Helioseismology

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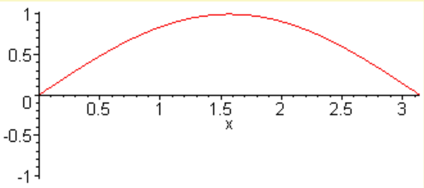
Yale University



“At first sight it would seem that the deep interior of the Sun and stars is less accessible to scientific investigation than any other region of the universe. Our telescopes may probe farther and farther into the depths of space; but how can we ever obtain certain knowledge of that which is being hidden behind substantial barriers? What appliance can pierce through the outer layers of a star and test the conditions within?”

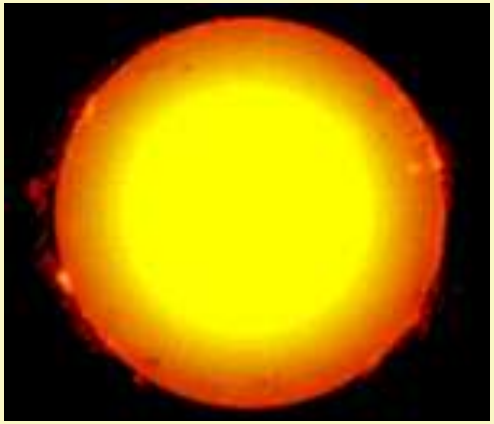
Sir Arthur Eddington in “*The internal constitution of stars*” (1926)

THE "APPLIANCE" IS ASTEROSEISMOLOGY

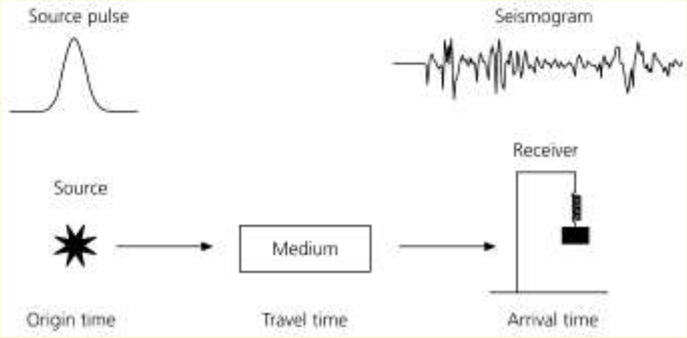
Aster: Classical Greek for star

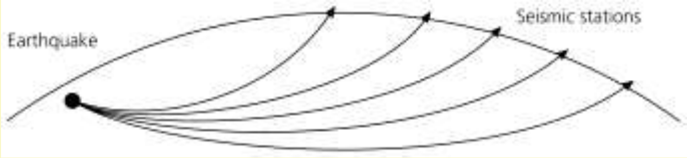
Seismos: Classical Greek for tremors

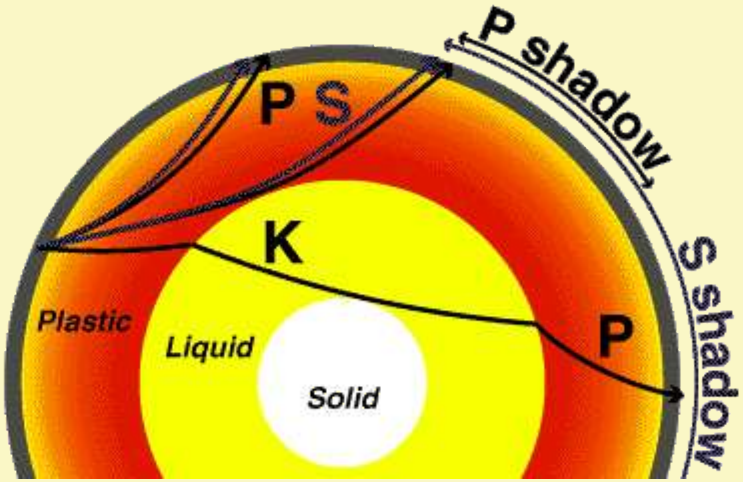
logos: Classical Greek for reasoning or discourse



Helioseismology is slightly different from geoseismology







Helioseismology usually uses "normal" modes

The Sun and the stars oscillates in *normal* modes.

Normal modes in an oscillating system are special solutions where all the parts of the system are oscillating with the same frequency (called "normal frequencies" or "allowed frequencies").

Stellar oscillations are *standing waves*.

A standing wave, also known as a stationary wave, is a wave that remains in a constant position.

This phenomenon can occur because the medium is moving in the opposite direction to the wave, or it can arise in a stationary medium as a result of interference between two waves travelling in opposite directions.

How do we study stellar interiors? We probe it with sound

Image courtesy D. Kutz

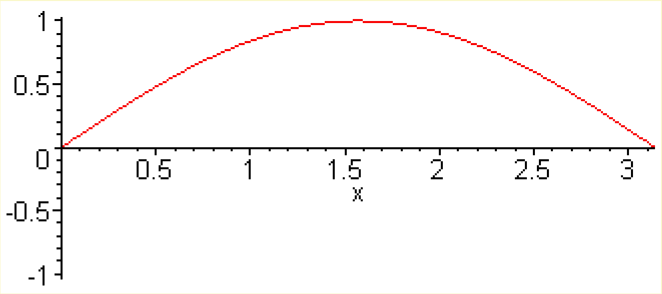
The speed of sound

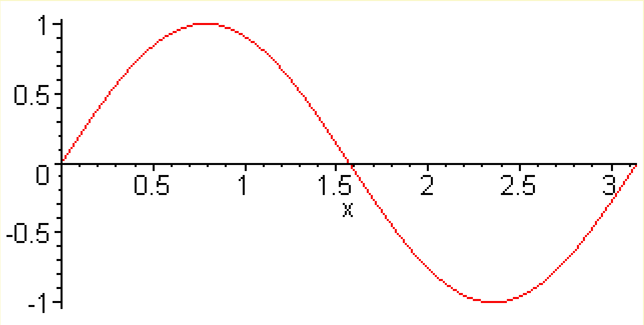
- Air = 343 m/s (20 C)
- Helium = 965 m/s
- Hydrogen = 1284 m/s
- Water = 1482 m/s (20 C)
- Granite = 6000 m/s
- Outer layers of the Sun = 10 km/s
- Core of the Sun = 550 km/s

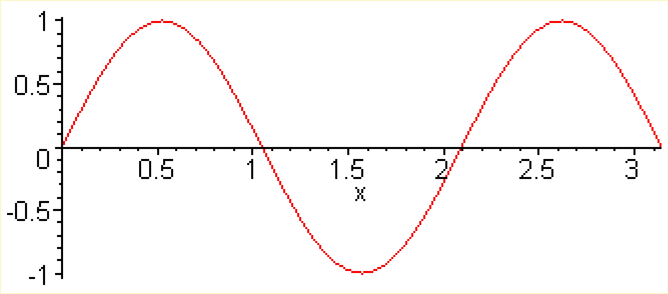
What about the Sun

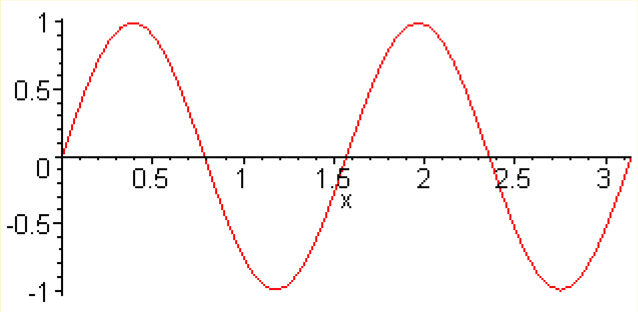
It rumbles at 3/1000 Hz

Strings: normal modes in 1D



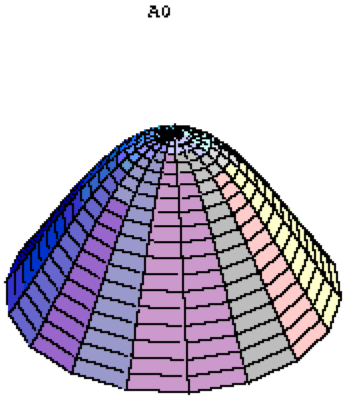






$$y_n(x,t) = A_n \sin\left(n\pi \frac{x}{L}\right)\cos(\omega_n t - \phi_n)$$

Vibrating drum: normal modes in 2D



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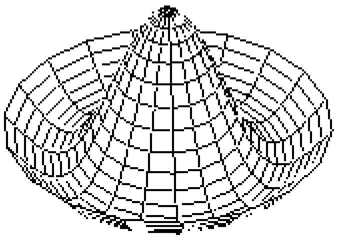
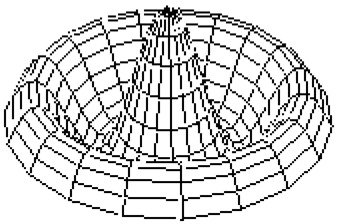
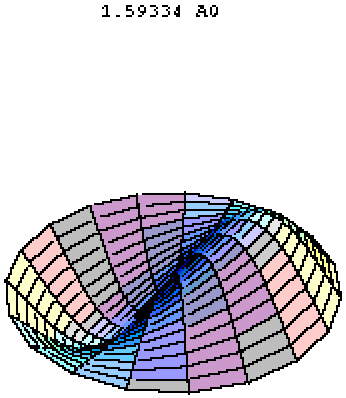


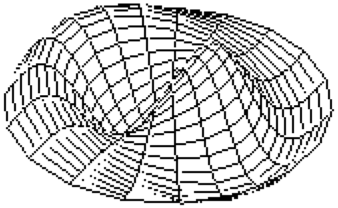
(0,2)



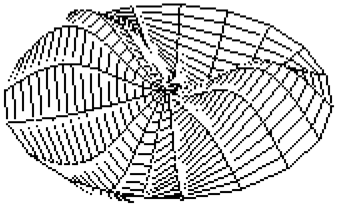
(0,3)



(1,1)



(1,2)



(2,1)

2D oscillations















Slide courtesy D. Kutz

The Sun: 3D example of normal modes

Need 3 numbers to specify mode.:

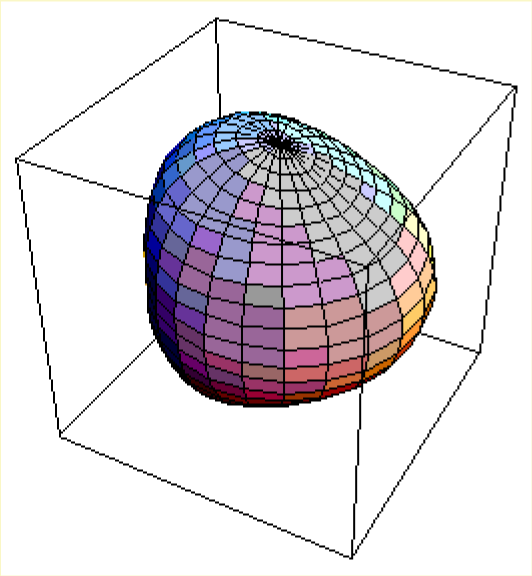
One for mode in the theta (θ) direction,

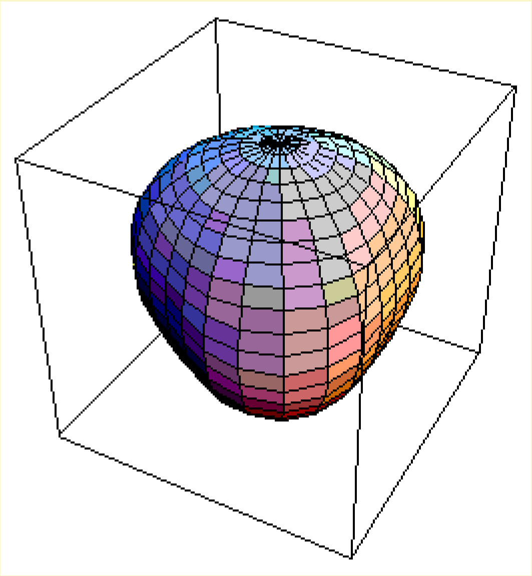
One for nodes in the phi (ϕ) direction,

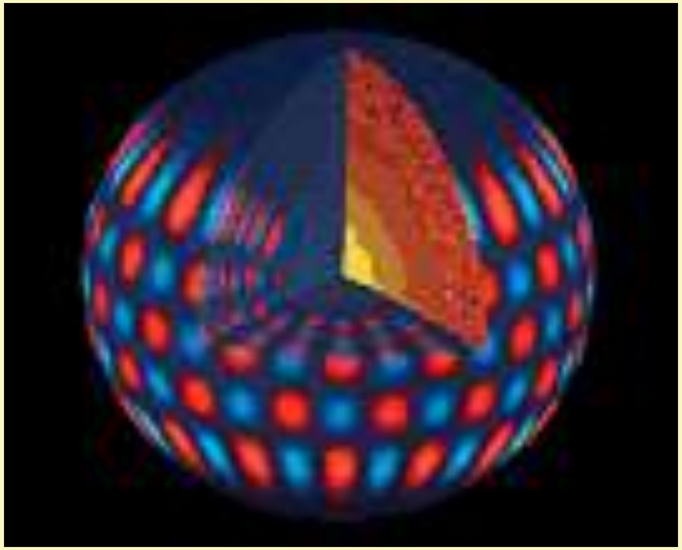
One for the nodes in the radial (r) direction.

(Spherical harmonics can describe the θ and ϕ part).

Normal modes in 3D







A Basic Course in Helioseismology

- The Sun oscillates in millions of different modes.
- The oscillations are linear and adiabatic.
- All observed modes are acoustic i.e., ***p***-modes or surface modes i.e. ***f***-modes.
- Each mode is characterized by three numbers:

(1) ***n***: the radial order, the number of nodes in the radial direction

(2) ***l***: the degree

(3) ***m***: the azimuthal order. m goes from **+*l*** to **-*l***

|***m***|= no. of node circles crossing a latitude

l-|***m***|=no. of node circles crossing a longitude.

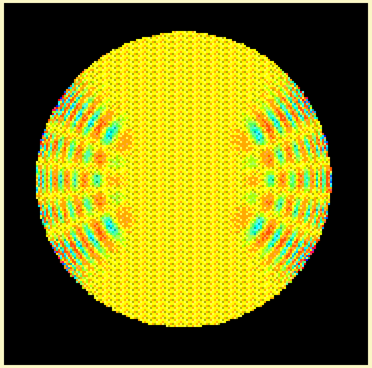
l=2, m=1

l=3 ,m=0

l=3, m=–3

- The three quantities used are *n, l, m*

- *n* is the number of nodes in the radial direction, i.e. towards the centre of the Sun



- *l* and *m* describe a checker-board pattern on the surface.

  $\sqrt{l(l+1)} = (2\pi R/\lambda)$ $[= l + 0.5]$

- The are a total *l* nodal planes intersecting the surface.
  |*m*| is the number of circles passing through the poles (like longitudes).
  m can have values from $-l$ to $+l$.

  There are *(l-m)* lines parallel to the equator (just like the latitudes).

  l is related to the horizontal wavelength of the mode on the surface:

- If the Sun were spherically symmetric and did not rotate, all modes with the same ***l*** and ***n*** but different ***m*** would have the same frequency.

- Rotation lifts this degeneracy, giving rise to "rotational splittings" of the modes:

$$D_{nlm} = \frac{\nu_{nlm} - \nu_{nl-m}}{2m} = \int_0^1 \int_0^1 dr \, d\cos\theta \, K_{nlm}(r,\theta)\Omega(r,\theta)$$

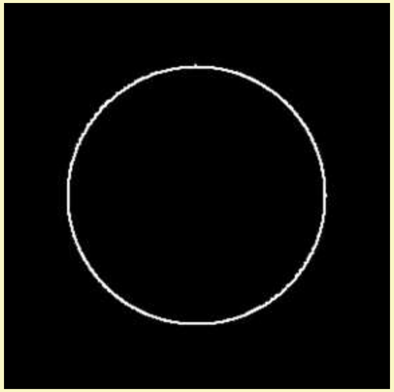
$$\nu_{nlm} = \nu_{nl} + \sum_{j=1}^{j_{max}} a_j(n,l)\mathcal{P}_j^{(l)}(m).$$

- The central frequency is used to determine the spherically symmetric structure of the Sun.

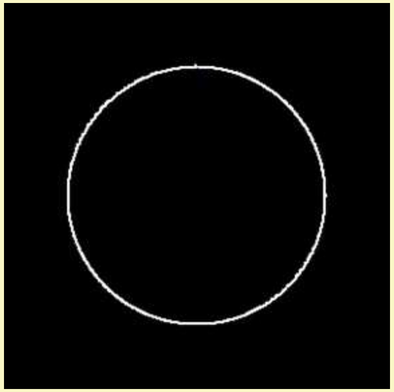
- Odd order splitting coefficients (corresponding to the symmetric part of the splitting coefficients) are used to determine the rotation rate inside the Sun.

- Even-order splitting coefficients (corresponding to the antisymmetric part of the splitting coefficients) are used to determine asphericity.

