi = 0. \quad (21)$$

We assume the same altitude dependence as in Leovy (1987) and a Gaussian distribution in latitude for the zonal wind

$$\bar{u}(\phi, z) = \bar{u}_0 \exp \left[-\frac{\phi^2}{2\phi_d^2} \right] \left(1 - \frac{1}{2} \lambda^2 z^2 \right). \quad (22)$$

Similar to $[-D, D]$ for the vertical domain of integration in Eq. (8), we set $[-\phi_b, \phi_b]$ to be the latitudinal domain over which the momentum source and sink terms are averaged. In this paper, we set $\phi_b = \pi/9$, which corresponds to the major portion of the equatorial superrotation being confined to $-20^\circ < \phi < 20^\circ$ latitude. Setting $\phi_d = \pi/4$ in Eq. (22) fits reasonably well with the meridional variation of $\bar{u}(\phi, z)$ near the cloud-top level within the meridional integral domain of $[-\phi_b, \phi_b]$ (Newman et al., 1984).

Given the meridional circulation and the prescribed form of the zonal jet, the momentum sources and sinks for the remaining terms in Eq. (2) can all be evaluated analytically. Together with the already derived expressions for III and IV, these expressions are listed as follows:

$$F_{\text{III}} \equiv \int_{-D}^D \rho \overline{w' \xi'} dz = \left(\frac{\pi \rho_0}{4k\beta^2 N^4 h} \right) A^2 G(\hat{u}), \quad (23)$$

$$F_{\text{IV}} \equiv \int_{-D}^D \rho \bar{w} \bar{u}_z dz = K_4^* \left(\frac{\beta \rho_0 h}{\gamma N} \right) A \hat{u}, \quad (24)$$

$$F_{\text{V}} \equiv \int_{-D}^D 2\Omega \rho \bar{w} dz = K_5^* \left(\frac{\rho_0 h \Omega}{\gamma N^2} \right) A, \quad (25)$$

$$F_{\text{VI}} \equiv \int_{-D}^D \alpha_R \rho \bar{u} dz = K_6^* (\beta \rho_0 N H h \alpha_R) \hat{u}, \quad (26)$$

$$F_{\text{VII}} \equiv \frac{1}{2\phi_b} \int_{-\phi_b}^{\phi_b} d\phi \int_{-D}^D 2\Omega \sin \phi \rho \bar{v} dz = J_7^* K_7^* \left(\frac{a \rho_0 \Omega}{\gamma N^2} \right) A, \quad (27)$$

$$F_{\text{VIII}} \equiv \frac{1}{2\phi_b} \int_{-\phi_b}^{\phi_b} d\phi \int_{-D}^D \frac{(\bar{u} \cos \phi)_\phi}{a \cos \phi} \rho \bar{v} dz = J_8^* K_8^* \left(\frac{\beta \rho_0 h}{\gamma N} \right) A \hat{u} \quad (28)$$

where the coefficients are tedious analytic functions of geometric parameters for the domain (Zhu, 2005). For the standard setting of $h = H$, $\delta = D/h = 1.5$, $\varepsilon = 1/4$ and $lh = 1/3$, we have

$$K_4^* = 0.0404, \quad K_5^* = 3.603, \quad K_6^* = 4.037, \\ K_7^* = 0.347, \quad K_8^* = 11.22. \quad (29)$$

$$J_7^* = \begin{cases} 0.0368, & \phi_c = 0, \\ 0.0196, & \phi_c = \pi/6, \end{cases} \quad J_8^* = \begin{cases} 0.186, & \phi_c = 0, \\ 0.099, & \phi_c = \pi/6. \end{cases} \quad (30)$$

The first case (i) in Eq. (30) with $\phi_c = 0$ corresponds to Venus with a near zero (-2°) inclination and to the equinox seasons for Titan and Earth. The second case (ii) with $\phi_c = \pi/6$ corresponds to the northern hemisphere summer solstice for Titan and Earth, where the maximum solar heating occurs near 30° latitude. Because of the very small contribution to the momentum budget (below, Table 2) or the large uncertainty in α_R , the small seasonal changes of the coefficients in Eqs. (24)–(26) have been neglected.

