Large-scale electron acceleration by parallel electric fields during magnetic reconnection

J Egedal, A Le, J Ng, O Ohia, A Vrublevskis, P Montag, W Daughton & VS Lukin

MIT, PSFC, Cambridge, MA

PIC simulations

Fluid simulations
Outline

• Description of Magnetic Reconnection

• Spacecraft Observations from the Earth’s Magnetotail

• Kinetic Model for the Electrons

• Fixing the Fluid Equations (The Equations of State)

• Large Scale Electron Acceleration

• A New Experiment is Needed

• Conclusions
Coronal Mass Ejections

Movie from NASA’s Solar Dynamics Observatory (SDO)
Space Weather

The Solar Wind affects the Earth’s environment
The Earth’s Magnetic Shield

Before reconnection

During reconnection

Magnetopause
Cusp
Solar Wind
Bow Shock

Magnetosheath
Magnetotail
Plasmasheet
Neutral point
Plasmasphere
Reconnection: A Long Standing Problem

*Simplest model for reconnection:*

\[ \mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j} \quad [\text{Sweet-Parker (1957)}] \]

*Sweet-Parker: \( L >> \delta \):*

\[ t_{sp} = \sqrt{t_R t_A} = \sqrt{\frac{\mu_0 L^2}{\eta}} \sqrt{\frac{L}{v_A}} \]

Unfavorable for fast reconnection

Two months for a coronal mass ejections
Plasma Kinetic Description

The collisionless Vlasov equation:

\[
\left( \frac{\partial}{\partial t} + v \frac{d}{dt} f_j(x, v, t) = 0 \right) \cdot \nabla_v f_j = 0
\]

\[
n_j = \int f_j d^3v \quad \quad J_j = q_j \int v f_j d^3v
\]

+ Maxwell’s eqs.

Vlasov-Maxwell system of equations

Can be solved numerically (PIC-codes)
Fluid Formulation (Conservation Laws)

mass:

\[ \frac{\partial n}{\partial t} + \frac{\partial (nu_j)}{\partial x_j} = 0, \]

momentum:

\[ mn \left( \frac{\partial u_j}{\partial t} + u_k \frac{\partial u_j}{\partial x_k} \right) + \frac{\partial P_{jk}}{\partial x_k} - en(E_j + \epsilon_{jkl}u_kB_l) - F^\text{coll}_j = 0, \]

energy:

\[ \frac{\partial P_{jk}}{\partial t} + \frac{\partial}{\partial x_l} \left( P_{jk}u_l + Q_{jkl} \right) + \frac{\partial u_j}{\partial x_l} P_{lk} - \frac{e}{m} \epsilon_{jlm}B_mP_{lk} - G^\text{coll}_{jk} = 0 \]

Isotropic (scalar) pressure is the standard closure!

\[ p = nT \]

Add Maxwell’s eqs to complete the fluid model

\[ \Rightarrow \text{Fast Reconnection!} \]
Two-Fluid Simulation

GEM challenge (Hall reconnection)
\[ \mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{(\mathbf{j} \times \mathbf{B})}{ne} \]  
[Birn,…, Drake,… Bhattacharjee, et al. (2001)]

\[ E + v \times B = (j \times B)/ne \]

Aspect ratio: 1 / 10

\[ v_{in} \sim \frac{v_{A}}{10} \]
Two-Fluid vs Kinetic Simulations

Isotropic pressure

Fluid: Isotropic Pressure

Out of plane current

Kinetic Simulation

Fully Kinetic Simulation

Particle In Cell (PIC) simulation,
Outline

- Description of Magnetic Reconnection
- Spacecraft Observations from the Earth’s Magnetotail
- Kinetic Model for the Electrons
- Fixing the Fluid Equations (The Equations of State)
- Large Scale Electron Acceleration
- A New Experiment is Needed
- Conclusions
Wind Spacecraft Observations in Distant Magnetotail, 60\(R_E\)

- Measurements within the ion diffusion region reveal:
  
  Strong anisotropy in \(f_e\)

\[ p_\parallel > p_\perp \]

[Øieroset et al., PRL (2001)]
Observed Electron Heating

1-10 keV beam like electron distributions often observed

Flat-top distributions typical in the exhaust; related beams?