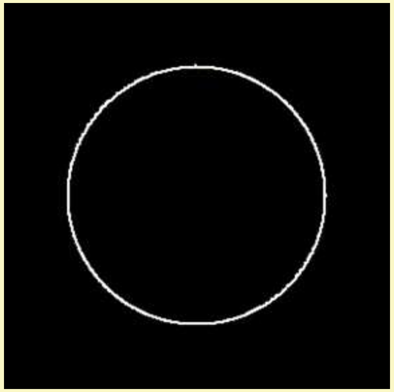
Ray paths within the Sun



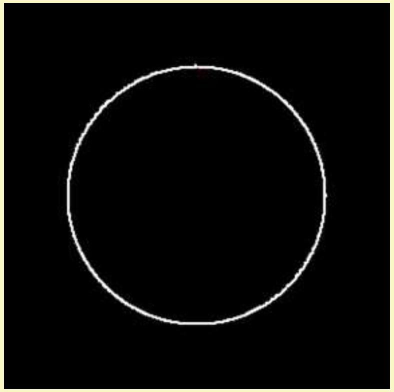
l=0



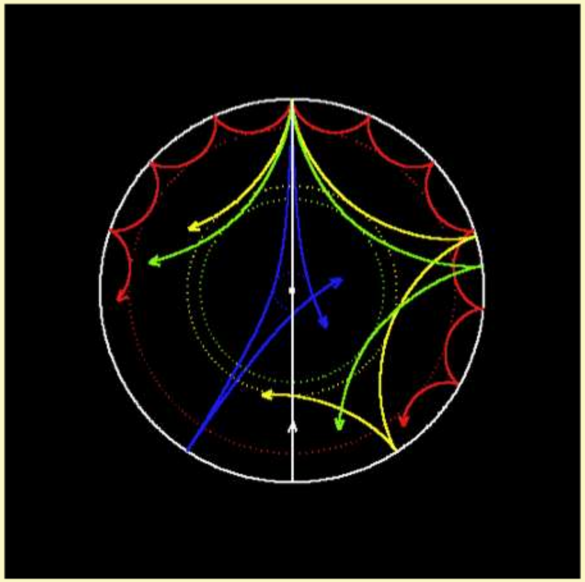
l=2

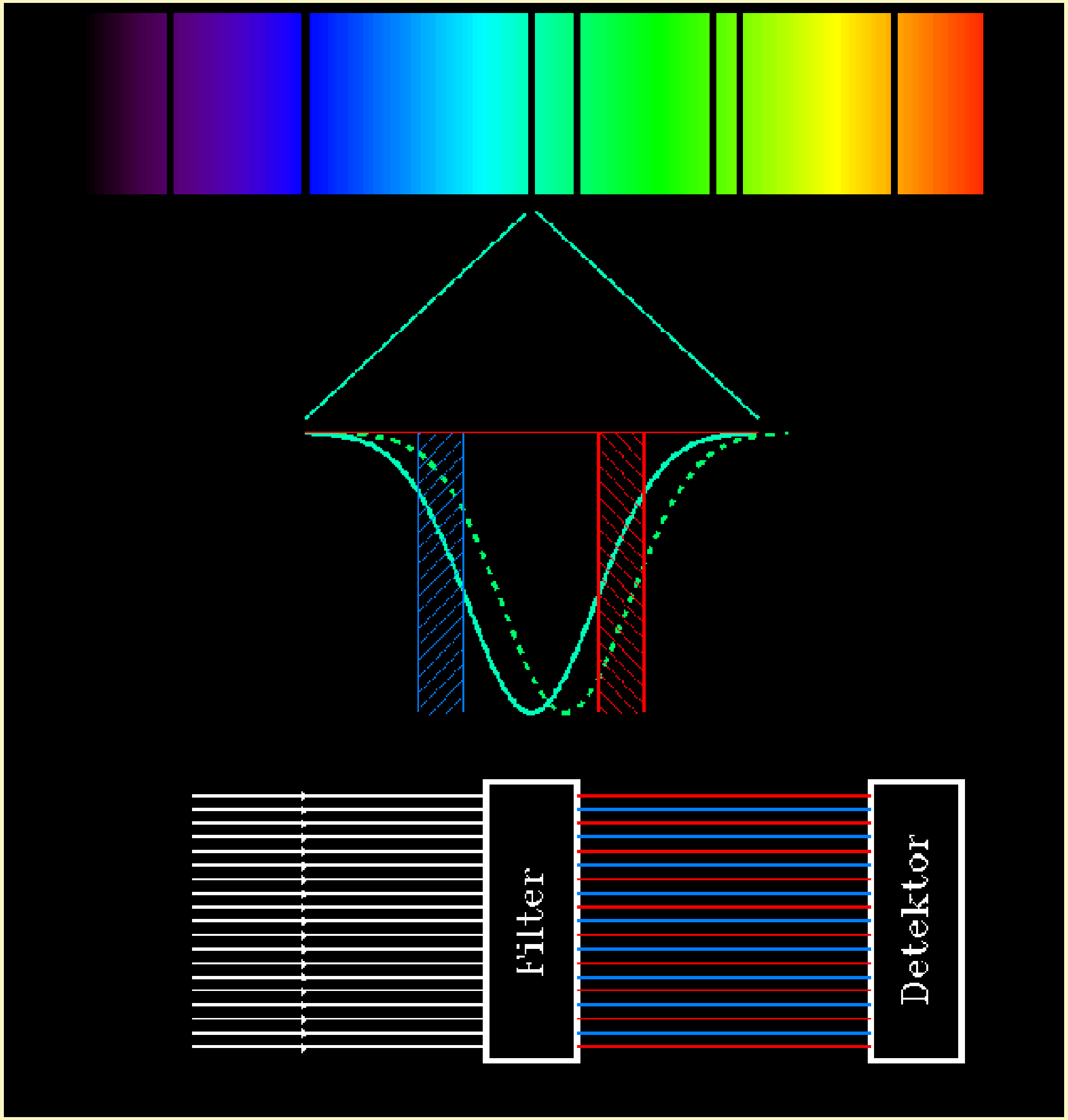


l=25

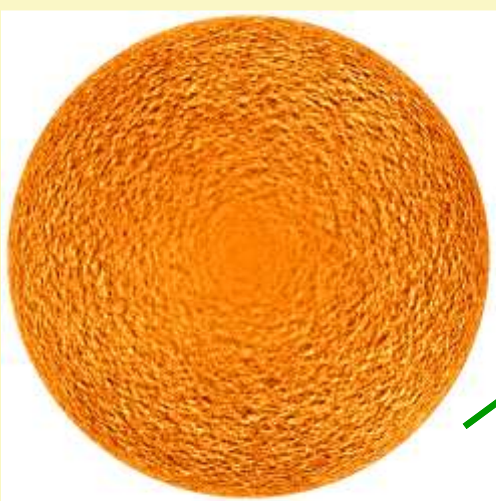


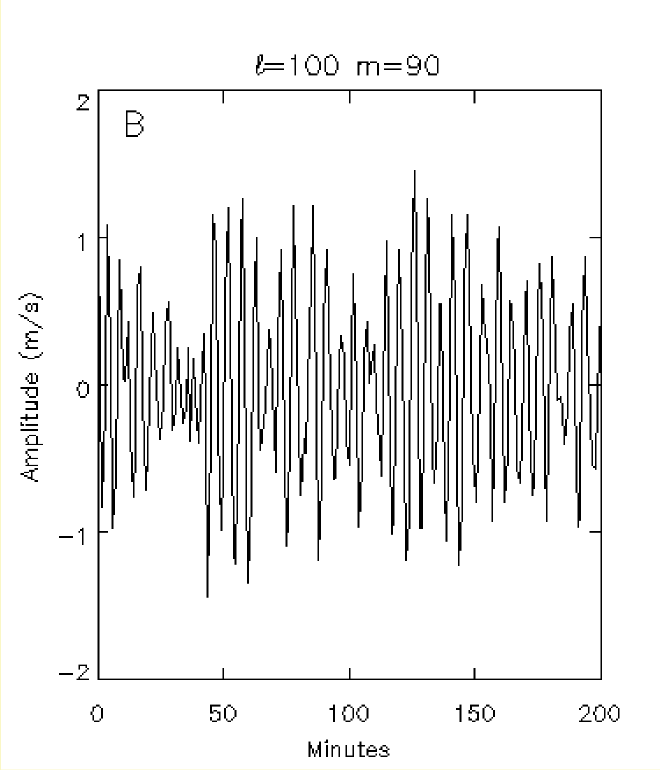
l=75




Filter
Detektor

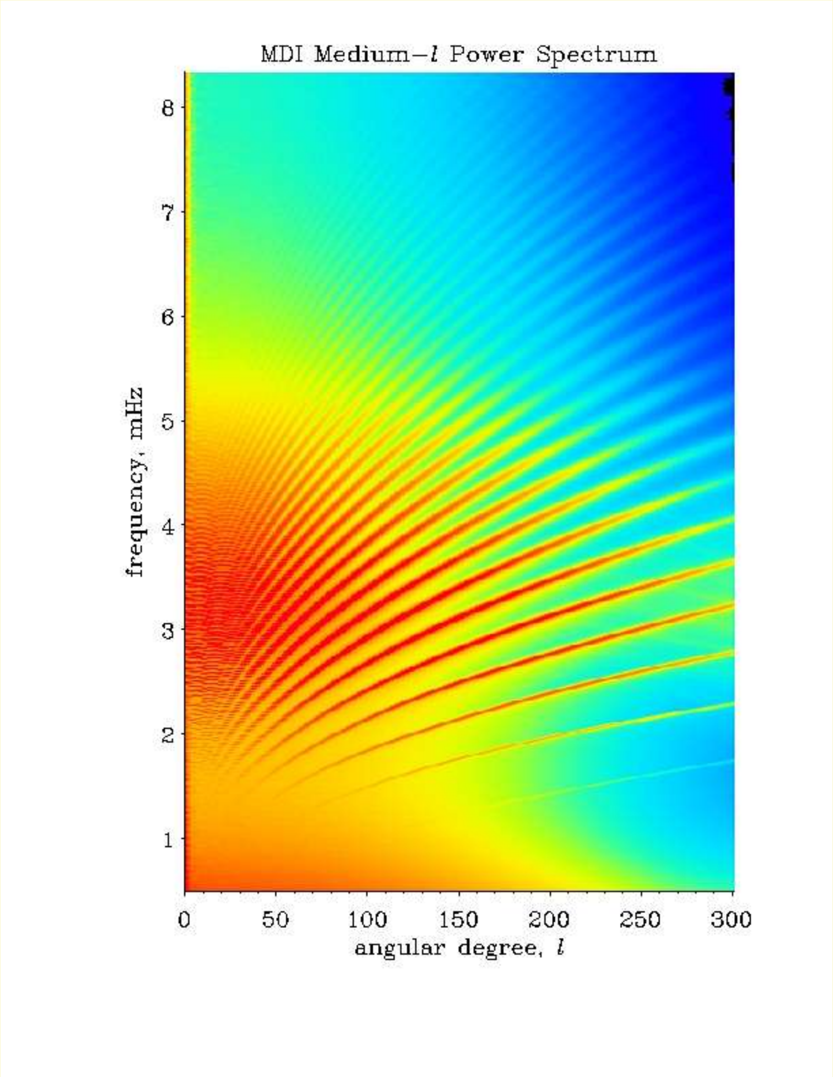
What do solar observations look like?



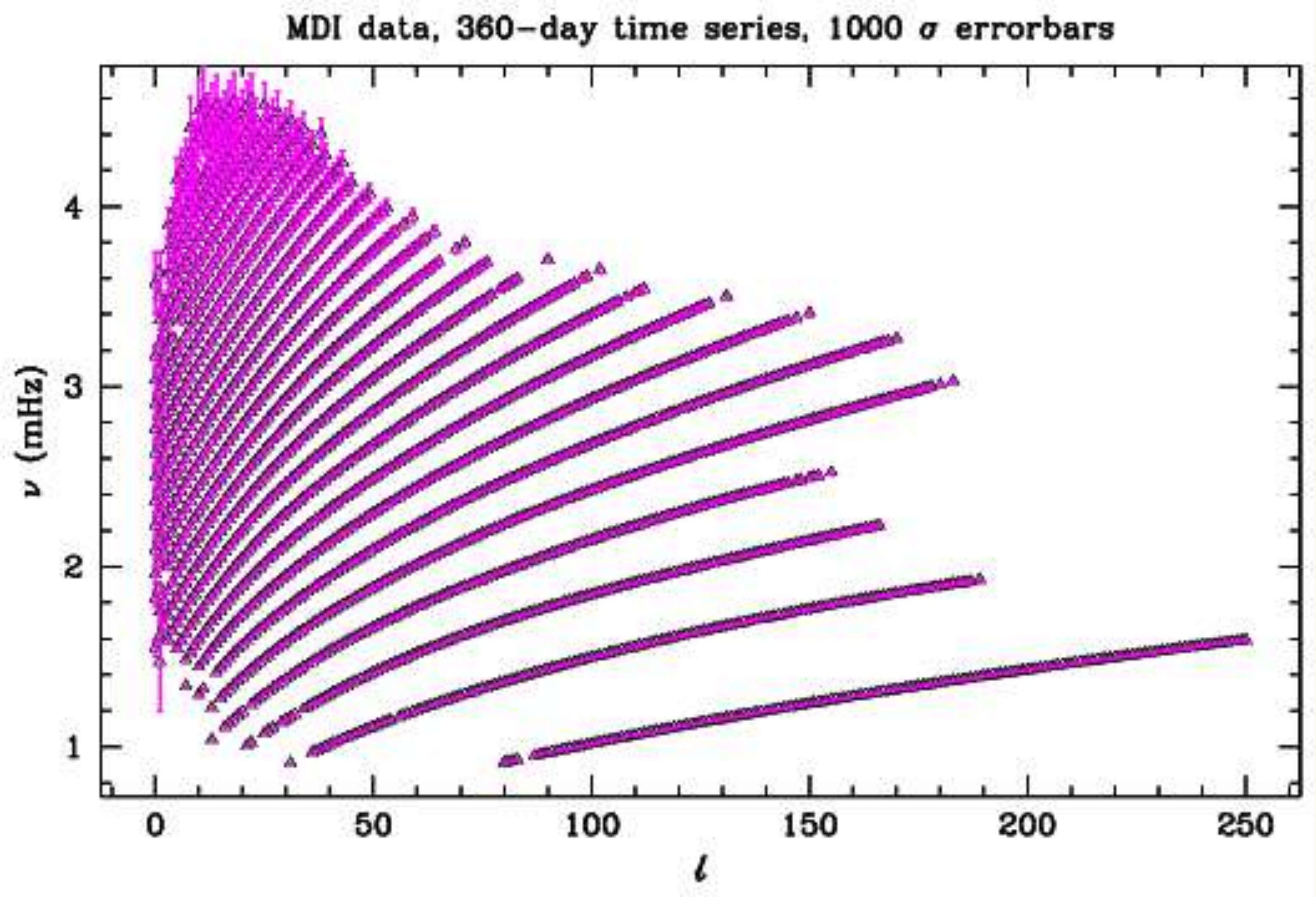


ℓ=100 m=90
B
Amplitude (m/s)
2
1
0
−1
−2
0
50
100
150
200
Minutes

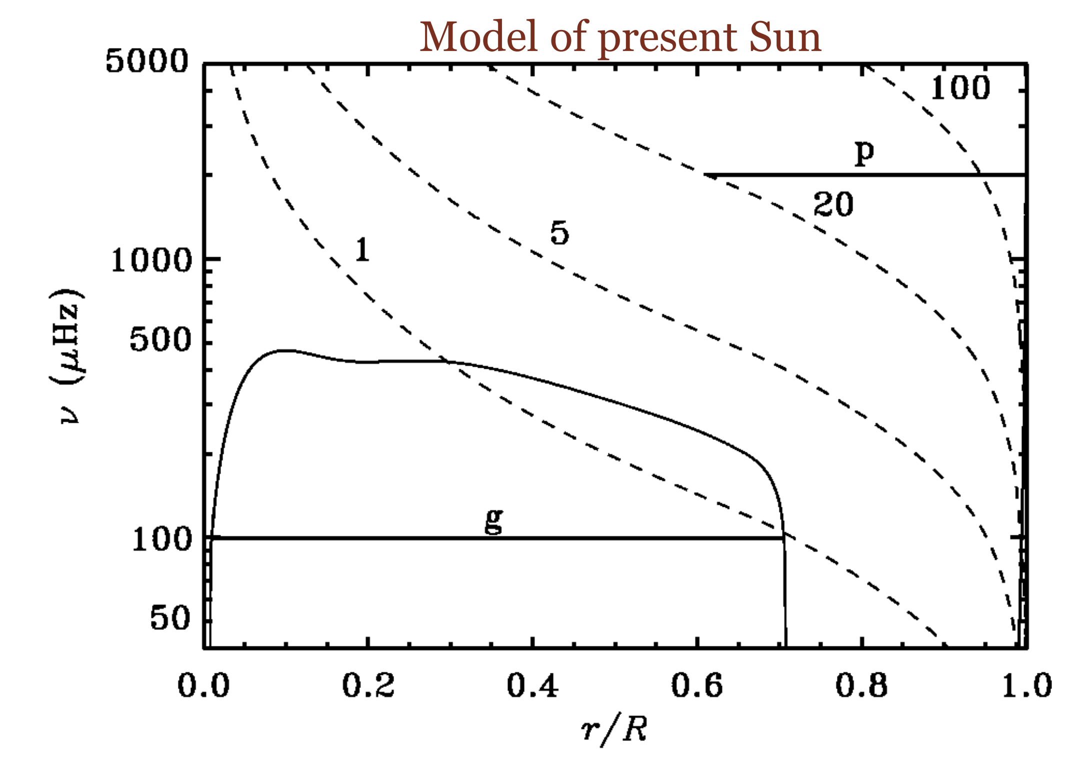
A Solar Power Spectrum



A sample of helioseismic data



Describing the modes



$$\frac{d^2\xi_r}{dr^2} \simeq -\frac{\omega^2}{c^2}\left(\frac{S_l^2}{\omega^2}-1\right)\left(\frac{N^2}{\omega^2}-1\right)\xi_r$$

Eigenfunction oscillates as function of *r* when

$\omega^2 > S_l^2, N^2$ **p modes**

$\omega^2 < S_l^2, N^2$ **g modes**

$$S_l^2 = \frac{l(l+1)c^2}{r^2}$$

$$N^2 = g\left(\frac{1}{\Gamma_1}\frac{d\ln p}{dr} - \frac{d\ln \rho}{dr}\right) \simeq \frac{g^2\rho}{p}(\nabla_{ad} - \nabla + \nabla_\mu)$$

P-modes: Equidistant in frequency

$$\nu_{nl} \simeq \Delta\nu\left(n + \frac{\ell}{2} + \alpha\right) + \epsilon_{nl}, \text{ where}$$

$$\Delta\nu = \left[2\int_0^R \frac{dr}{c}\right]^{-1}$$

G-modes: Equidistant in period

$$P_{n+1,1} \approx P_{n,1} + \frac{P_0}{\sqrt{2}}, \text{ where}$$

$$P_0 = 2p^2\left(\int_0^{r_c} \frac{N}{r}\right)^{-1}$$

Modelling the Sun

For most parts, stars are **spherically symmetric**, i.e., their internal structure is only a function of radius and not of latitude or longitude.

This means that we can express the properties of stars using a set of 1D equations, rather than a full set of 3D equations. The main equations concern the following physical principles:

(1) Conservation of Mass
(2) Conservation of momentum
(3) Thermal equilibrium
(4) Transport of energy
(5) Nuclear reaction rates
(6) Change of abundances by various processes
(7) Equation of state

An overview of the equations

(1) $$dm = 4\pi r^2 \rho \, dr$$

(2) $$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

(3) $$\frac{dl}{dm} = \varepsilon + \varepsilon_g = \varepsilon \boxed{- C_P \frac{dT}{dt} + \frac{\delta}{\rho}\frac{dP}{dt}}$$

(4) $$\frac{dT}{dm} = -\frac{GmT}{4\pi r^4 P}\nabla$$

(5) $$\frac{\partial X_i}{\partial t} = \frac{m_i}{\rho}\left[\sum_j r_{ji} - \sum_k r_{ik}\right], \quad i = 1....N$$

There are 5 equations in 6 unknowns *(r, P, T, ρ , l, X_i)*
Need a relation to connect ρ to P,T,X_i – the Equation of State.

Some salient points:

Equations (1) and (2) are mechanical and are connected to the other equations through ρ. Thus, if we have an independent prescription for ρ, we can solve these equations without reference to others.

How do we model the Sun?

- We have two constraints at $t=4.57$ Gyr:

(1) The luminosity of the Sun

(2) The radius of the Sun

- We have two "free" parameters to play with

(1) The initial helium abundance, Y_0, i.e., the helium abundance of the Sun at $t=0$.