Table 2
Coefficients in Eq. (31) and the solution of \hat{u} and \bar{u}_0 (m s^{-1})

	Venus	Titan	Earth	Titan-E
<i>(a) case (i): $\phi_c = 0$</i>				
χ_1	3.343×10^{-3}	145.9	45.47	1.742
χ_0	-6.588×10^{-7}	2.216×10^{-3}	-0.0155	2.216×10^{-4}
\hat{u}	4.063	-1.519×10^{-5}	3.416×10^{-4}	0.617
\bar{u}_0	118	-0.0026	0.038	106
<i>(b) case (ii): $\phi_c = \pi/6$</i>				
χ_1	2.292×10^{-3}	144.4	45.06	1.598
χ_0	-2.567×10^{-7}	3.959×10^{-3}	-4.995×10^{-3}	3.939×10^{-4}
\hat{u}	4.471	-2.741×10^{-5}	1.109×10^{-4}	0.653
\bar{u}_0	130	-0.0047	0.012	113

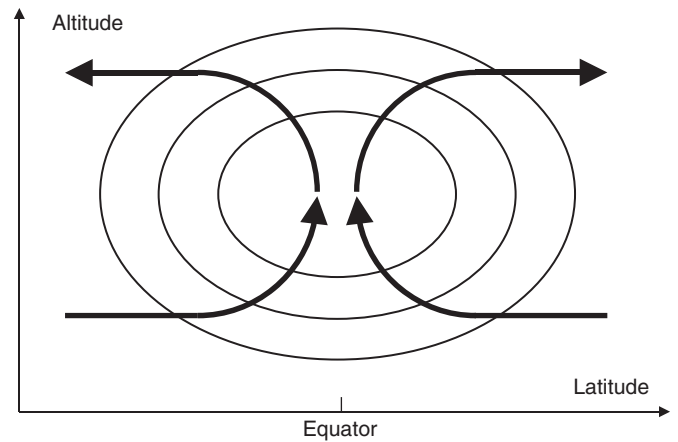


Fig. 4. Schematic diagram showing the physical mechanism of the advection of the horizontal shear of zonal wind by the horizontal branches of the Hadley circulation. The incoming branches of the Hadley circulation cause the convergence of the momentum depleted air mass from the outside into the region of the superrotational jet, whereas the outgoing branches cause the divergence of the momentum surplus air mass from the jet center to the outside region.

F_{V} and F_{VI} are both momentum sinks due to the vertical advection of planetary vorticity and parameterized Rayleigh friction, respectively. From Eqs. (2) and (27) we note that the Coriolis acceleration by the horizontal branches of the Hadley circulation is positive. This is because that our calculation of F_{VII} is averaged in both latitude and altitude. The resulting F_{VII} represents a small asymmetric difference between the retrograde acceleration of the incoming flow and the prograde acceleration of the outgoing flow, which yields a small and positive difference in the net acceleration.

As for F_{VIII} , the incoming branches of the Hadley circulation cause the convergence of momentum-depleted air masses from the outside into the region of the superrotating jet, whereas the outgoing branches cause the divergence of momentum-surplus air masses from the jet center to the outside region (see schematic diagram in Fig. 4). Note that, as a first approximation, the peaking \bar{w} of vanishing divergence near the jet center does not contribute a momentum sink to the jet, as shown in

Fig. 1c. Since \bar{v} and \bar{w} are scaled according to the aspect ratio (a/h) with comparable effects on the momentum transport, we should expect a much greater contribution to the momentum sink from VIII than IV. Comparing Eq. (24) with Eq. (28), we obtain the ratio of the two momentum sink terms, $F_{\text{VIII}}/F_{\text{IV}} = J_8^* K_8^*/K_4^*$, to be 53.2 and 28.3 for cases (i) and (ii), respectively.