Cluster observations

Cluster observations on 2001-10-01, [L.-J. Chen, JGR (2008)]
Observed Electron Heating

Cluster observations on 2001-10-01.

Lobe: \( T_e \sim 0.1 \text{ keV} \)

Inflow: \( T_{e\perp} \sim 0.1 \text{ keV}, \ T_{e\parallel} \sim 1 \text{ keV} \)

Exhaust: \( T_{e\perp} \sim T_{e\parallel} \sim 10\text{keV} \)

Reproduced from Egedal, …, Chen et al. JGR (2010)
Outline

- Description of Magnetic Reconnection
- Spacecraft Observations from the Earth’s Magnetotail
- Kinetic Model for the Electrons
- Fixing the Fluid Equations (The Equations of State)
- Large Scale Electron Acceleration
- A New Experiment is Needed
- Conclusions
Wind Spacecraft Observations in Distant Magnetotail, 60$R_E$

- Measurements within the ion diffusion region reveal:
  Strong anisotropy in $f_e$
  
  $p_{\parallel} > p_{\perp}$
Electrons in an Expanding Flux Tube

Magnetic moment:

\[ \mu = \frac{mv_{\perp}^2}{2B} \]

⇒ mirror force:

Expanding Flux tube

- Trapped electron

- \( -\mu \nabla B \)

- \( -\mu \nabla B \)

\[ v_{\perp} \]

\[ v_{\parallel} \]

Trapped

Passing

Trapped

Passing
Electrons in an Expanding Flux Tube

Trapped:
\[ \mathcal{E}_\perp = \mu B = \mathcal{E}_\infty B / B_\infty \]
\[ \implies \mathcal{E}_\infty = \mu B_\infty \]

Passing:
\[ \mathcal{E} = \mathcal{E}_\infty + e\Phi_\parallel \]
\[ \implies \mathcal{E}_\infty = \mathcal{E} - e\Phi_\parallel \]

Vlasov:
\[ \frac{df}{dt} = 0 \]
\[ f(x, v) = f_\infty(\mathcal{E}_\infty) \]

\[ \Phi_\parallel(x) = \int_x^\infty E \cdot dl \]

\[ \mathcal{E}_\parallel(\infty) = \mathcal{E} - e\Phi_\parallel - \mu B_\infty = 0 \]

J. Egedal et al., JGR (2009)
Wind Spacecraft Observations in Distant Magnetotail, 60R_E

\[ f(x, v) = \begin{cases} 
  f_\infty(\mathcal{E} - e\Phi_\parallel) , & \text{passing} \\
  f_\infty(\mu B_\infty) , & \text{trapped}
\end{cases} \]

\[ \Phi_\parallel \sim 1 \text{ kV} \]
Formal derivation using an “ordering”

The drift kinetic equation:

$$\frac{\partial f}{\partial t} + (v_\parallel + v_D) \cdot \nabla f + \left[ \mu \frac{\partial B}{\partial t} + e(v_\parallel + v_D) \cdot E \right] \frac{\partial f}{\partial E} = 0$$

Boundary conditions:

$$B = B_\infty$$
$$f = f_\infty(E_\parallel, E_\perp)$$

Ordering:

$$\nabla_\parallel \sim \frac{1}{L} \quad , \quad \nabla_\perp \sim \frac{1}{d} \quad , \quad \frac{\partial}{\partial t} \sim \frac{v_D}{d}$$
$$\frac{d}{L} \sim \delta \quad , \quad \frac{v_D}{v_t} \sim \delta^2 \quad , \quad \frac{E_\parallel}{E_\perp} \sim \delta$$
Formal derivation, passing electrons

Passing electrons, lowest order equation: 
\[ v_\parallel \cdot \nabla f + e(v_\parallel \cdot E) \frac{\partial f}{\partial \mathcal{E}} = 0 \]

Integrate along characteristics (field lines):

\[ f(\mathcal{E}_\parallel, \mathcal{E}_\perp) = f_\infty(\mathcal{E} - e\Phi^\pm_\parallel - \mu B_\infty, \mu B_\infty) \]

where 
\[ \mathcal{E} = \mathcal{E}_\parallel + \mathcal{E}_\perp , \quad \Phi^\pm_\parallel = \int_{x}^{\pm \infty} E \cdot dl_\parallel \]