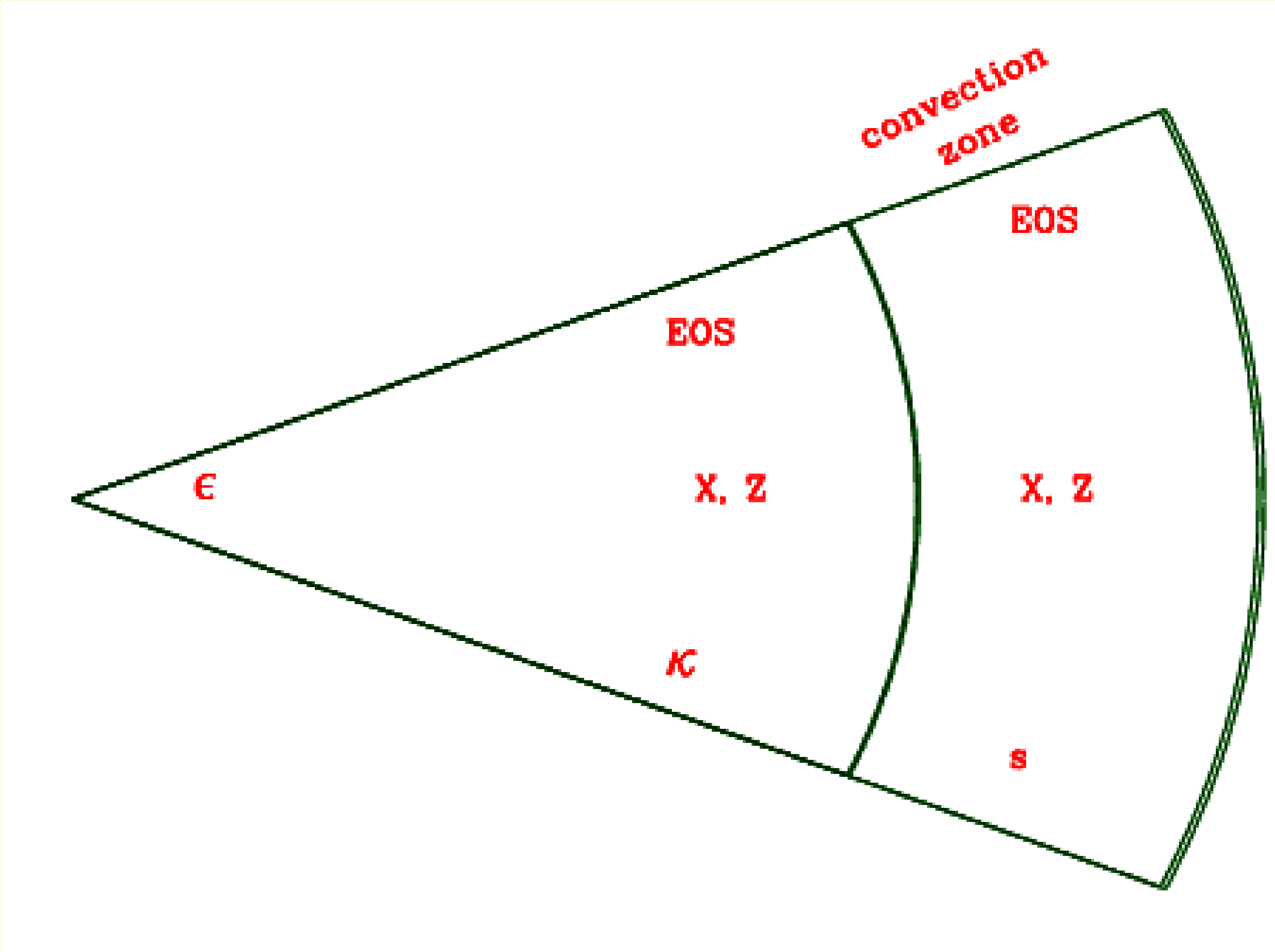
(2) The mixing length parameter α

- We can, therefore, iterate. Start with a given Y_0 and a, evolve till 4.57 Gyr. Test how close the luminosity and radius is to $1L_\odot$ and $1R_\odot$. Find corrections to Y_0 and α, evolve again, repeat till convergence is reached. Also need to change initial Z to get observed Z/X today.

Standard Solar Models

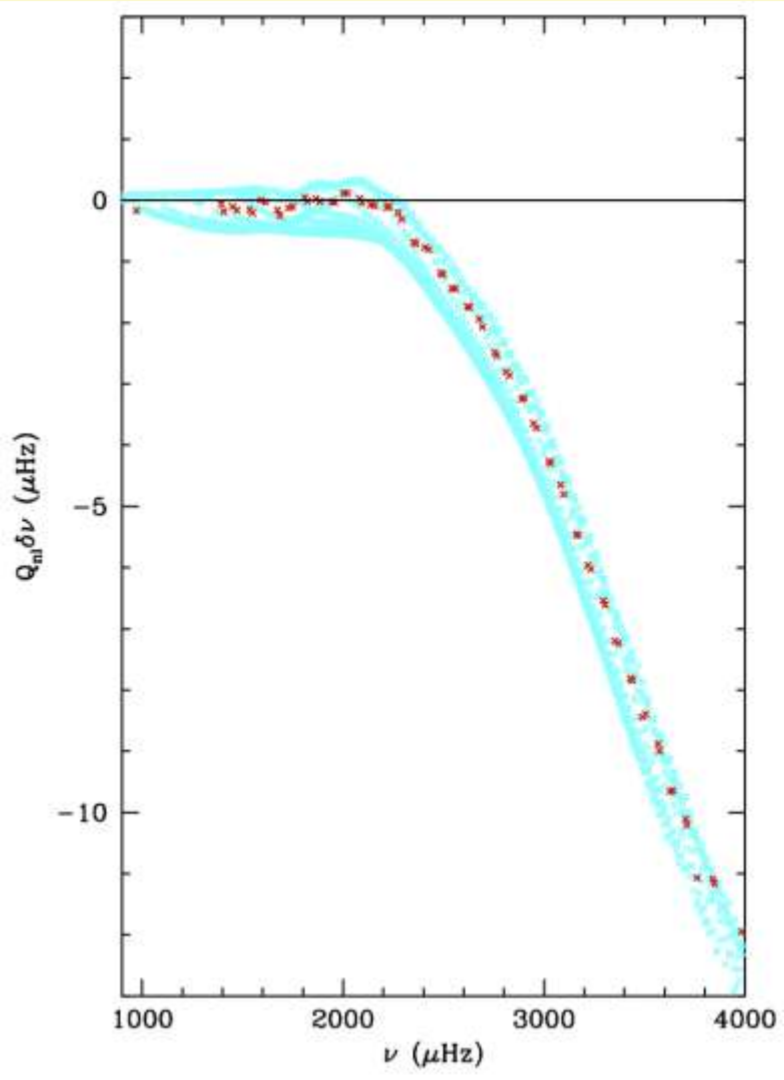
A standard solar model is one where the physical inputs are not changed to bring the model in better agreement with the Sun. The input physics (nuclear reaction rates, diffusion coefficients, opacity tables, equation of state) are input as they are. The agreement or otherwise between the Sun and the model is a indication of how good the input parameters.

The Schematic Sun: What input affects where?

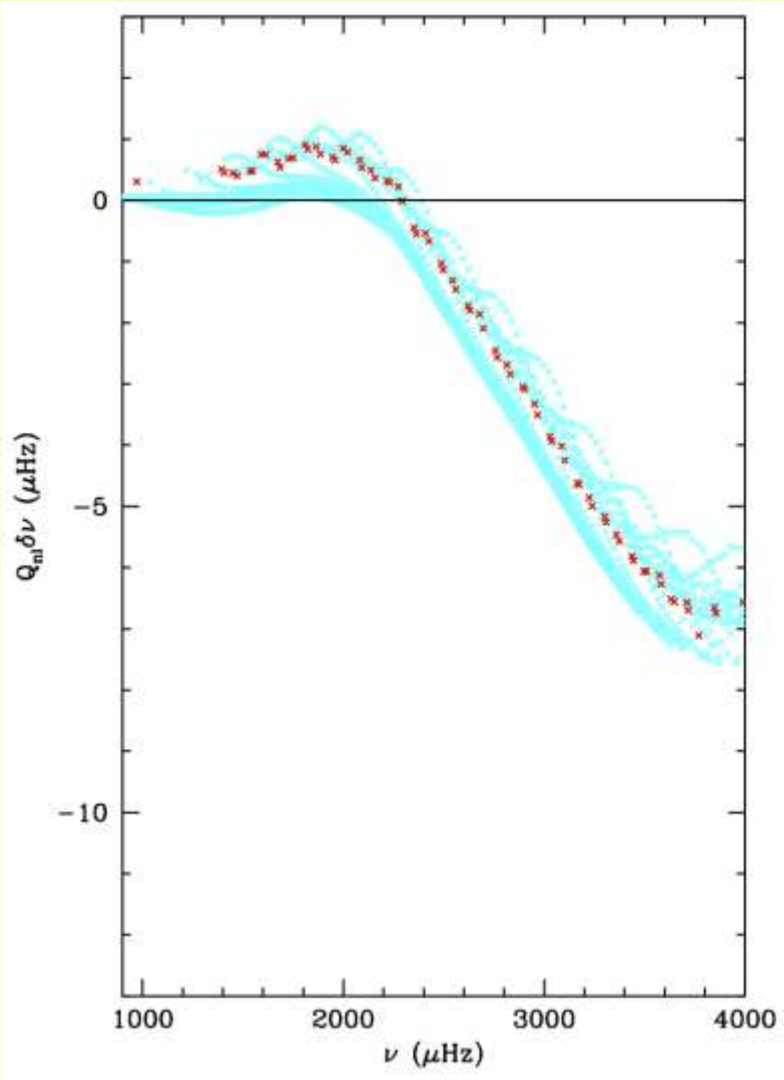


Given a model, we can calculate its frequencies

Comparing Frequencies:

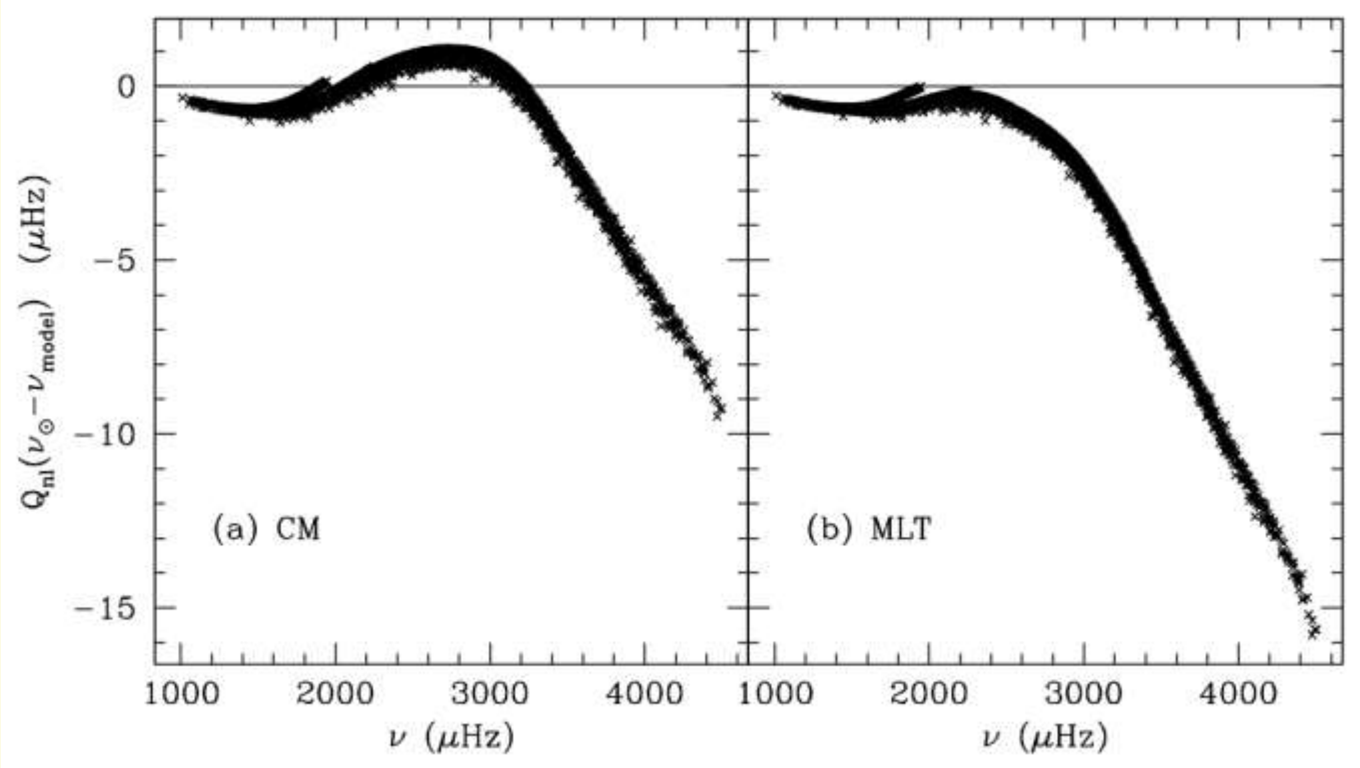


Model S



Model BSB05

Complication: the Surface Term

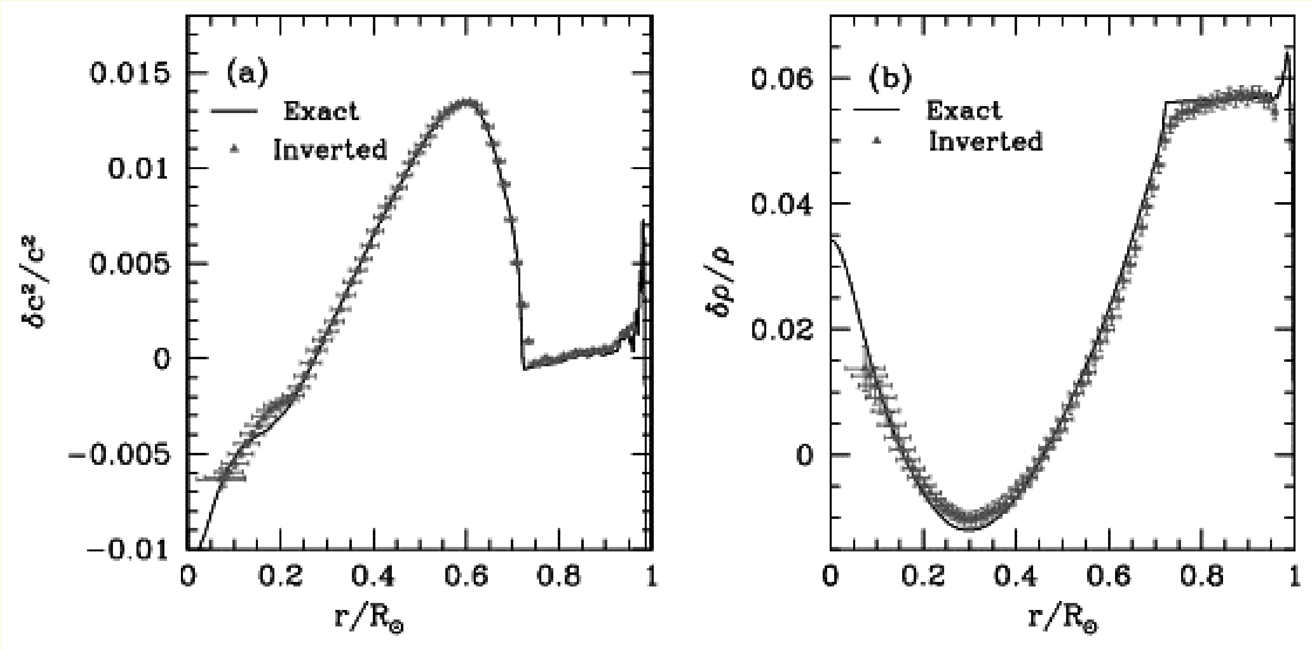


Relationship between structure and frequencies:

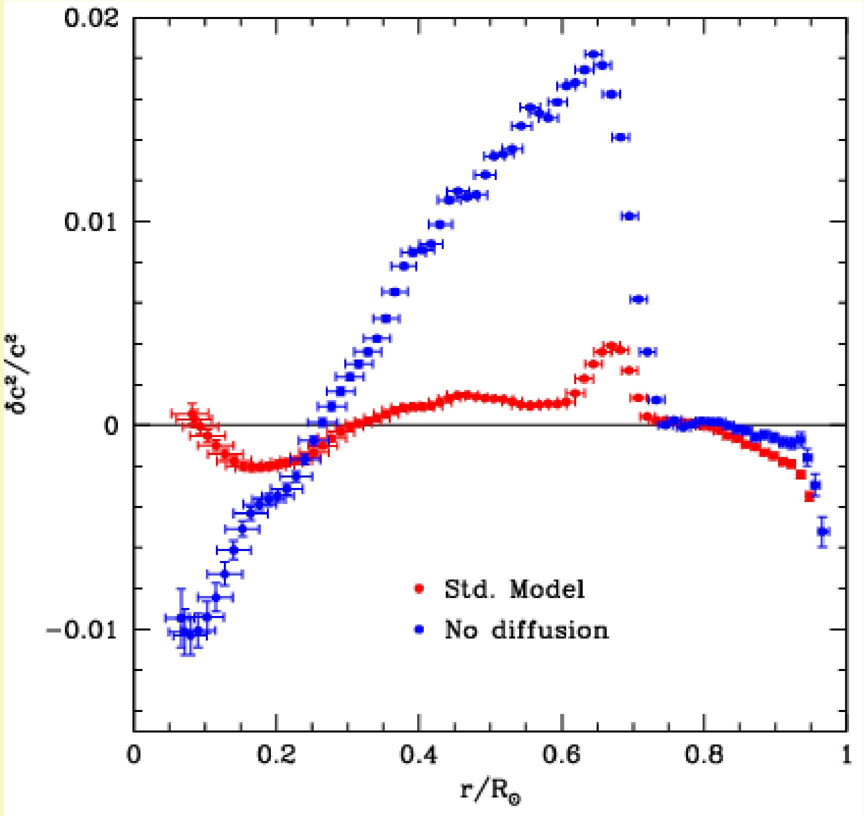
$$-\omega^2 \rho \vec{\xi} = \nabla \left(c^2 \rho \nabla \cdot \vec{\xi} + \nabla p \cdot \vec{\xi} \right) - \vec{g} \nabla \cdot (\rho \vec{\xi}) - G \rho \nabla \left(\int_v \frac{\nabla \cdot (\rho \vec{\xi}) d^3 \vec{r'}}{|\vec{r} - \vec{r'}|} \right)$$

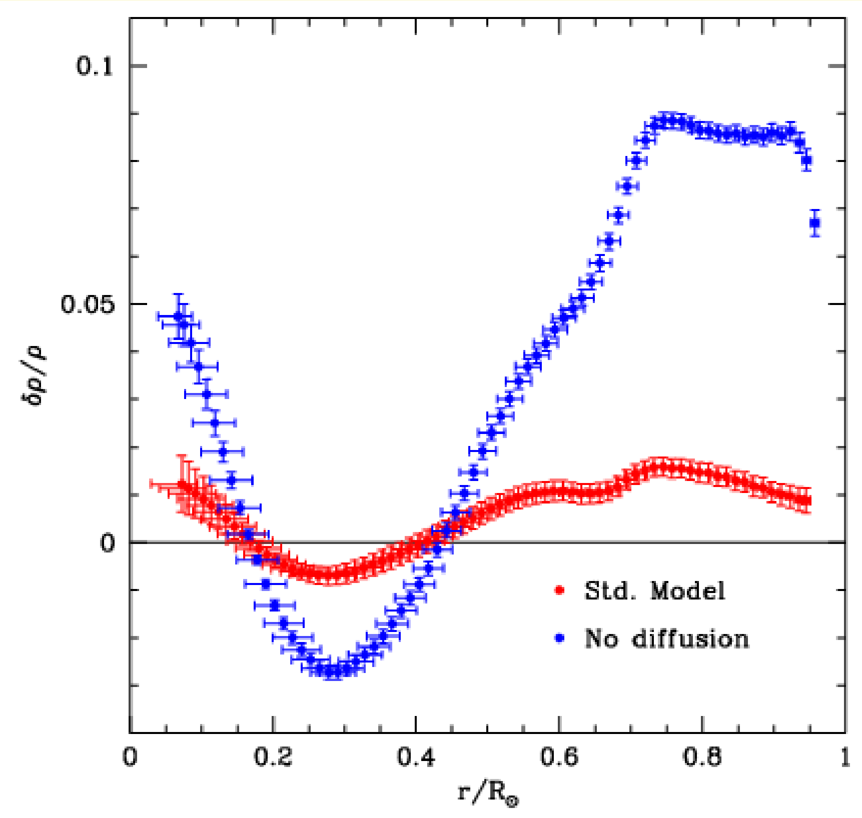
A Hermitian Eigenvalue problem, therefore use the variational principle:

$$\frac{\delta \omega_i}{\omega_i} = \int K^i_{c^2,\rho}(r) \frac{\delta c^2}{c^2} dr + \int K^i_{\rho,c^2}(r) \frac{\delta \rho}{\rho} dr + \frac{F_{\text{surf}}(\omega_i)}{I_i}$$