By collecting all the terms and substituting Eqs. (23)–(28) into Eq. (2) we obtain

$$\chi_1 \hat{u} + \chi_0 = G(\hat{u}), \quad (31)$$

where

$$\chi_1 = \frac{4k\beta^3 N^3 h^2}{\pi\gamma A} (K_4^* + J_8^* K_8^* + \gamma K_6^* N^2 H A^{-1} \alpha_R), \quad (32)$$

$$\chi_0 = \frac{4k\beta^2 N^2 h \Omega}{\pi\gamma A} (K_5^* h - J_7^* K_7^* a), \quad (33)$$

and $G(\hat{u})$ is given by Eq. (15). Given the values of χ_1 and χ_0 for different planetary atmospheres and/or under different seasonal conditions, Eq. (31) can be easily solved for \hat{u} , from which the momentum budget of all the terms in Eq. (2) can be explicitly evaluated. The physical definitions of various basic parameters have been listed following Eqs. (1), (2) and (7).

One critical parameter is the amplitude of the semi-diurnal tidal heating A . In Leovy (1987), A was derived by detailed calculations of radiative forcing of Venus' atmosphere. Here, let us take a more general view of planetary atmospheres and relate A to a few more fundamental astronomical parameters. Assuming a fraction (η) of the absorbed solar radiation is deposited in the atmosphere at a reference pressure p_r with a characteristic depth of H_r , then A can be expressed as

$$A = \frac{\gamma_L \kappa g (1 - a_p) \eta S}{p_r H_r d_{\text{sp}}^2}, \quad (34)$$

where $\kappa (= R/c_p)$ is the ratio of gas constant R to the specific heat at constant pressure c_p , g is gravitational acceleration, $S (= 1366 \text{ W m}^{-2}$; Liou, 2002) is the solar constant at 1 Astronomical Unit (AU), a_p is the planetary albedo, and d_{sp} is the Sun–planet distance in AU. In Table 1, we list various parameters that are needed for calculating A for Venus, Titan and Earth. Also listed in Table 1 are the squared buoyancy frequency N^2 , planet radius a and rotation rate Ω for evaluating the coefficients χ_1 and χ_0 in Eq. (31). The last two rows in Table 1 show values for the critical velocity \bar{u}_{0c} and velocity of maximum pumping \bar{u}_{0x} . The critical velocity is the velocity of the jet center below which there exists no stable solution for equatorial superrotation. The velocity of maximum pumping yields the maximum value of $G(\hat{u})$ on the rhs of Eq. (31). We set $H_r = 2H$ and set the Rayleigh frictional coefficient $\alpha_R = 7.74 \times 10^{-7} \text{ s}^{-1}$, which corresponds to a relaxation time of 15 Earth-days. The rationale of choosing this value and its effect on the main conclusions will be further discussed below. We adopt the minimum value of

$\gamma_L (= 3.1)$ in the numerator of Eq. (34) based on the premise discussed above that the semi-diurnal heating amplitude is solely determined by solar heating. The zonal mean heating rate at the jet center A/γ , as defined in Eq. (31), is a factor $\gamma_L/\gamma (= 25)$ smaller than the one used by Leovy (1987), suggesting a zonal mean quasi-radiative equilibrium.

Using the parameters given in previous sections and those listed in Table 1, we can calculate the coefficients in Eq. (31). The derived values are given in Tables 2a and b together with the solution of \hat{u} and the zonal velocity at the jet center \bar{u}_0 for the seasonal cases of equinox (i) and northern hemisphere summer solstice (ii), respectively. Two columns are presented for Titan's atmosphere. The first column corresponds to the nominal parameter settings for Titan's atmosphere shown in Table 2. Under these settings, Titan's atmosphere does not have an equatorial superrotation at the 1-hPa pressure level where the maximum solar heating occurs (Table 2). The last column in Table 2 (Titan-E) is derived using an enhanced amplitude of the semi-diurnal tidal heating A . This is done by setting $p_r = 10 \text{ Pa} (= 0.1 \text{ hPa})$ in Eq. (34) while evaluating A with the rest of the parameters unchanged. This choice of an enhanced A leads to an equatorial superrotation of $\sim 110 \text{ m s}^{-1}$ in Titan's stratosphere.