Quasi-neutrality at boundaries: 
\[ n = \int f d^3v = n_\infty \]

\[ \int_{-\infty}^{\infty} E \cdot dl_\parallel = 0. \quad \Rightarrow \quad \Phi^+ = \Phi^- = \Phi_\parallel \]
Formal derivation, trapped electrons

Use full equation with solution: \[ f = g(\mu, J, \phi_J) \]
\[ = f_\infty \left( E_{||\infty}[\mu, J, \phi_J], E_{\perp\infty}[\mu, J, \phi_J] \right) \]

2\textsuperscript{nd} adiabatic invariant: \[ J = \oint v_{||} dl \]

\[ E_{\perp\infty} = h_1(\mu, J, \phi_J) = \mu B_\infty \]

\[ E_{||\infty} = h_2(\mu, J, \phi_J) \approx 0 \]

Thus, for Maxwellian \( f_\infty \):

\[ f(x, v) = \begin{cases} 
  f_\infty(E - e\Phi_{||}) , & \text{passing} \\
  f_\infty(\mu B_\infty) , & \text{trapped}
\end{cases} \]

Only electrons with small parallel energy will be caught in the magnetic and electric well as it develops slowly compared to the electron transit time, \( \tau \sim L/v_t \).

\[ E_{||\infty} \leq \mu \frac{\partial B}{\partial t} \tau + e \frac{\partial E_{||}}{\partial t} \tau L \]
\[ \approx \mu B \frac{v_d L}{d v_t} + e E_{||} \frac{v_d L}{d v_t} \]
\[ \approx \delta (T_e + e\Phi_{||}) \]
Kinetic Model $\Rightarrow$ Fluid Closure

Theoretical distribution:

\[ f(x, v) = \begin{cases} 
  f_{\infty}(\varepsilon - e\Phi_{\parallel}), & \text{passing} \\
  f_{\infty}(\mu B_{\infty}), & \text{trapped}
\end{cases} \]
Outline

• Description of Magnetic Reconnection
• Spacecraft Observations from the Earth’s Magnetotail
• Kinetic Model for the Electrons
• Fixing the Fluid Equations (The Equations of State)
• Large Scale Electron Acceleration
• A New Experiment is Needed
• Conclusions
Kinetic Model $\rightarrow$ Fluid Closure

$$f_p(\mathcal{E}) = f_\infty(\mathcal{E}) e^{e \Phi_\parallel/T_\infty}$$

$$\Phi_\parallel < 0, \quad B > B_\infty \quad \implies \quad \text{no trapping}$$

$$n = n_\infty e^{e \Phi_\parallel/T_\infty} < n_\infty,$$

$$\frac{e \Phi_\parallel}{T_\infty} = \log \left( \frac{n}{n_\infty} \right)$$

$$p_\parallel = p_\perp = nT_\infty$$

$$\tilde{p}_\parallel = \tilde{p}_\perp = \tilde{n}$$
Kinetic Model $\rightarrow$ Fluid Closure

\[ f_t = \frac{n_\infty}{(\sqrt{\pi} v_t)^3} e^{-\frac{v_{\perp}^2}{v_t^2} \frac{B_\infty}{B}} \]

\[ f_p(\mathcal{E}) = f_\infty(\mathcal{E}) e^{e\Phi_\parallel/T_\infty} \]

\[ e\Phi_\parallel \gg T_\infty, \quad B < B_\infty \]

(ignoring contributions from passing electrons)

\[ n \simeq \int_{-v_\phi}^{v_\phi} dv_\parallel \int_0^\infty 2\pi v_\perp dv_\perp f_t = \frac{2}{\sqrt{\pi}} \frac{v_\phi}{v_t} \frac{B}{B_\infty} n_\infty \quad \implies \quad \frac{v_\phi}{v_t} \simeq \sqrt{\frac{\pi}{2}} \frac{\tilde{n}}{B} \]

\[ p_\parallel \simeq m_e \int_{-v_\phi}^{v_\phi} v_\parallel^2 dv_\parallel \int_0^\infty 2\pi v_\perp dv_\perp f_t = \frac{1}{3} \frac{v_\phi^2}{v_t} m_e n \quad \implies \quad \tilde{p}_\parallel \simeq \frac{\pi}{6} \frac{\tilde{n}^3}{B^2} \]