Solar Structure



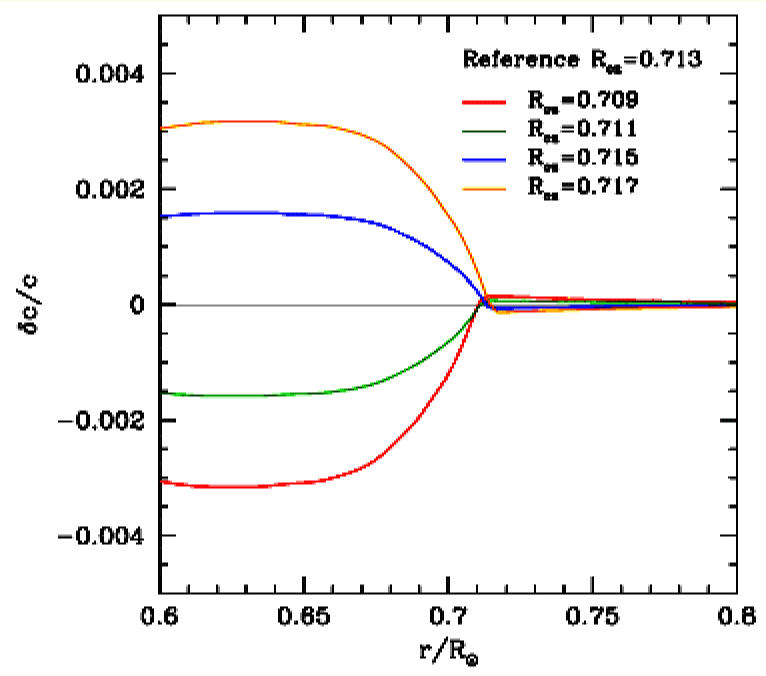


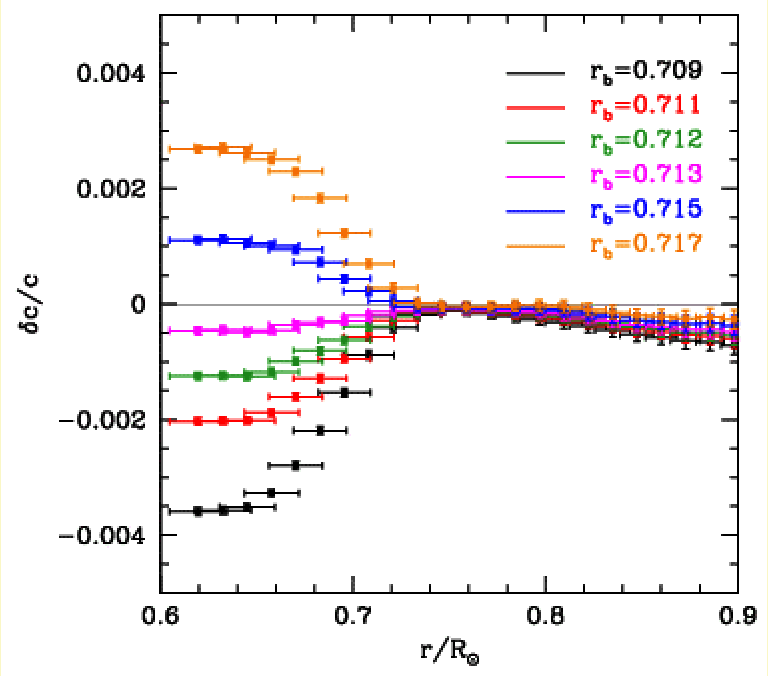
Models from Basu, Pinsonneault & Bahcall 2000

Diffusion is essential to get a good solar model

Models without diffusion have the convection zone boundary at the wrong place.

How do we know where the solar CZ base is?



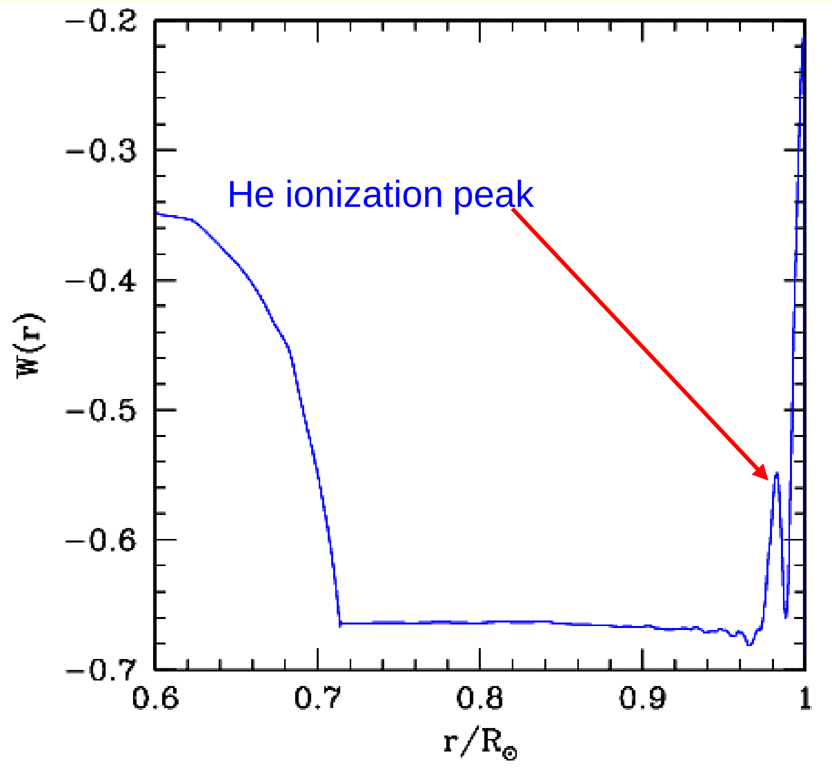


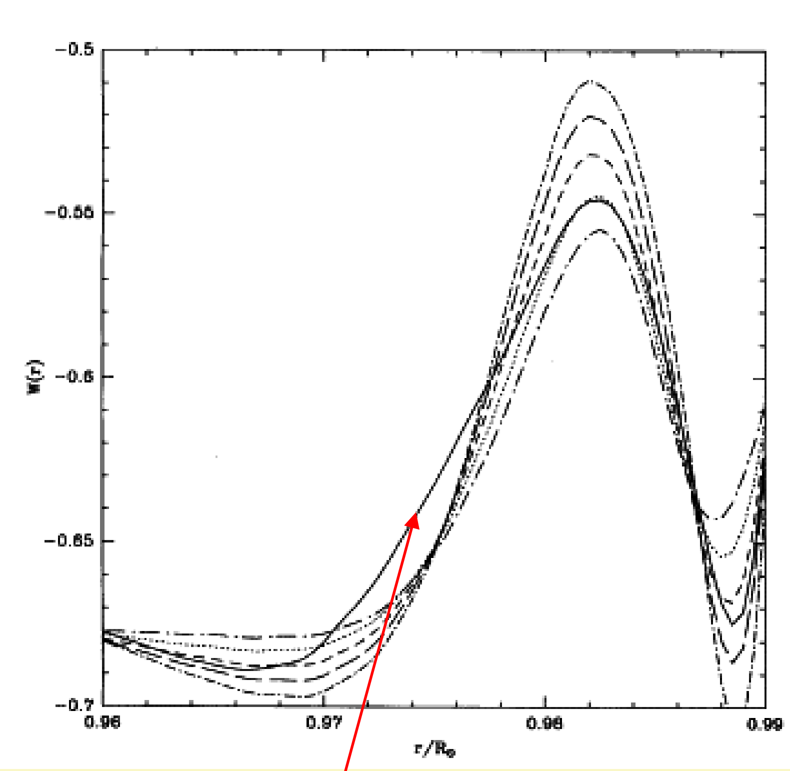
Solar CZ base is at 0.7134 ± 0.001 Rsun

Models without diffusion have the wrong helium abundance

The helium abundance (Y) of the solar envelope is 0.249 ± 0.003

$$W(r) = \frac{r^2}{Gm}\frac{dc^2}{dr} = \frac{1-\gamma_r-\gamma}{1-\gamma_{c^2}}$$

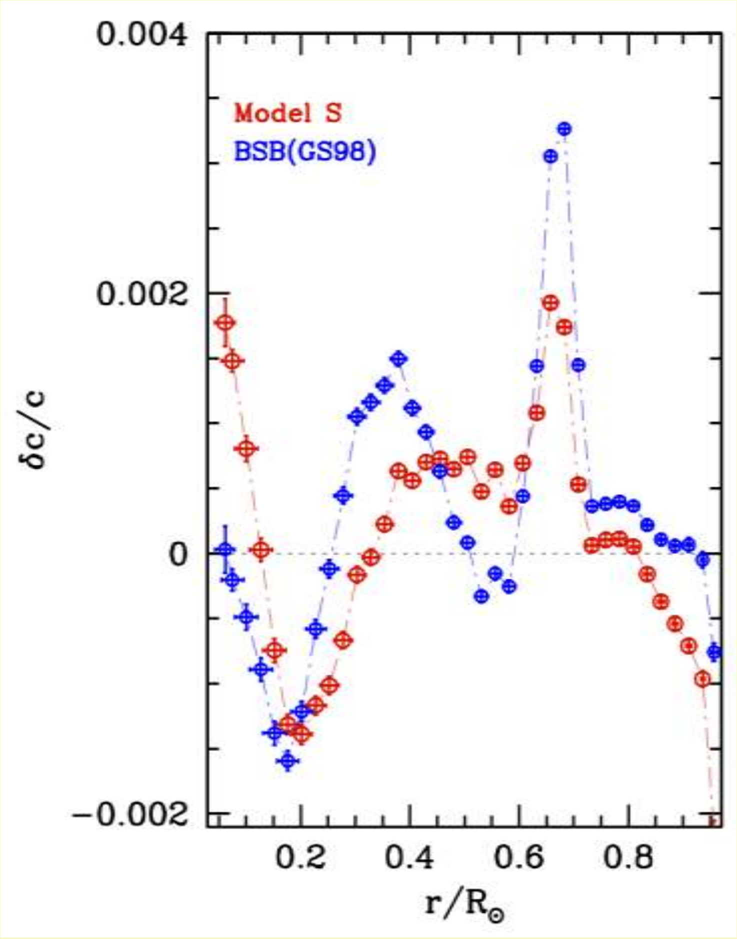


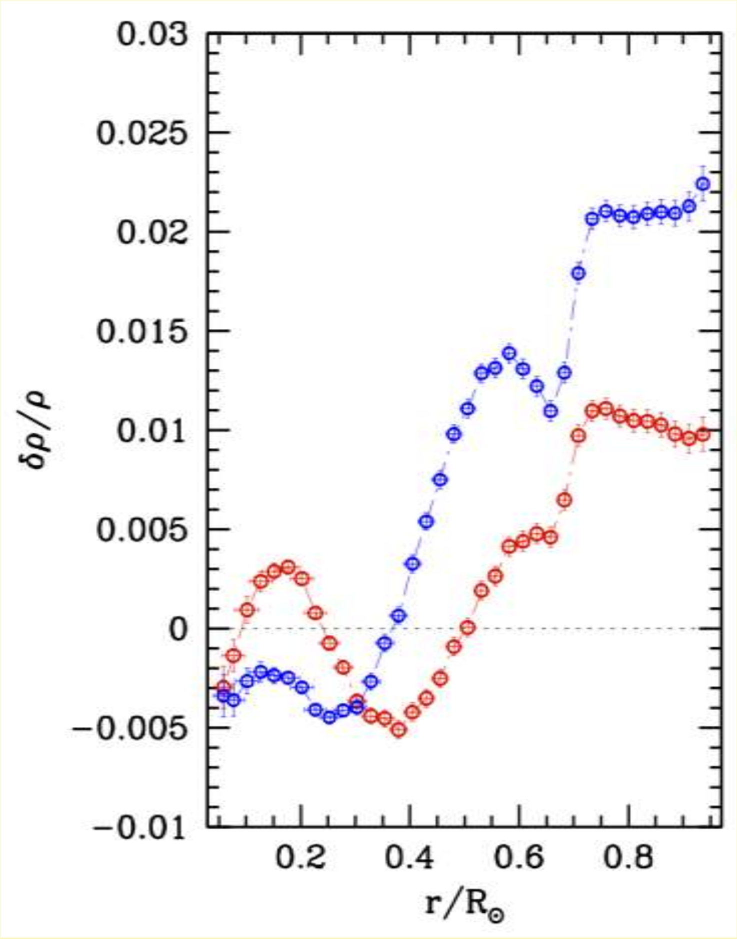


From Basu & Antia, 1995

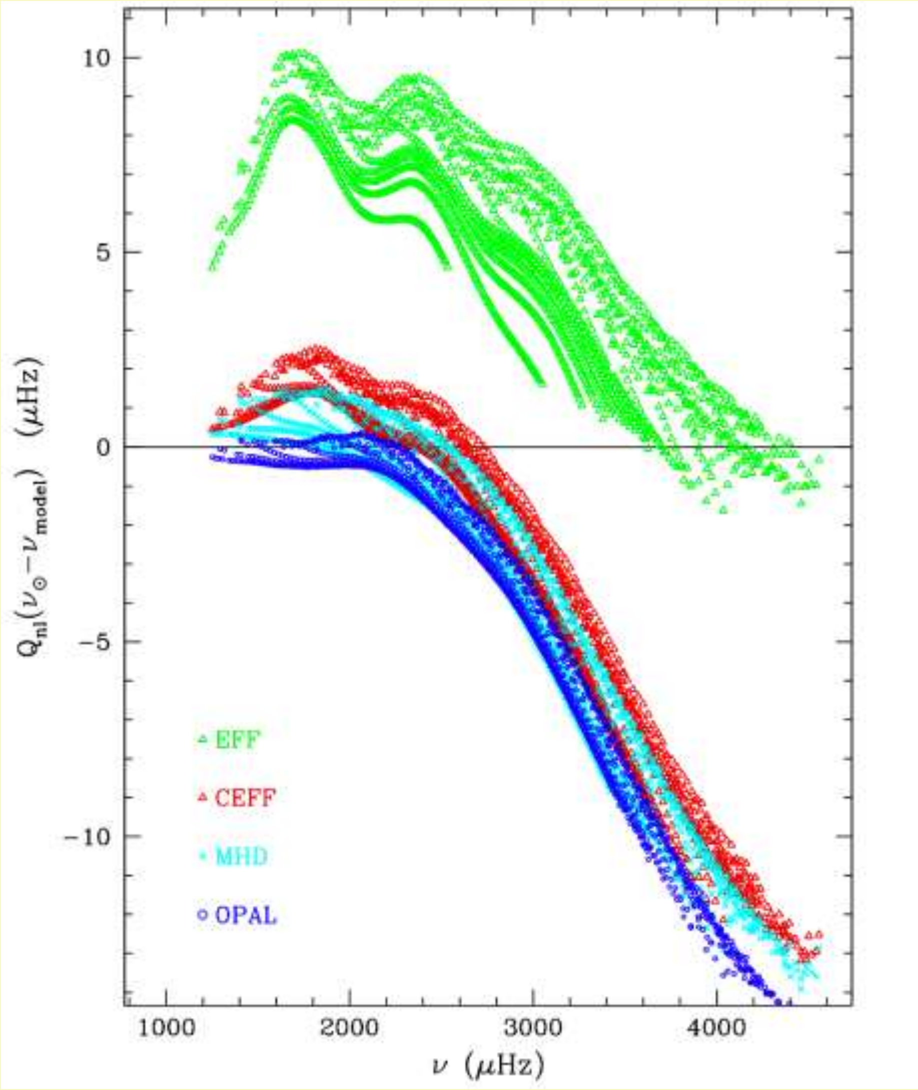
Observations

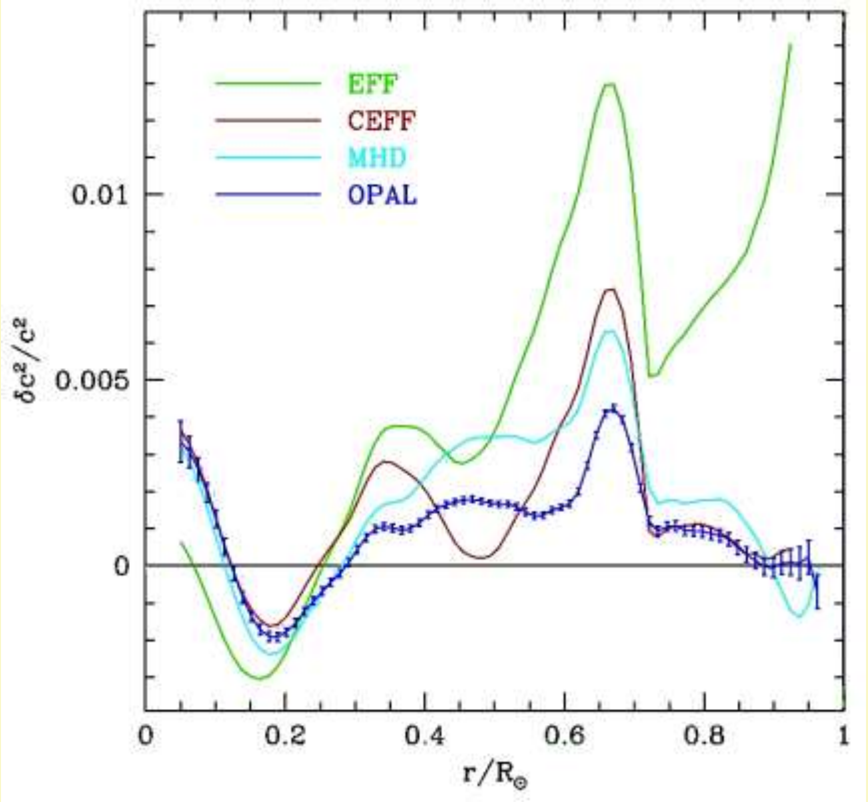
What about the two models we saw earlier?