The solved winds in Table 2 lead to our main conclusion, that an equatorial superrotation of 118 m s^{-1} can be maintained at Venus' cloud-top level, whereas there is no equatorial superrotation in Titan's stratosphere centered at the 1-hPa level ($\sim 185 \text{ km}$). However, if the major stratospheric heating layer is lifted to the 0.1-hPa level ($\sim 288 \text{ km}$), then Titan's stratosphere could have an equatorial superrotation of $\sim 110 \text{ m s}^{-1}$ centered around the 0.1-hPa heating layer.

Substituting the solved \hat{u} from Table 2 into Eqs. (23)–(28) we obtain the contributions of the individual momentum sources and sinks. In Table 3, we list the relative contributions of each term for Venus' atmosphere and the case of enhanced heating for Titan's stratosphere. The numbers in parentheses are negative and represent retrograde acceleration. We have excluded the two cases shown by the two center columns in Table 2 that correspond to stable solutions with a vanishingly small zonal wind. Even though these solutions are mathematically steady and

Table 3
Relative momentum budget integrated over the model domain of all terms in Eq. (2), (i) and (ii) denote equinox and northern hemisphere summer solstice, respectively

	Venus (i)	Titan-E (i)	Venus (ii)	Titan-E (ii)
III	1	1	1	1
VII	6.3×10^{-5}	9.3×10^{-6}	4.5×10^{-5}	5.1×10^{-6}
IV	(0.013)	(0.003)	(0.019)	(0.004)
V	(1.5×10^{-5})	(5.5×10^{-4})	(2.0×10^{-5})	(5.7×10^{-4})
VI	(0.315)	(0.819)	(0.459)	(0.893)
VIII	(0.672)	(0.177)	(0.522)	(0.103)

stable, they are most likely unphysical or unimportant because the terms neglected while driving Eq. (2) by scale analysis could become dominant. For example, the transient term ($\partial\bar{u}/\partial t$), say, ultimately induced by absorption of solar radiation in other regions, may support a much stronger oscillatory zonal flow, as seen in the Earth's stratosphere.

From Table 3, we see that pumping by thermal tides, III, is the sole dominant momentum source for the maintenance of equatorial superrotation. The momentum sinks are almost entirely (>98%) shared by two processes: (i) parameterized Rayleigh friction, VI, and (ii) advection of the horizontal wind shear by the horizontal branches of the Hadley circulation, VIII. Since Hadley circulation is thermally driven, its strength is linearly dependent on the radiative forcing (A/γ), which in turn is inversely proportional to the square of the Sun–planet distance (d_{sp}), as shown in Eqs (16), (19) and (34). As a result, given a fixed value of the Rayleigh friction coefficient (α_R), the relative contribution from VIII decreases with the increasing d_{sp} . For equatorial superrotation on Venus, the momentum sink due to advection of the horizontal wind shear (VIII) is about a factor two greater than the sink due to parameterized Rayleigh friction (VI). On the other hand, at Saturn's orbit where $d_{sp} = 9.57$, VIII is a factor 5–10 smaller than VI for Titan's equatorial superrotation with an enhanced heating rate.

Since the magnitude of α_R is crucial to the above conclusion, we first discuss the rationale that the same value of α_R can be used here for different planets with significantly different astronomical parameters. We have already pointed out that α_R is a parameterization of the drag force imposed on the mean flow by unresolved eddies. Such a force could be considered an internal property of the fluid due to, say, instability of the flow or the cascade of kinetic energy from large-scale motions to small-scale ones in a 3D turbulence. Therefore, we expect it to be not very sensitive to fundamental astronomical parameters such as d_{sp} or a .

One advantage of having analytic formulations for evaluating all the terms in the mean zonal momentum equation is that the parameter dependence of different physical processes is given explicitly. Specifically, we show that even under a hypothetical extreme case of vanishingly small α_R , it is still impossible to get a stable equatorial superrotation for Titan's stratosphere centered at the 1-hPa level, which will be demonstrated in the next section.