\[ \frac{e\Phi_\parallel}{T_\infty} \simeq \frac{\pi}{4} \frac{\tilde{n}^2}{B^2} \]

\[ p_\perp \simeq nT_\infty \frac{B}{B_\infty} \quad \implies \quad \tilde{p}_\perp \simeq \tilde{n} \tilde{B} \]
Kinetic Model \( \rightarrow \) Fluid Closure (EoS)

\[
f(x, v) = \begin{cases} 
  f_{\infty}(E - e\Phi_{\parallel}), & \text{passing} \\
  f_{\infty}(\mu B_{\infty}), & \text{trapped}
\end{cases}
\]

\[
\int \ldots d^3v
\]

\[
\begin{align*}
  n &= n(B, \Phi_{\parallel}) \\
  p_{\parallel} &= p_{\parallel}(B, \Phi_{\parallel}) \\
  p_{\perp} &= p_{\perp}(B, \Phi_{\parallel})
\end{align*}
\]

Eliminate \( \Phi_{\parallel} \) \( \rightarrow \)

\[
\begin{align*}
  p_{\parallel} &= p_{\parallel}(n, B) \\
  p_{\perp} &= p_{\perp}(n, B)
\end{align*}
\]

Transition from Boltzmann to double adiabatic CGL-scaling

[ G Chew, M Goldberger, F E Low, 1956]

A. Le et al., PRL (2009)
The Acceleration Potential in a Kinetic Simulation

eΦₚ / Tₑ

Trapping potential, 0 - 8
Confirmed in Kinetic Simulations

EoS previously confirmed in 2D simulations, now also in 3D simulations.
New EoS Now Implemented in Two-Fluid Code

New code implemented by O Ohia using the HiFi framework developed in part by VS Lukin

Standard two-fluid equations

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}_i) = 0
\]

\[
m_i n \left( \frac{\partial \mathbf{V}_i}{\partial t} + \mathbf{V}_i \cdot \nabla \mathbf{V}_i \right) = \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbf{P} + m_i n v_i \nabla^2 \mathbf{V}_i
\]

\[
\frac{\partial}{\partial t} \left( \frac{p_i}{n \Gamma} \right) = -\nabla \cdot \frac{p_i}{n \Gamma}
\]

\[
\frac{\partial \mathbf{B}'}{\partial t} = -\nabla \times \mathbf{E}'
\]

\[
\mathbf{E}' + \mathbf{V}_i \times \mathbf{B} = \frac{1}{n_e} \left( \mathbf{J} \times \mathbf{B}' - \nabla \cdot \mathbf{P}_e \right) + \eta R \mathbf{J} - \eta_H \nabla^2 \mathbf{J}
\]

\[
\mathbf{B}' = (1 - d_e^2 \nabla^2) \mathbf{B}
\]

\[
\mu_0 \mathbf{J} = \nabla \times \mathbf{B}
\]

Anisotropic pressure model

\[
\bar{P} = p_i \bar{I} + \bar{P}_e = p_i \bar{I} + p_{\parallel} \bar{I} + (p_{\parallel} - p_{\perp}) \frac{BB}{B^2}
\]

\[
p_{*\parallel} = \frac{n_*}{1 + \alpha/2} + \frac{\alpha \pi}{6 + 3/\alpha}
\]

\[
p_{*\perp} = \frac{n_*}{1 + \alpha} + \frac{n_* B_*}{1 + 1/\alpha}
\]

where \( \alpha = n_*^3/B_*^2 \) and for any quantity \( Q \), \( Q_* = Q/Q_\infty \)
New *EoS* Implemented in Two-Fluid Code

Out of plane current

Isotropic pressure

Anisotropic pressure

Kinetic Simulation

FLUID: ISOTROPIC PRESSURE

FLUID: NEW CLOSURE

FULLY KINETIC SIMULATION

Ohia et al., PRL 2012 (in press)
EoS for anti-parallel reconnection?

The electrons are magnetized in the inflow region:
Electron distributions in the layer  [J. Ng et al., PRL2011]
Electron distributions in the layer [J. Ng et al., PRL 2011]

The triangular shape + rotation yield finite $\partial_x P_{xy}$ balancing the reconnection electric field, $E_y$!