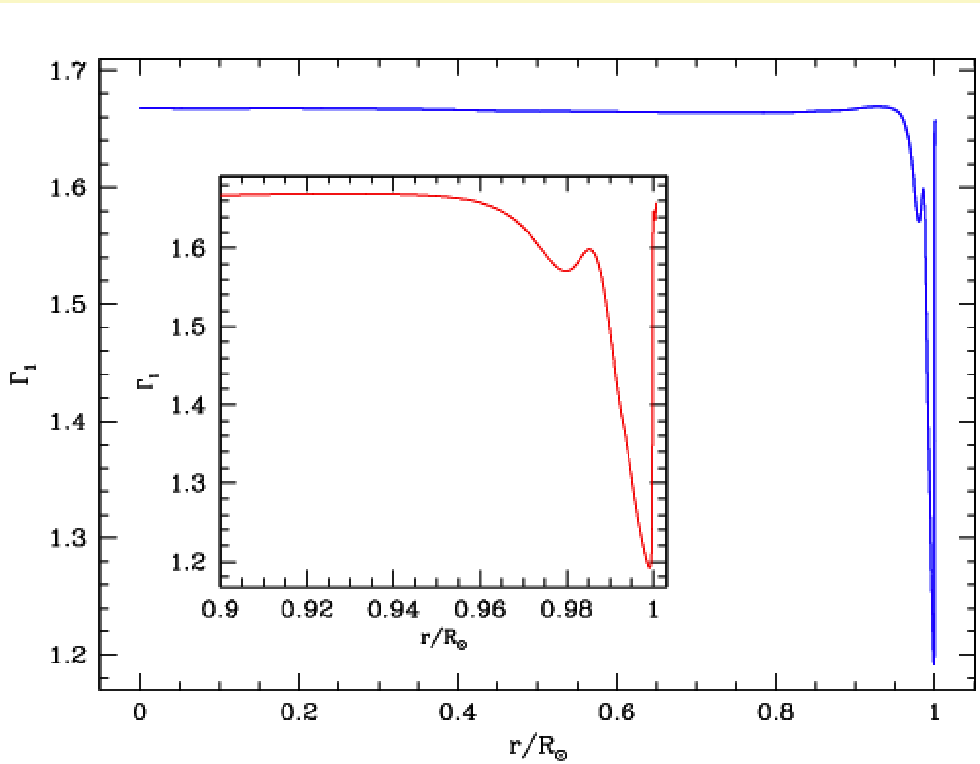


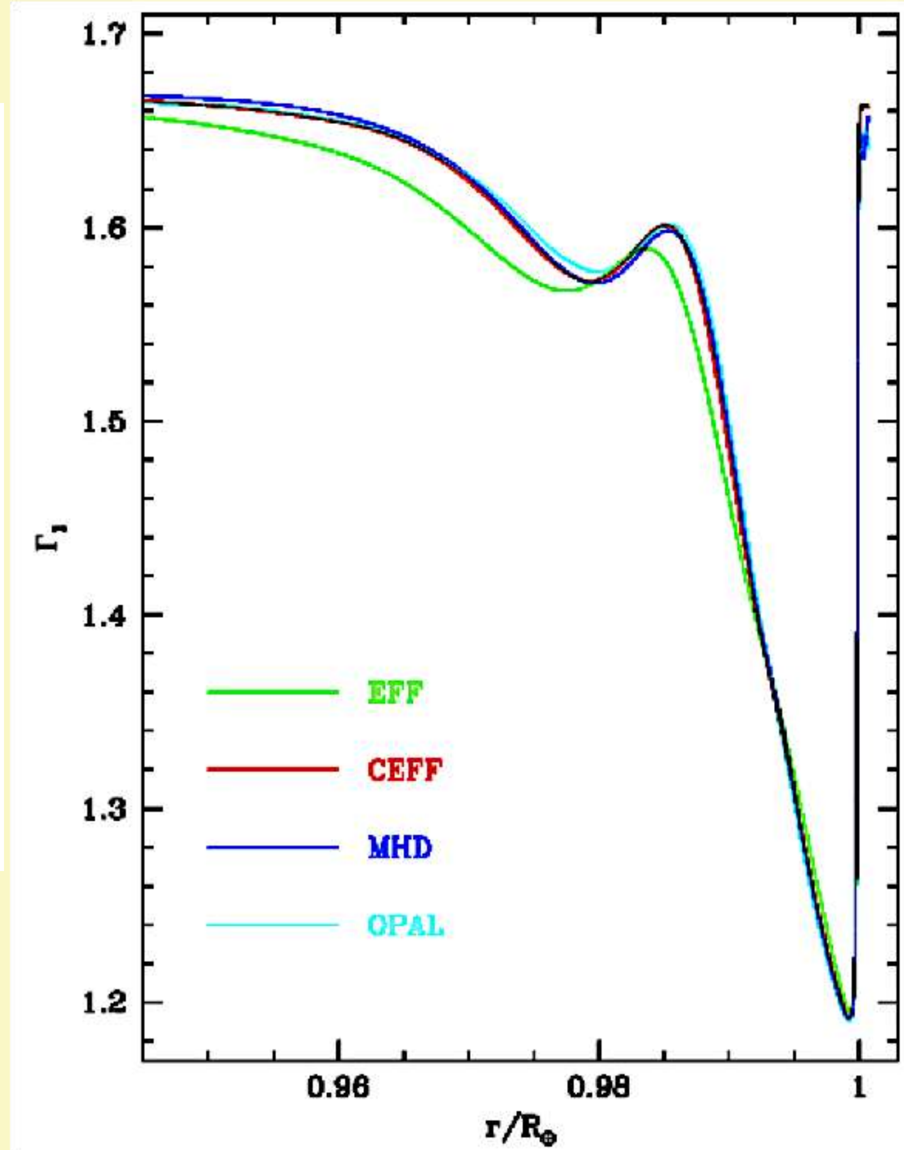
Using the Sun as a Laboratory: The stellar EOS



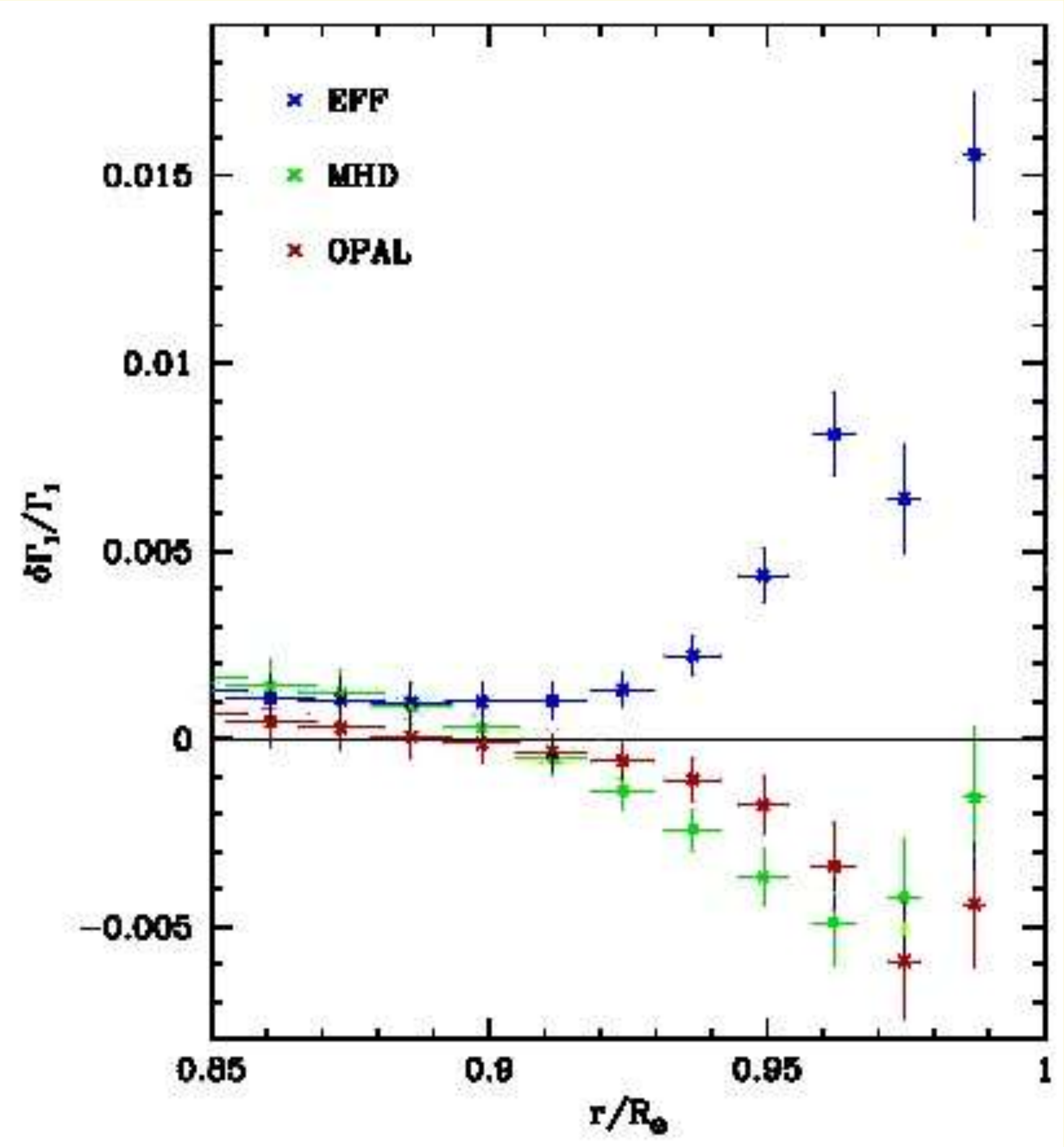


MORE DIRECT WAYS OF TESTING EOS:



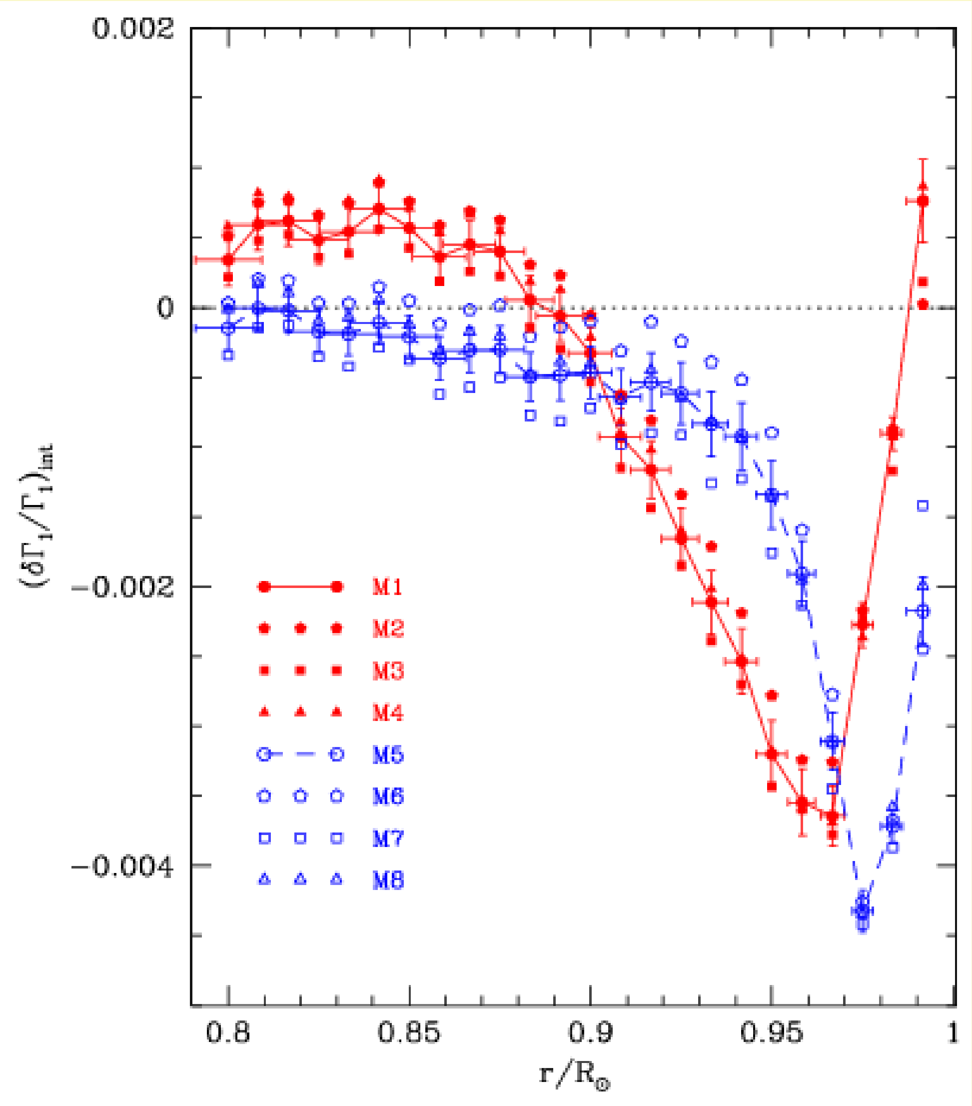


$$G_1 = \left(\frac{\partial \ln P}{\partial \ln r}\right)_s$$



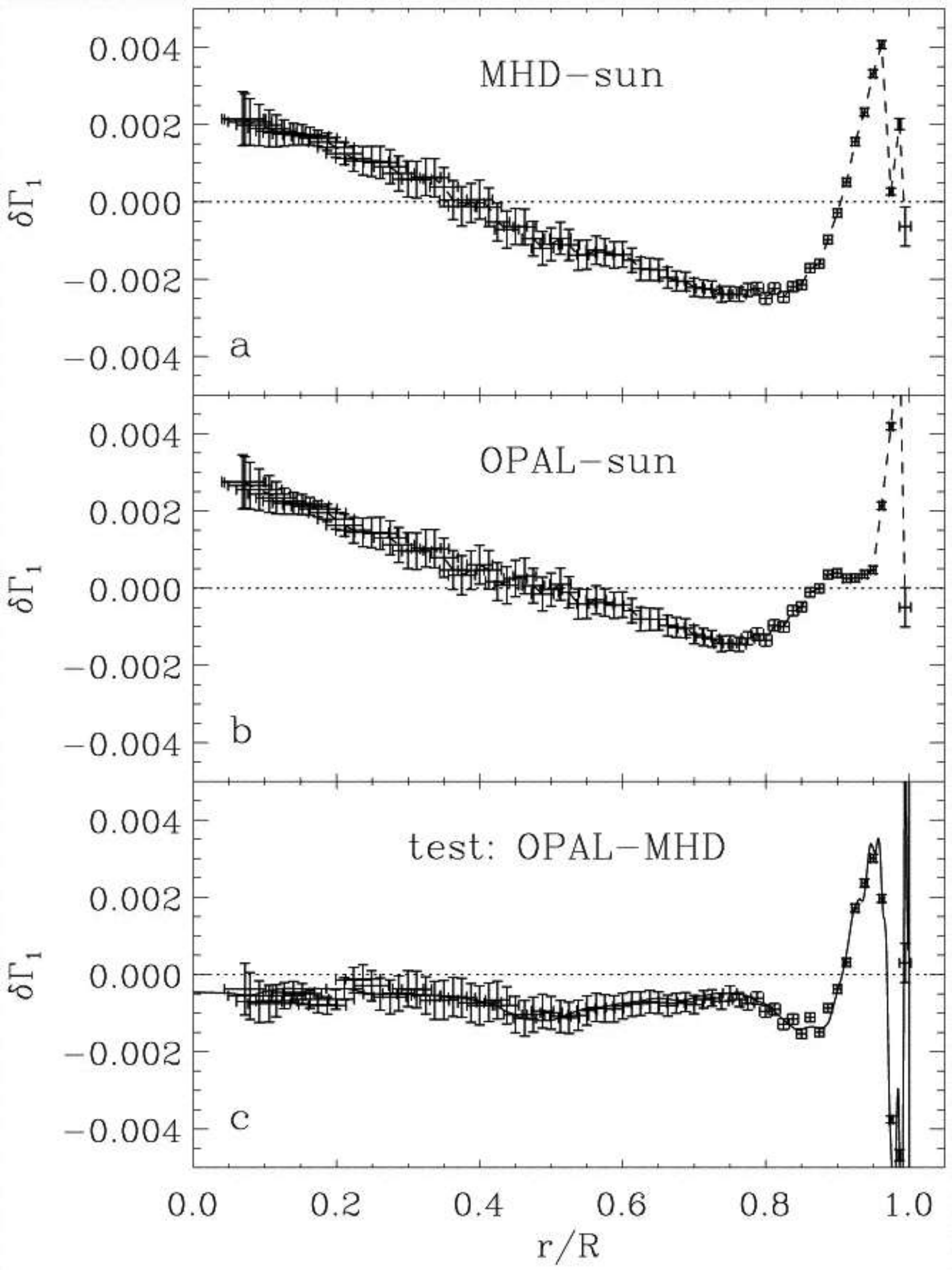
$$\frac{d\Gamma_1}{\Gamma_1} = \left(\frac{\partial \ln \Gamma_1}{\partial \ln p}\right)_{r,Y} \frac{dp}{p} + \left(\frac{\partial \ln \Gamma_1}{\partial \ln r}\right)_{p,Y} \frac{dr}{r} + \left(\frac{\partial \ln \Gamma_1}{\partial Y}\right)_{r,p} dY + \left(\frac{d\Gamma_1}{\Gamma_1}\right)_{\mathrm{int}}$$

OPAL v/s MHD



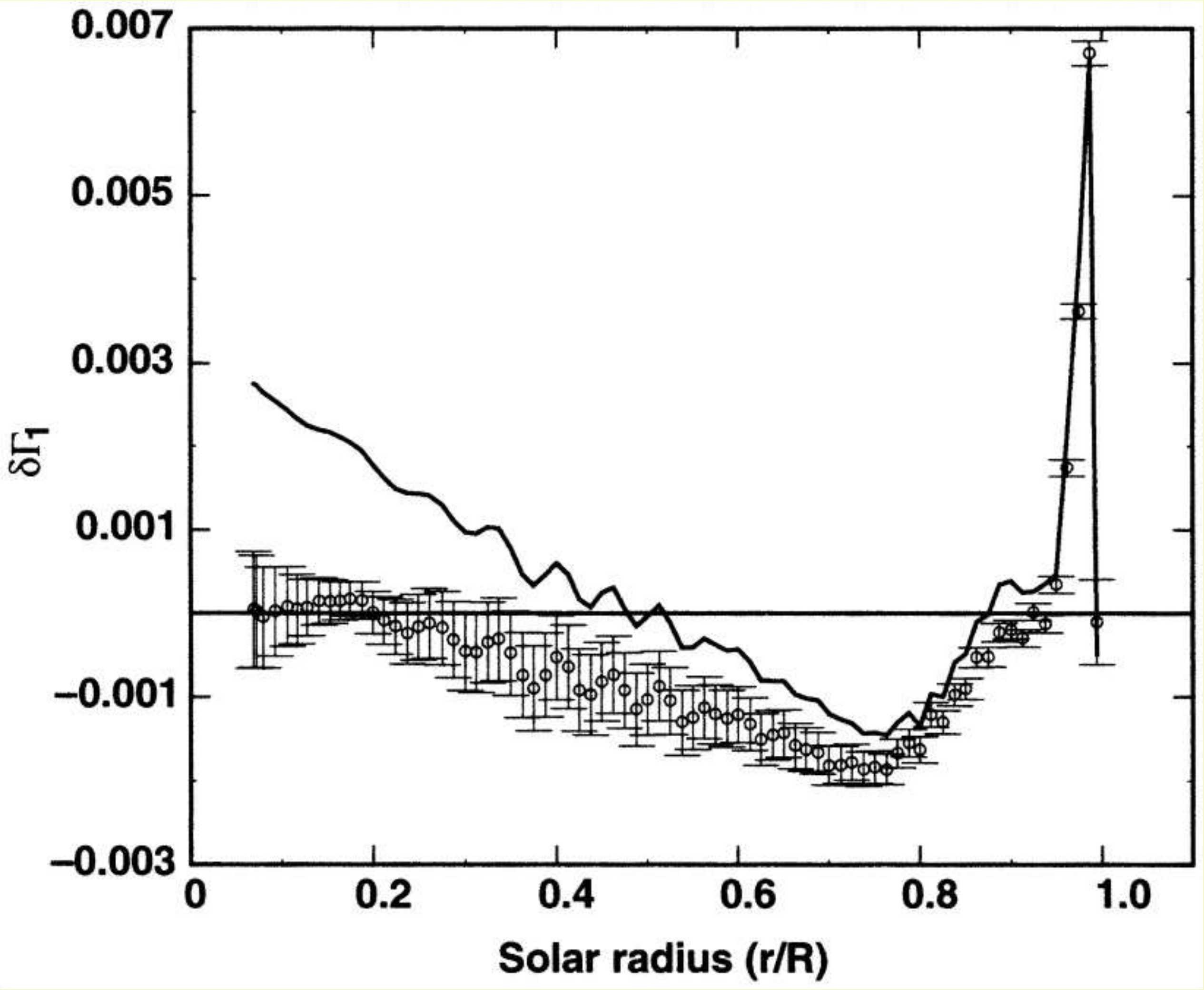
Differences in how ionized hydrogen is treated.