5. Physical reasons for existence and non-existence of equatorial superrotation of a rotating planetary atmosphere

Our model for a steady and stable equatorial superrotation is based on the mean zonal momentum Eq. (1) or its simplified version Eq. (2). The existence and the magnitude of superrotational winds result from a steady balance between momentum sources and sinks of various physical processes described by the different terms in the

equation. Table 3 shows that the only momentum source that leads to prograde acceleration is III, i.e., momentum pumping by thermal tides. Since eddy and/or molecular viscosity always exist in a flow and they often act as drag force that decelerates it, the essential difficulty in finding a mechanism for superrotation is to determine its momentum sources. In this regard, a possibly correct mechanism was discovered by Fels and Lindzen (1974). In Leovy (1987), superrotational wind was solved as a stable solution of an exact balance in *acceleration* between the momentum source proposed by Fels and Lindzen (1974) and a momentum sink (the vertical advection of the wind shear). Leovy's (1987) approach of analytically solving the superrotational wind by prescribing the wind and heating rate profiles makes it possible to also evaluate the remaining terms that could potentially be important. Following this idea, Table 3 shows that the momentum sinks that balance the source term for maintaining a steady and stable equatorial superrotation on Venus and possibly also on Titan are the advection of the horizontal shear by the horizontal branches of the Hadley circulation (VIII) and the drag force parameterized by the Rayleigh friction (VI).

Hide's theorem states that the equatorial superrotation of a planetary atmosphere can only be produced by up-gradient eddy transfer (Hide, 1969; Lindzen, 1990). The momentum pumping by thermal tides shown in Fig. 1a energetically converts the thermally forced wave motions into the zonal flow, which leads to the prograde acceleration at equator. Note that the current paper follows and extends Leovy's theory on Venus' equatorial superrotation at the cloud-top level where the maximum prograde acceleration occurs. On the other hand, the observed or GCM-simulated equatorial superrotations extend much deeper into the lower atmosphere. Therefore, it is plausible that the equatorial superrotation in the lower atmosphere can be partially maintained by the horizontal transport of the superrotational angular momentum from the mid-latitudes, which could be brought from the cloud-top region from the equator. In other words, it is the *maximum* prograde acceleration by tidal pumping at the jet center that ultimately drives the superrotational wind system of the whole equatorial atmosphere.

We have assumed the value of relaxation time ($1/\alpha_R$) of ~ 15 Earth-days and also made a case that a similar value for the Rayleigh friction coefficient is appropriate for the similar type of large-scale flows in slowly rotating planetary atmospheres that are expected to behave as 3D turbulence. For a jet with a velocity of 100 m s^{-1} at the center, this corresponds to a deceleration of $6.7 \text{ m s}^{-1} \text{ Earth-day}^{-1}$ at the jet center. Due to the very small radiative drive (A/γ) at Saturn's orbit, a stable superrotational jet cannot be found at the 1-hPa level in Titan's stratosphere with frictional drag (VI) being the major momentum sink in the momentum budget. On the other hand, recent observations by direct measurements suggest strong superrotational winds in Titan's stratosphere *centered* near the 1-hPa level (Kostiuk et al., 2001). Since Rayleigh frictional drag is only

an empirical parameterization, it is reasonable to ask if we can get an equatorial superrotation centered at the 1-hPa level if we completely eliminate frictional drag.

We first note from Table 2 that χ_0 is at least three orders in magnitude smaller than χ_1 . Therefore, for a solution of \hat{u} that is less than 10, the second term on the rhs of Eq. (31) can be neglected and Eq. (31) becomes