J. Ng et al., PRL (2011)
Outline

• Description of Magnetic Reconnection
• Spacecraft Observations from the Earth’s Magnetotail
• Kinetic Model for the Electrons
• Fixing the Fluid Equations (The Equations of State)
• Large Scale Electron Acceleration
• A New Experiment is Needed
• Conclusions
Analytic Model for Electron Jets

The magnetic tension is balanced by pressure anisotropy:

$$p_{\parallel}(n, B) - p_{\perp}(n, B) = B^2/\mu_0$$

Use EoS to get scaling laws:

$$\beta_e = \frac{\text{plasma pressure}}{\text{magnetic pressure}}$$
Observed Electron Heating

1-10 keV beam like electron distributions often observed

Flat-top distributions typical in the exhaust; related beams?


Observed Electron Heating

Cluster observations on 2001-10-01.

Lobe: \( T_e \sim 0.1 \) keV

Inflow: \( T_{e\perp} \sim 0.1 \) keV, \( T_{e\parallel} \sim 1 \) keV

Exhaust: \( T_{e\perp} \sim T_{e\parallel} \sim 10 \) keV

For this event \( \beta_e \sim 0.003 \)
New simulation with $\beta_e \sim 0.003$

- 320 $d_i$ long, 180 billion particles!
New simulation with $\beta_e \sim 0.003$

- 320 $d_i$ long, 180 billion particles!

J. Egedal et al., Nature Physics (2012)
Flat-top Distributions

- 320 $d_i$ long, 180 billion particles!

J. Egedal et al., Nature Physics (2012)
Outline

• Description of Magnetic Reconnection
• Spacecraft Observations from the Earth’s Magnetotail
• Kinetic Model for the Electrons
• Fixing the Fluid Equations (The Equations of State)
• Large Scale Electron Acceleration
  • A New Experiment is Needed
• Conclusions
**EoS for Anti-Parallel Reconnection?**

The electrons are magnetized in the inflow region:

Pitch angle diffusion is controlled by:

\[ \kappa = \sqrt{R_B / \rho_e} \]

Depends on \( B_g \) and \( m_i/m_e \)
Scan in $B_g$ and $m_i/m_e$ ($\beta_e = 0.03$)

Pitch angle diffusion
Scan in $B_g$ and $m_i/m_e$
Need Experiment for 3D Reconnection Study

Not accessible to 3D codes!

Daughton et al. [2011].
Requirements on New Experiment

- Large normalized size of experiment: \( L / d_i \sim 10 \) (high \( n \), large \( L \))
- Low collisionallity to allow \( p_\parallel \gg p_\perp \): \( \tau_{ei} v_A > d_i \) (low \( n \), high \( T_e \), high \( B \))
- Low electron pressure: \( \beta_e < 0.05 \) (low \( n \), \( T_e \), high \( B \))
- Manageable loop voltage: \( 0.1 v_A B_{\text{rec}} (2\pi R) < 5 \text{kV} \) (high \( n \), low \( B \))
- Variable guide field: \( B_g = 0 - 4 B_{\text{rec}} \)
- Symmetric inflows

Experimental window available in Hydrogen or Helium plasma with

\[ n \sim 10^{18} \text{ m}^{-3}, \quad T_e \sim 15 \text{ eV}, \quad B_{\text{rec}} \sim 15 \text{ mT}, \quad L \sim 2 \text{ m} \]
Flare heating by parallel E-fields?

**Ohm’s law:**

\[-enE_\parallel = \hat{b} \cdot (\nabla \cdot \mathbf{p})\]

Before reconnection: \( p = nT_e \Rightarrow e\Phi_\parallel \sim T_e \log(n/n_0) \)

During reconnection:

\[ P_\parallel \propto \frac{n^3}{B^2} \]

\[ \Rightarrow e\Phi_\parallel \approx T_e \frac{(n/n_0)^2}{(B/B_0)^2} \]

\((n/n_0) \approx 10, (B/B_0) \approx 0.5\)

\[ e\Phi_\parallel \approx 400T_e \]
Conclusions

• A new analytic model for electron the electron distribution function was inspired by the VTF experiment and has been confirmed in kinetic simulations.

• The model has been applied as a closure to the fluid equations and has helped explain electron energization in spacecraft observations.

• Long current layer can be driven by the pressure anisotropy for magnetotail conditions and their investigation requires a new experimental facility.