Conditions in the solar core



From Elloitt & Kosovichev 1998

Corrected EOS



From Nayfonov & Rogers 2002

Even more fundamental physics: The Solar Neutrino Problem

Cosmic Gall

Neutrinos, they are very small.
They have no charge and have no mass
And do not interact at all.
The earth is just a silly ball
To them, through which they simply pass,
Like dustmaids down a drafty hall
Or photons through a sheet of glass.

They snub the most exquisite gas,
Ignore the most substantial wall,
Cold-shoulder steel and sounding brass,
Insult the stallion in his stall,
And scorning barriers of class,
Infiltrate you and me! Like tall
And painless guillotines, they fall
Down through our heads into the grass.

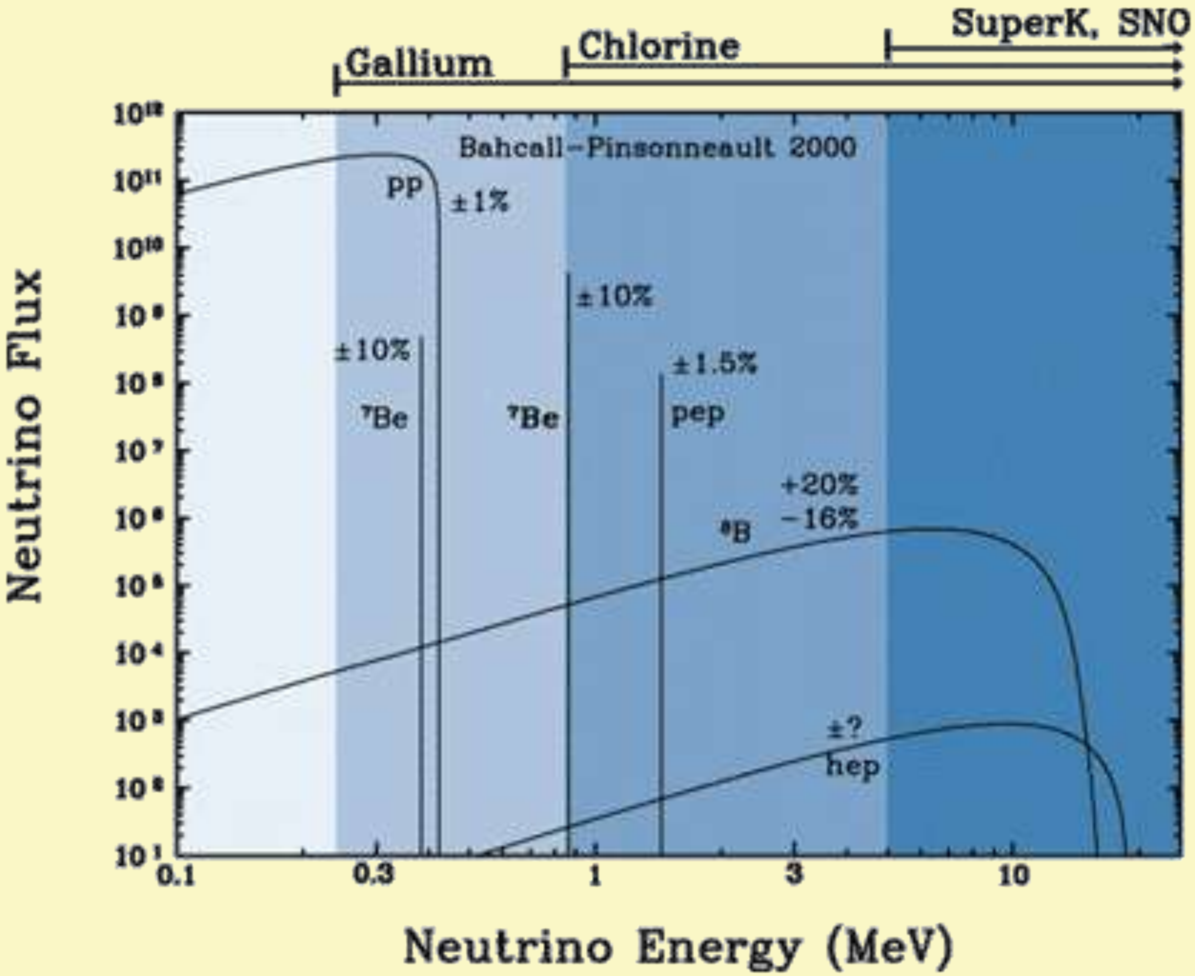
At night, they enter at Nepal
And pierce the lover and his lass
From underneath the bed — you call
It wonderful; I call it crass.

John Updike, 1960

REACTION	TERM (%)	ν ENERGY (MeV)
$p + p \rightarrow {}^2H + e^+ + \nu_e$	(99.96)	≤ 0.423
or		
$p + e^- + p \rightarrow {}^2H + \nu_e$	(0.44)	1.445
${}^2H + p \rightarrow {}^3He + \gamma$	(100)	
${}^3He + {}^3He \rightarrow \alpha + 2p$	(85)	
or		
${}^3He + {}^4He \rightarrow {}^7Be + \gamma$	(15)	
${}^7Be + e^- \rightarrow {}^7Li + \nu_e$	(15)	0.863 90%; 0.385 10%
${}^7Li + p \rightarrow 2\alpha$		
or		
${}^7Be + p \rightarrow {}^8B + \gamma$	(0.02)	
${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$		< 15
${}^8Be^* \rightarrow 2\alpha$		
or		
${}^3He + p \rightarrow {}^4He + e^+ + \nu_e$	(0.00003)	< 18.8

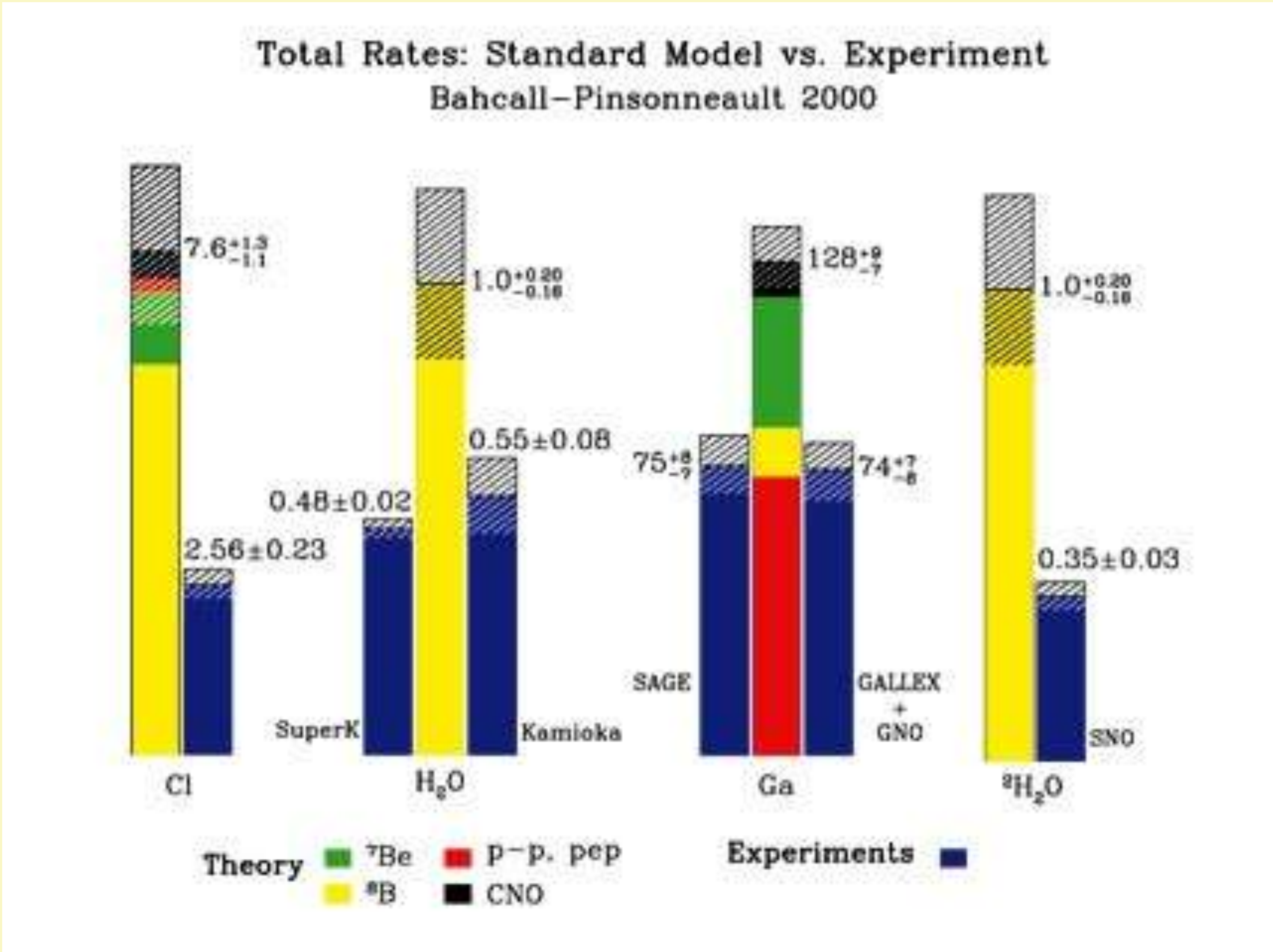
Neutrino terminations from BP2000 solar model. Neutrino energies include solar corrections: J. Bahcall, Phys. Rev. C, 56, 3391 (1997).

THE SOLAR NEUTRINO SPECTRUM

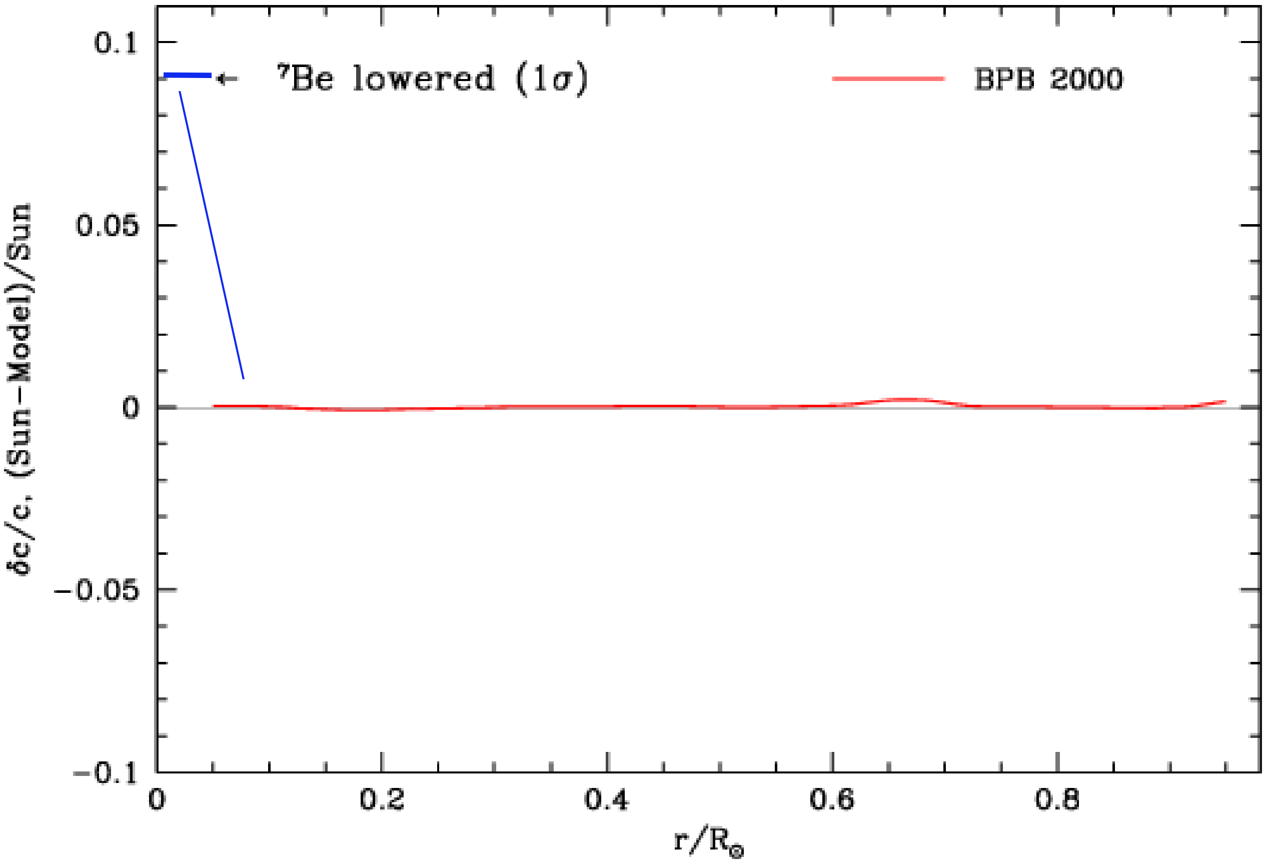


Viewgraph courtesy J. N. Bahcall

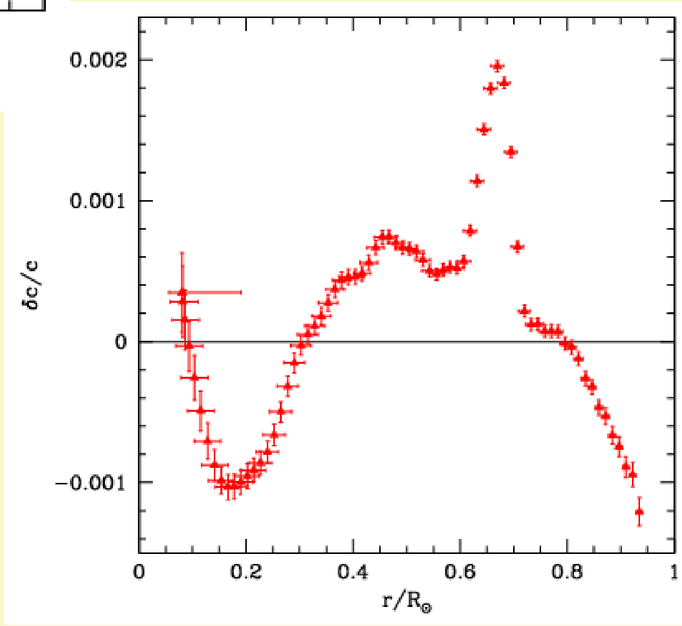
The Solar Neutrino Problem



Viewgraph courtesy J. N. Bahcall

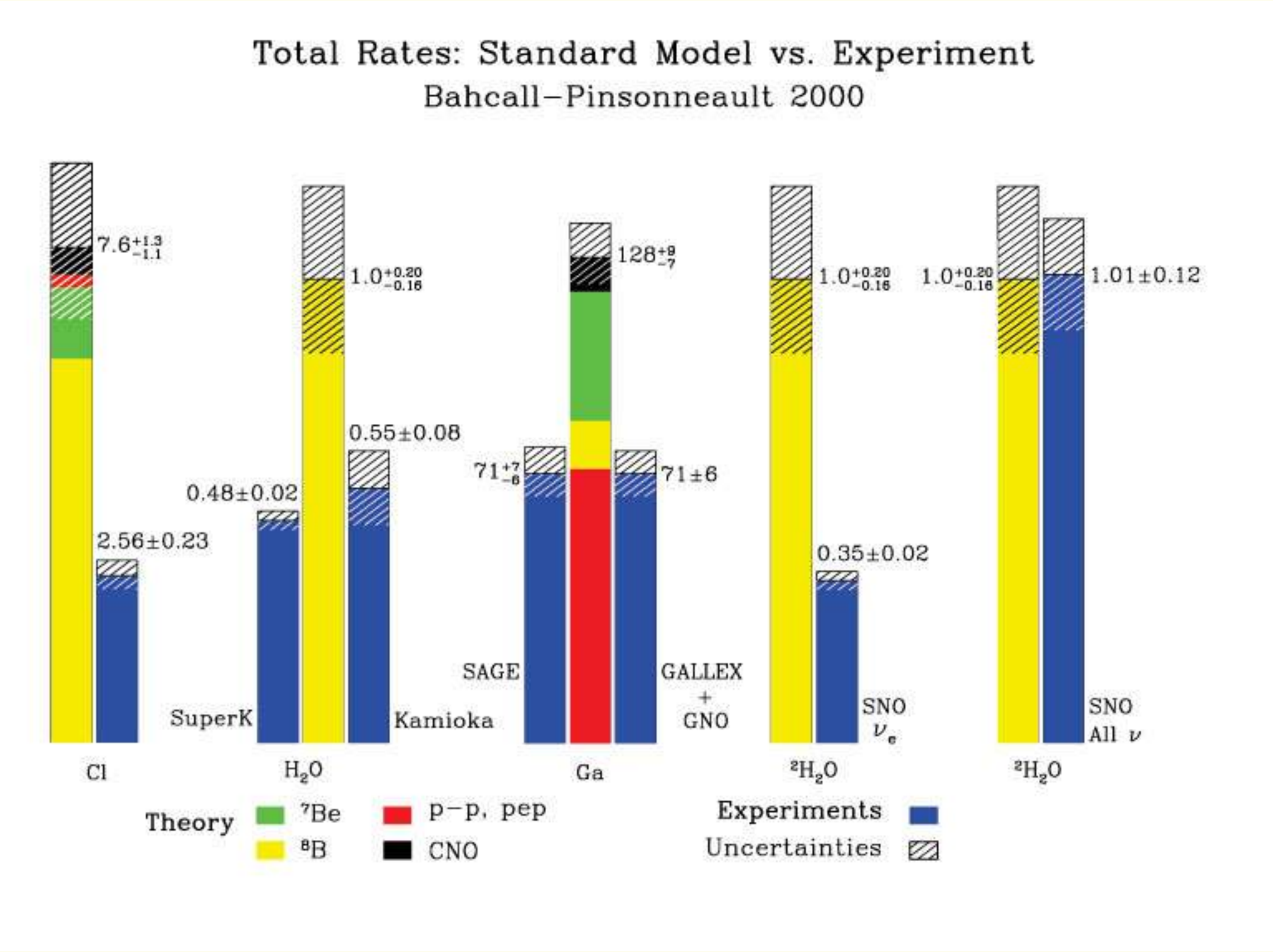


^{7}Be lowered (1σ)
BPB 2000
δc/c, (Sun−Model)/Sun
0.1
0.05
0
−0.05
−0.1
0
0.2
0.4
0.6
0.8
r/R$_\odot$



δc/c
0.002
0.001
0
−0.001
0
0.2
0.4
0.6
0.8
1
r/R$_\odot$

The Solar Neutrino Picture Now



Our current obsession? OPACITIES!

A problem with the solar heavy-element abundances.