$$\chi_1 \hat{u} = G(\hat{u}). \quad (35)$$

In general, the above equation has either one root near $\hat{u} = 0$ or three roots with the largest one representing a stable solution of superrotation as shown in Fig. 3. A special case is when two larger roots merge, which corresponds to the case of $y_1 = \chi_1 \hat{u}$ being tangent to $y_2 = G(\hat{u})$. The solution of Eq. (35) plus $\chi_1 = G'(\hat{u})$ gives the critical values of $\hat{u}_c = 0.5$ and $\chi_{1c} = 2.03$. There is no solution for superrotation below \hat{u}_c when χ_1 is greater than χ_{1c} . The existence of these critical values results from the nonlinear dependence of the pumping strength on the magnitude of the zonal wind. The dimensional values of \bar{u}_{0c} ($= \beta N h \hat{u}_c$) for different planets are listed in Table 1. We see from the two central columns in Table 2 that when χ_1 is greater than χ_{1c} , only vanishingly small solutions of \hat{u} exist. If we set $\alpha_R = 0$ in Eq. (32) for Titan and Earth, then the corresponding values of χ_1 in Table 2a will be reduced from 145.9 and 45.47 to 3.152 and 0.900, respectively. Therefore, Titan's atmosphere with a standard setting of heating centered at the 1-hPa level will not have equatorial superrotation ($3.152 > \chi_{1c}$) even under the hypothetically unrealistic specification of vanishingly small frictional drag ($\alpha_R = 0$) that, if applied to Earth's atmosphere, would induce an equatorial superrotation.

In summary, for steady and stable equatorial superrotation of a rotating planetary atmosphere the acceleration induced by the thermal forcing is primarily balanced by two momentum sinks imposed locally on the jet: (i) the parameterized Rayleigh frictional drag and (ii) the meridional advection of wind shear by the horizontal branches of Hadley circulation. Though the strengths of the momentum sinks are linearly dependent on those of the thermal forcing and/or zonal wind, the momentum source is nonlinearly dependent on the heating rate and the zonal wind. As a result, there exists a critical value that characterizes the ratio of the momentum source to the momentum sinks, below which a steady and stable superrotational wind becomes non-existent.

6. Conclusions

In this paper, the zonal momentum budget for a rotating planetary atmosphere at the equator is examined analytically and comprehensively. Our results show that the prograde acceleration induced by thermally forced tides within an absorption layer is the predominant momentum source for the maintenance of a steady and stable equatorial superrotation centered at the heating layer. Constrained by the zonal mean continuity equation and

measurements of Venus' meridional wind, it is found that the direct momentum sinks to balance the momentum source are (i) the advection of the horizontal shear by the returning branches of the Hadley circulation and (ii) the frictional drag imposed on the superrotational jet.

Since we have studied the problem analytically, the parameter dependence of the equatorial superrotation on various external and internal parameters of a planetary atmosphere becomes explicit. The important parameters included in the final Eqs. (31)–(34) are: (i) Sun–planet distance, (ii) radius of the planet, (iii) rotation rate, (iv) inclination of the equatorial plane, (v) gravity, (vi) atmospheric scale height, (vii) atmospheric buoyancy frequency, (viii) Rayleigh friction coefficient, (ix) albedo, and (x) the pressure level at which the deposition of the solar radiation occurs.

Applying the theory to the stratospheres of Venus and Titan, the main results are: (i) Venus equatorial superrotation of 118 m s^{-1} results primarily from a balance between the momentum source of pumping by thermal tides and the momentum sink of meridional advection of wind shear by horizontal branches of the Hadley circulation; (ii) no solution is found for Titan's stratospheric equatorial superrotation centered at the 1-hPa level; (iii) however, if the main absorption layer of the solar radiation in Titan's stratosphere is lifted from 1 hPa ($\sim 185 \text{ km}$) to 0.1 hPa ($\sim 288 \text{ km}$), an equatorial superrotation of $\sim 110 \text{ m s}^{-1}$ centered at 0.1-hPa could be maintained. Titan's equatorial superrotation results mainly from a balance between the momentum source of tidal pumping and the momentum sink of frictional drag.

Acknowledgements

This research was supported by NASA Grants NAG5-11962 to The Johns Hopkins University Applied Physics Laboratory. The author thanks Conway B. Leovy, Theodore G. Shepherd and several anonymous reviewers for making many constructive comments and suggestions on the original and revised manuscripts, which has led to a significant improvement of the manuscript. Editorial assistance from Dr. Steven A. Lloyd is also greatly appreciated.

References

- Andrews, D.G., Holton, J.R., Leovy, C.B., 1987. Middle Atmosphere Dynamics. Academic Press, New York, 489pp.
- Baker, N.L., Leovy, C.B., 1987. Zonal winds near Venus' cloud top level: a model study of the interaction between the zonal mean circulation and the semidiurnal tide. *Icarus* 69, 202–220.
- Batchelor, G.K., 1967. An Introduction to Fluid Dynamics. Cambridge Univ. Press, Cambridge, 615pp.
- Charney, J.G., 1971. Geostrophic turbulence. *J. Atmos. Sci.* 28, 1087–1095.
- Crisp, D., 1989. Radiative forcing of the Venus mesosphere. II. Thermal fluxes, cooling rates, and radiative equilibrium temperature. *Icarus* 77, 391–413.

- Del Genio, A.D., Rossow, W.B., 1990. Planetary-scale waves and the cyclic nature of cloud top dynamics on Venus. *J. Atmos. Sci.* 47, 293–318.
- Del Genio, A.D., Zhou, W., 1996. Simulation of superrotation on slowly rotating planets: Sensitivity to rotation and initial condition. *Icarus* 120, 332–343.
- Drazin, P.G., Reid, W.H., 1981. *Hydrodynamic Stability*. Cambridge Univ. Press, Cambridge, 527pp.
- Fels, S.B., Lindzen, R.S., 1974. The interaction of thermally excited gravity waves with mean flows. *Geophys. Fluid Dyn.* 6, 149–191.
- Fritts, D.C., 1984. Gravity wave saturation in the middle atmosphere: a review of theory and observation. *Rev. Geophys. Space Sci.* 22, 275–308.
- Gage, K.S., Nastrom, G.D., 1986. Theoretical interpretation of atmospheric wavenumber spectra of wind and temperature observed by commercial aircraft during GASP. *J. Atmos. Sci.* 43, 729–740.
- Gierasch, P., 1975. Meridional circulation and the maintenance of the Venus atmospheric rotation. *J. Atmos. Sci.* 32, 1038–1044.
- Goody, R.M., Yung, Y.L., 1989. *Atmospheric Radiation: Theoretical Basis*, Second ed. Oxford Univ. Press, New York and Oxford, 519pp.
- Hide, R., 1969. Dynamics of the atmospheres of the major planets with an appendix on the viscous boundary layer at the rigid bounding surface of an electrically-conducting rotating fluid in the presence of a magnetic fluid. *J. Atmos. Sci.* 26, 841–853.
- Holton, J.R., 1975. *The Dynamic Meteorology of the Stratosphere and Mesosphere*. Amer. Meteorol. Soc., Boston, 218pp.
- Holton, J.R., 1982. The role of gravity wave induced drag and diffusion in the momentum budget of the mesosphere. *J. Atmos. Sci.* 39, 791–799.
- Holton, J.R., 2004. *An Introduction to Dynamic Meteorology*, Fourth ed. Elsevier Academic Press, New York, 535pp.
- Hou, A.Y., Fels, S.B., Goody, R.M., 1990. Zonal superrotation above Venus' cloud base induced by the semidiurnal tide and the mean meridional circulation. *J. Atmos. Sci.* 47, 1894–1901.
- Hourdin, F., Talagrand, O., Sadourny, R., Courtin, R., Gautier, D., McKay, C.P., 1995. Numerical simulation of the general circulation of the atmosphere of Titan. *Icarus* 117, 358–374.
- Iga, S.-I., Matsuda, Y., 2005. Shear instability in a shallow water model with implications for the Venus atmosphere. *J. Atmos. Sci.* 62, 2514–2527.
- James, I.N., 1994. *Introduction to Circulating Atmospheres*. Cambridge University Press, Cambridge, 422pp.
- Kostiuk, T., Fast, K.E., Livengood, T.A., Hewagama, T., Goldstein, J.J., Espenak, F., Buhl, D., 2001. Direct measurement of winds on Titan. *Geophys. Res. Lett.* 28, 2361–2364.