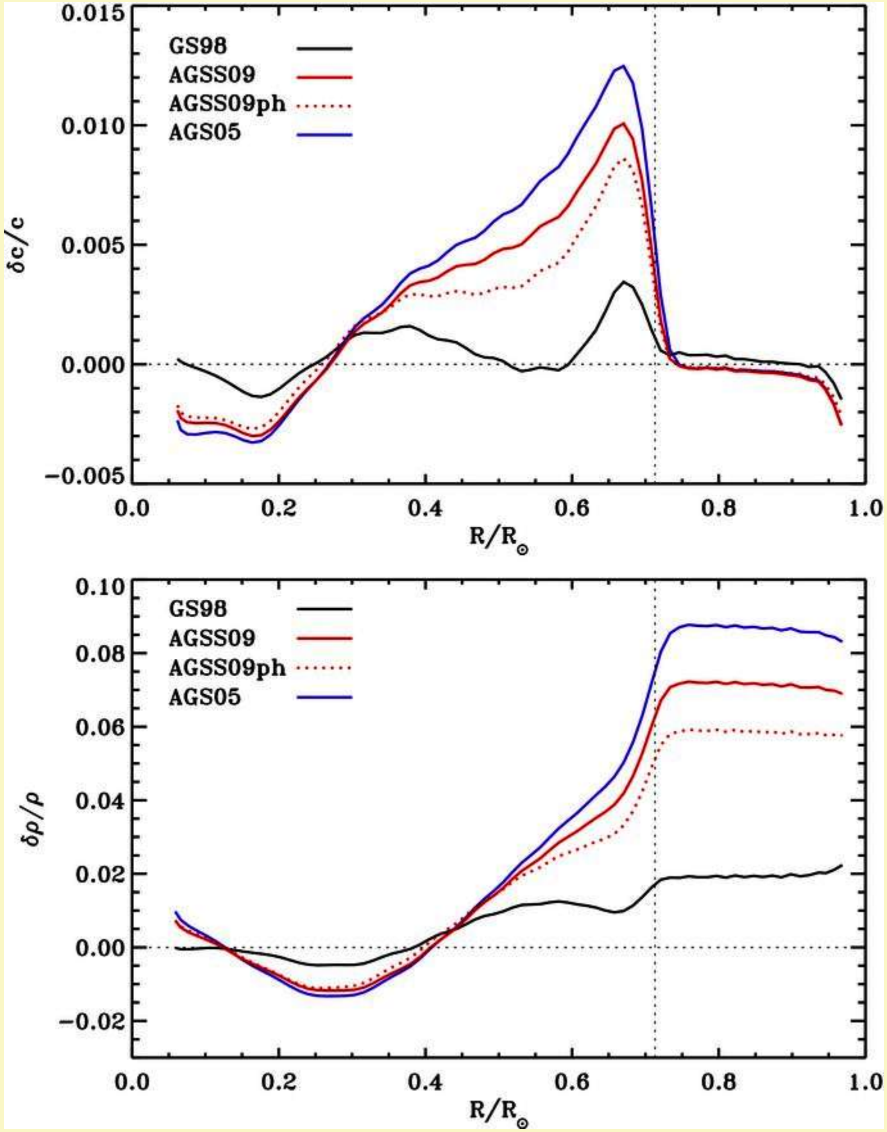
Grevesse & Sauval (1998)
Z/X=0.023

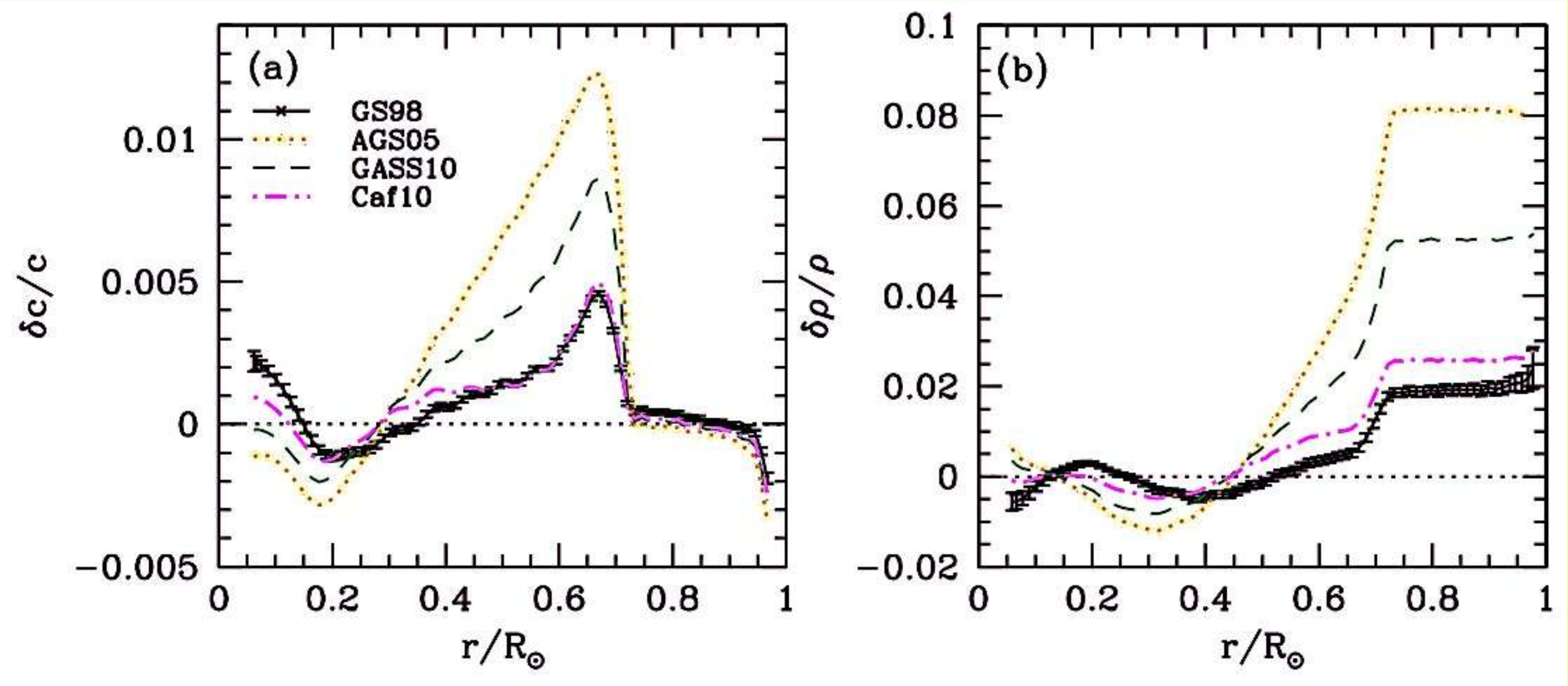
Asplund et al (2004,2005)
Z/X=0.0165
O, C, N all reduced.

Asplund et al. (2009) (met)
Z/X=0.0178

Asplund et al. (2009), Grevesse et al. (2010) (ph)
Z/X=0.0181

Caffau et al. (2011):
Z/X=0.0209





(a)
GS98
AGS05
GASS10
Caf10
δc/c
0.01
0.005
0
−0.005
0
0.2
0.4
0.6
0.8
1
r/R⊙
(b)
δρ/ρ
0.1
0.08
0.06
0.04
0.02
0
−0.02
0
0.2
0.4
0.6
0.8
1
r/R⊙

Solar rotation

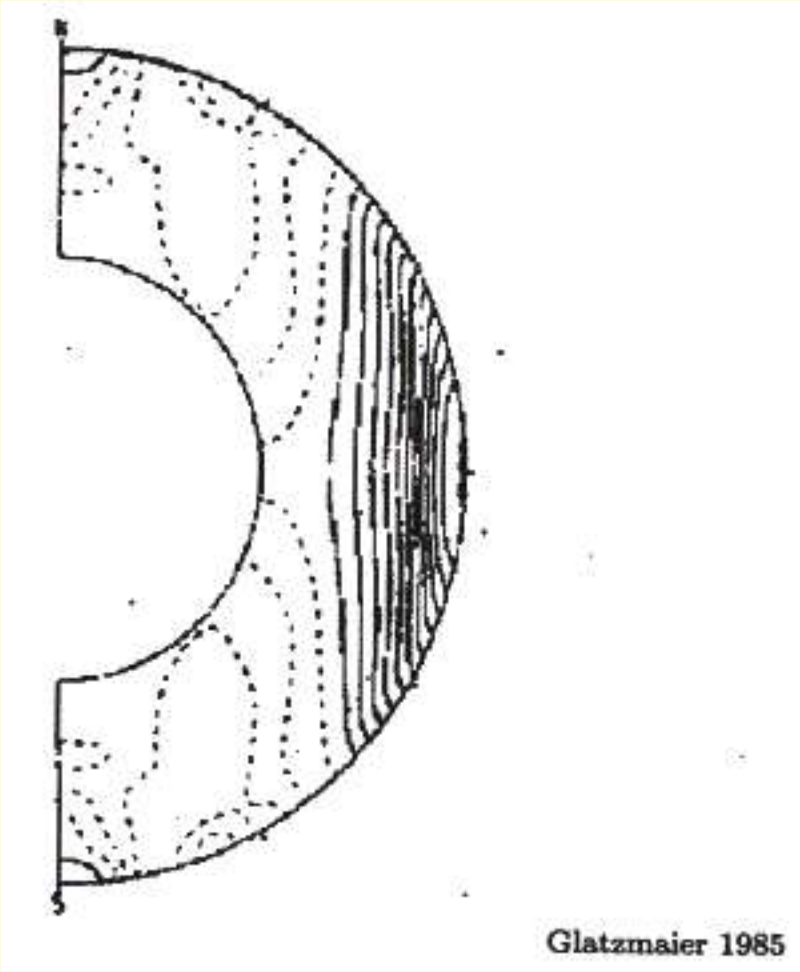
- If the Sun were spherically symmetric and did not rotate, all modes with the same ***l*** and ***n*** but different ***m*** would have the same frequency.

- Rotation lifts this degeneracy, giving rise to "rotational splittings" of the modes:

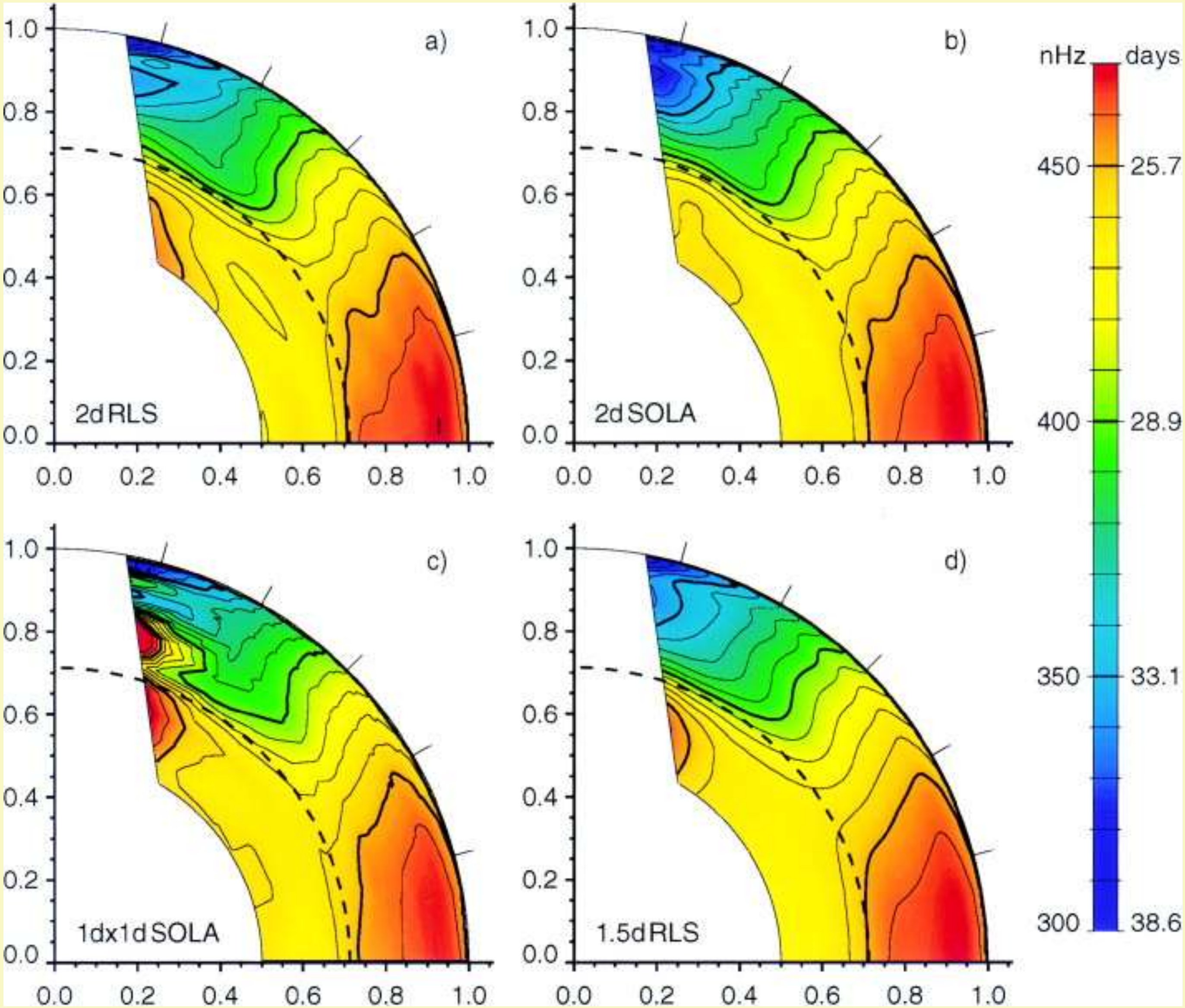
$$D_{nlm} = \frac{\nu_{nlm} - \nu_{nl-m}}{2m} = \int_0^1 \int_0^1 dr\ d\cos\theta\ K_{nlm}(r,\theta)\Omega(r,\theta)$$

$$\nu_{nlm} = \nu_{nl} + \sum_{j=1}^{j_{max}} a_j(n,l)\mathcal{P}_j^{(l)}(m).$$

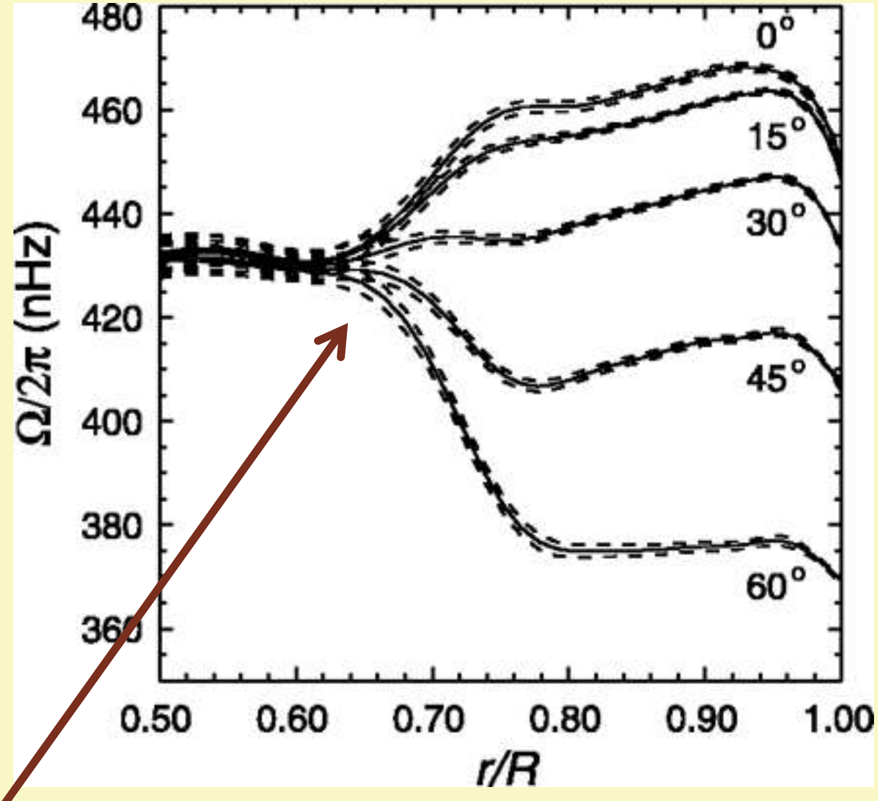
Expected rotation profile



Solar Rotation from helioesismology



The Tachocline



The tachocline

Mathematical modeling of solar magneto-dynamics

Authors:

Spiegel, Edward A.;
Zahn, Jean-Paul

Final Report, 1 Nov. 1988 - 31 Oct. 1991 Columbia Univ., New York, NY. Astrophysics Lab.

…We have isolated the probable seat of the solar cycle in the shear layer recently detected by helioseismology just below the convection zone. We call this layer the solar **tachyline** because of certain analogies to the oceanic thermocline…

Astron. Astrophys. 265, 106–114 (1992)

The solar tachocline

E. A. Spiegel [1] **and J.-P. Zahn** [1,2]

[1] Astronomy Department, Columbia University, New York, NY 10027, USA
[2] Observatoire Midi-Pyrénées, 14 avenue E. Belin, F-31400 Toulouse, France

Received June 5, accepted July 20, 1992

Abstract. Acoustic sounding of the Sun reveals that the variation of angular velocity with latitude is independent of depth in the convection zone. By contrast, deep within the radiative zone, the rotation appears to be rigid. The transition between the two rotation laws occurs in a thin, unresolved layer that we here call the *tachocline*. This paper is an examination of the structure and previous evolution of this layer. We assume that the stress exerted by the convection zone is prescribed, much as oceanographers take the wind stress on the sea surface as given. We conclude that the helioseismic observations are best rationalized by a scenario in which, after an initial adjustment or spindown period, the subconvective rotation settles into a quasisteady state with a turbulent boundary layer. In the tachocline, the advection of angular momentum is controlled by horizontal turbulence. If this turbulence is intense enough, the tachocline is thin and is unresolved.

Key words: turbulence – Sun: interior, rotation

Characterizing the tachocline

Basu (1997)

$$W(r) = W_c + \frac{dW_d}{1+\exp\left[\left(r_d - r\right)/w\right]};$$

W_c = value of W deep in the interior

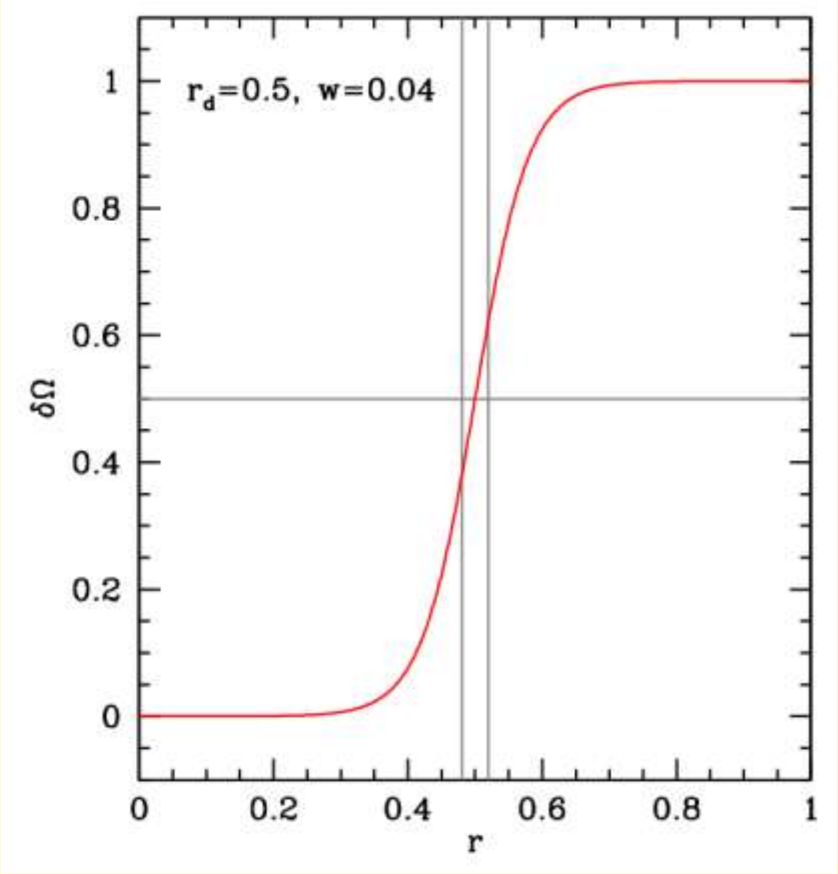
w = half width of the transition layer

r_d = mid-point of transition region.