- Kuo, H.L., 1949. Dynamic instability of two-dimensional nondivergent flow in a barotropic atmosphere. *J. Meteor.* 6, 105–122.
- Leovy, C.B., 1985. Zonal wind in the stratosphere of Titan. In: *The Atmospheres of Saturn and Titan*, Proc. Int. Workshop, 16–19 September 1985, ESA SP-241, Alpbach, Austria, pp. 95–98.
- Leovy, C.B., 1987. Zonal winds near Venus' cloud top level: an analytic model of the equatorial wind speed. *Icarus* 69, 193–201.
- Lighthill, M.J., 1978. *Waves in Fluids*. Cambridge University Press, London and New York, 504pp.
- Lindzen, R.S., 1981. Turbulence and stress due to gravity wave and tidal breakdown. *J. Geophys. Res.* 86, 9707–9714.
- Lindzen, R.S., 1990. *Dynamics in Atmospheric Physics*. Cambridge Univ. Press, Cambridge, 310pp.
- Liou, K.N., 2002. *An Introduction to Atmospheric Radiation*, Second ed. Academic Press, New York, 583pp.
- Luz, D., Hourdin, F., 2003. Latitudinal transport by barotropic waves in Titan's stratosphere. I. General properties from a horizontal shallow-water model. *Icarus* 166, 328–343.
- Murgatroyd, R.J., Singleton, 1961. Possible meridional circulations in the stratosphere and mesosphere. *Quart. J. Roy. Meteor. Soc.* 87, 125–135.
- Newman, M., Schubert, G., Kliore, A.J., Patel, I.R., 1984. Zonal winds in the middle atmosphere of Venus from Pioneer Venus radio occultation data. *J. Atmos. Sci.* 41, 1901–1913.
- Rossow, W.B., Williams, G.P., 1979. Large-scale motion in the Venus stratosphere. *J. Atmos. Sci.* 36, 377–389.
- Schubert, G., 1983. General circulation and dynamical state of the Venus atmosphere. In: Hunten, D.M., et al. (Eds.), *Venus*. Univ. of Arizona Press, Tucson, pp. 681–765.
- Schubert, G., Whitehead, J., 1969. Moving flame experiment with liquid mercury: possible implications for the Venus atmosphere. *Science* 163, 71–72.
- Tokano, T., Neubauer, F.M., Laube, M., McKay, C.P., 1999. Seasonal variation of Titan's atmospheric structure simulated by a general circulation model. *Planet. Space Sci.* 47, 493–520.
- Walterscheid, R.L., Schubert, G., Newman, M.J., Kliore, A.J., 1985. Zonal winds and the angular momentum balance of Venus atmosphere within and above the clouds. *J. Atmos. Sci.* 42, 1982–1990.
- Yamamoto, M., Takahashi, M., 2003. The fully developed superrotation simulated by a general circulation model of a Venus-like atmosphere. *J. Atmos. Sci.* 60, 561–574.
- Zhu, X., 2004. Radiative transfer in the middle atmosphere and planetary atmospheres. In: Zhu, X., et al. (Eds.), *Observation, Theory and Modeling of Atmospheric Variability*. World Scientific, Singapore, pp. 359–396.
- Zhu, X., 2005. Maintenance of equatorial superrotation in a planetary atmosphere: analytic evaluation of the zonal momentum budgets for the stratospheres of Venus, Titan and Earth. JHU/APL Technical Report, No. SRA-2005-01, The Johns Hopkins University Applied Physics Laboratory, Laurel, MD, USA, 66pp. [PDF file available upon request from the author: xun.zhu@jhuapl.edu].
- Zhu, X., Strobel, D.F., 2005. On the maintenance of thermal wind balance and equatorial superrotation in Titan's stratosphere. *Icarus* 176, 331–350.
- Zhu, X., Yee, J.-H., Strobel, D.F., Wang, X.-L., Greenwald, R.A., 1999. On the numerical modeling of middle atmosphere tides. *Quart. J. Roy. Meteorol. Soc.* 125, 1825–1857.