Thus, the rotation rate increases from a factor $1/(1+e)$ of its max. to $1/(1+(1/e))$ its max. in the range $r_d - w$ to $r_d + w$.

(I.e. w is the characteristic thickness between 0.377 to 0.622 of the transition).

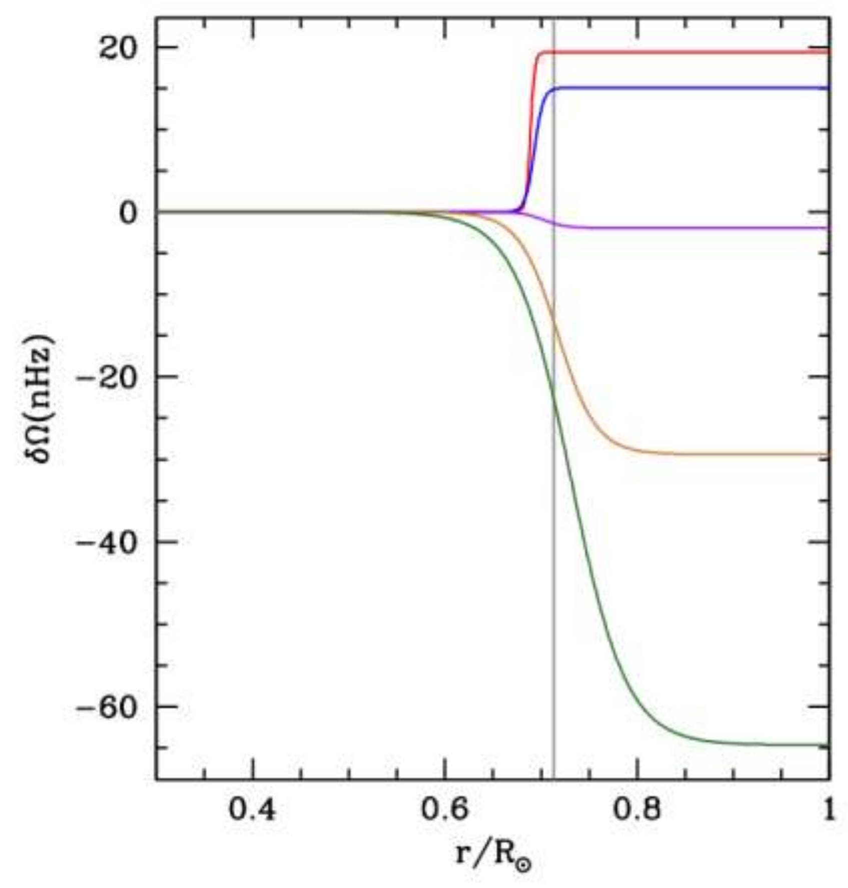
$dW = W_{sur} - W_c$



Averages

	Equator		15^o		30^o		45^o		60^o	
	MDI	GONG	MDI	GONG	MDI	GONG	MDI	GONG	MDI	GONG
Jump	19.38	18.82	15.07	14.41	-1.93	-1.06	-29.39	-30.32	-64.64	-68.92
Posi-tion	0.689	0.692	0.693	0.694	0.703	0.697	0.717	0.702	0.731	0.707
Width	0.0022 (0.006)	0.0024 (0.006)	0.0046 (0.011)	0.0040 (0.010)	0.0111 (0.028)	0.0084 (0.021)	0.0201 (0.072)	0.0144 (0.051)	0.0289 (0.089)	0.0204 (0.062)

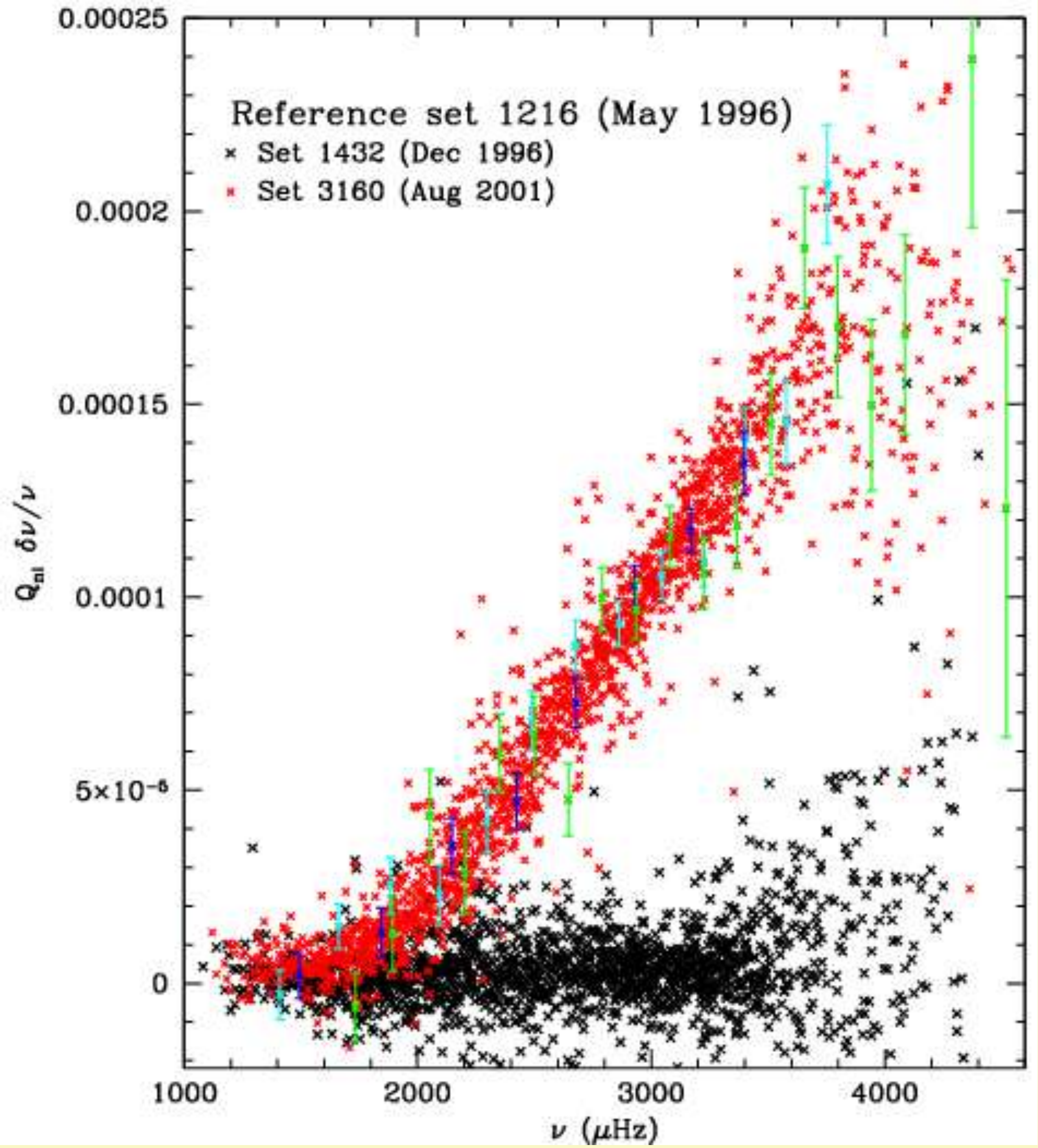
- The tachocline is prolate.
- The thickness of the tachocline is higher at higher latitudes.



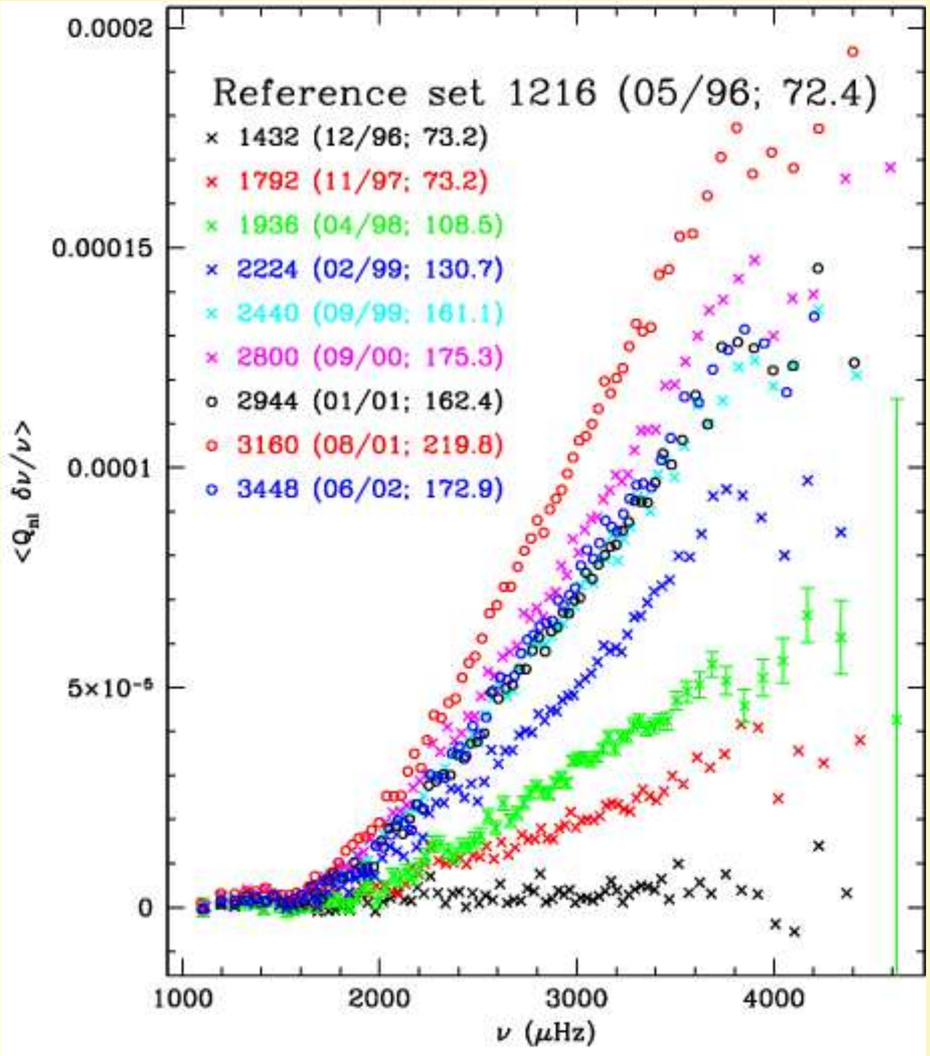
20
0
−20
−40
−60
δΩ(nHz)
0.4
0.6
0.8
1
r/R$_\odot$

The Solar Cycle and Helioseismology

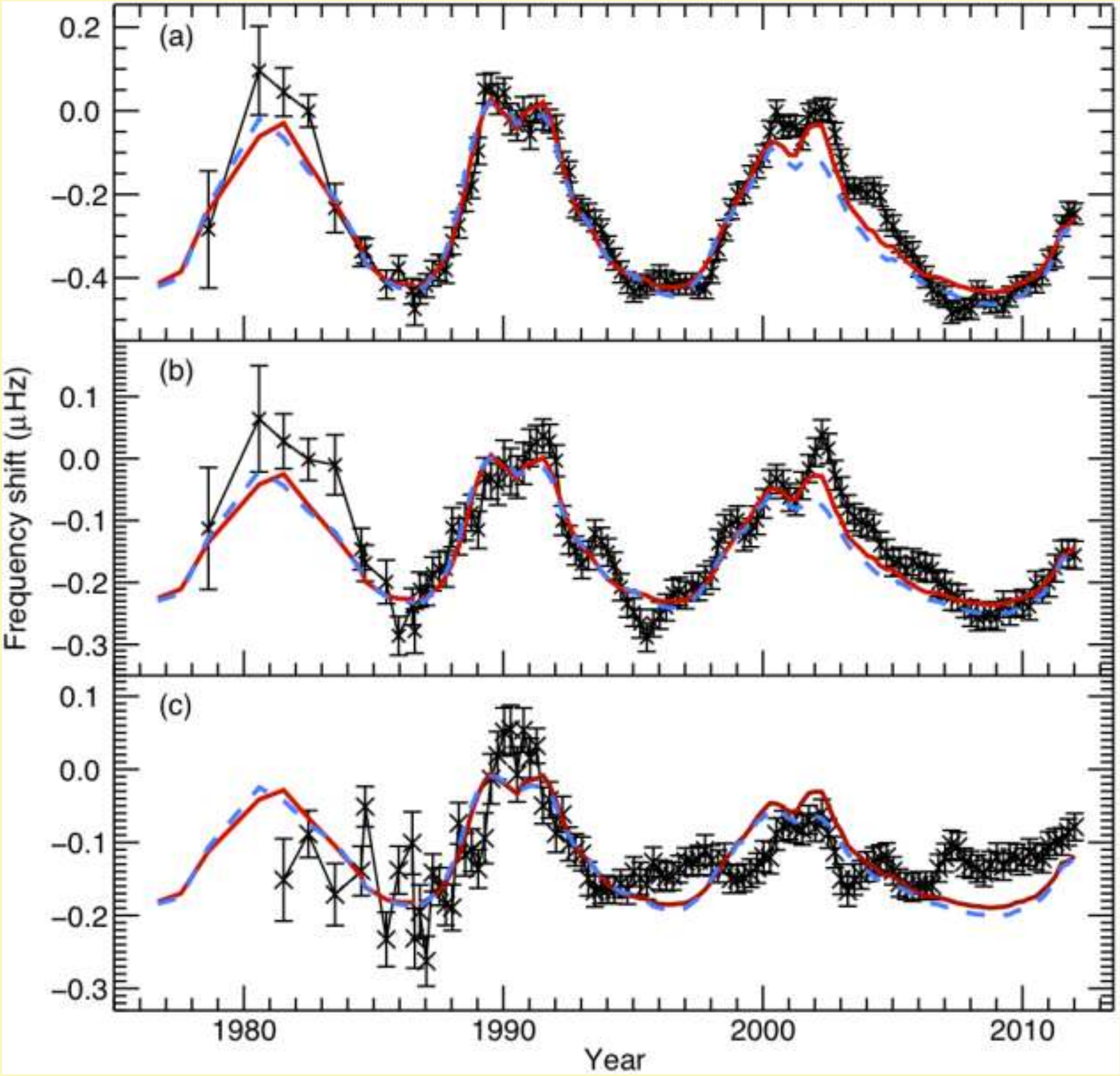
Change in frequencies



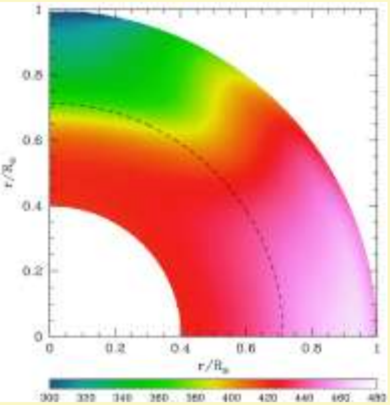
Change in frequencies as a function of solar activity

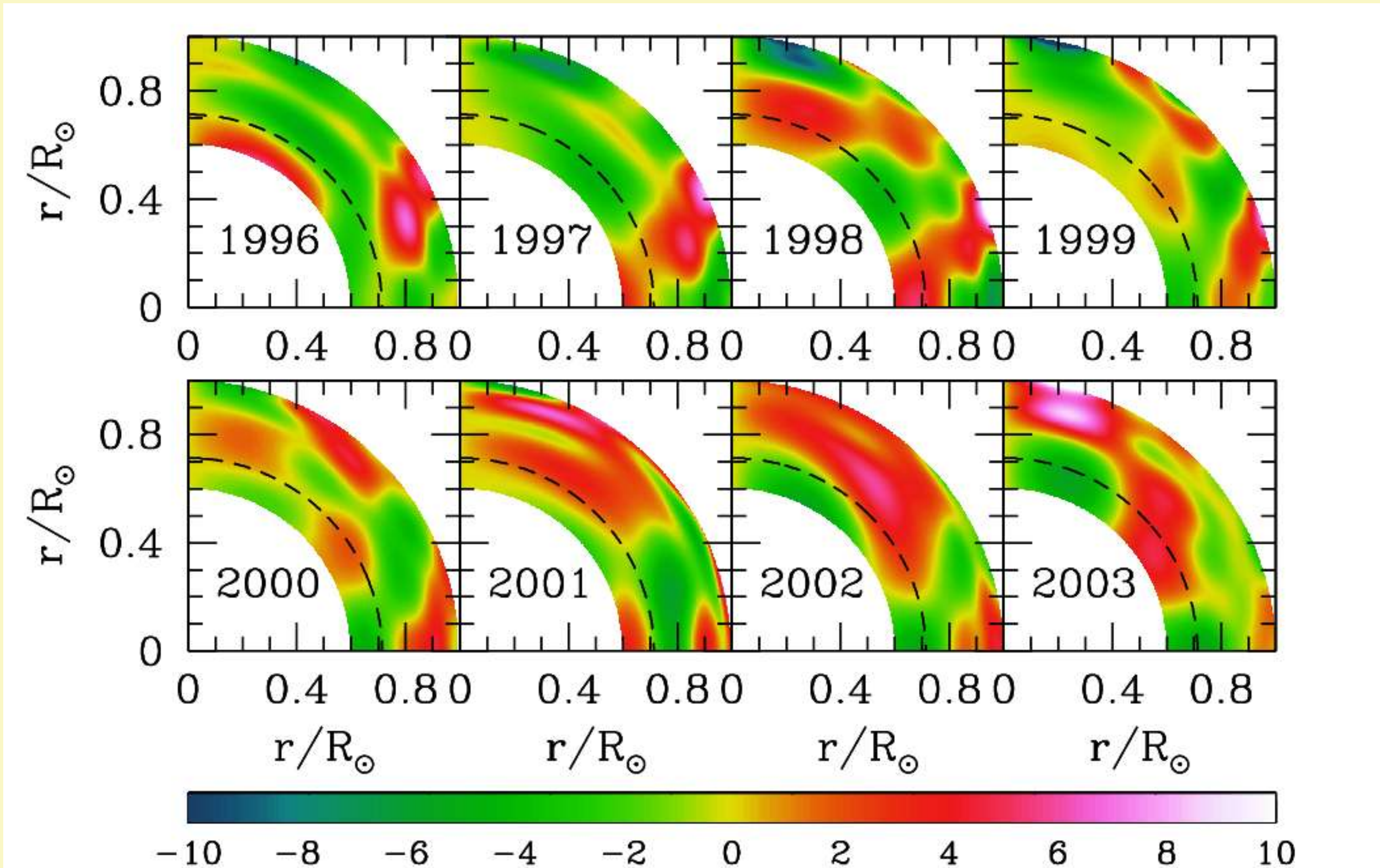


The solar cycle in frequencies



Changes in solar rotation: The zonal flows

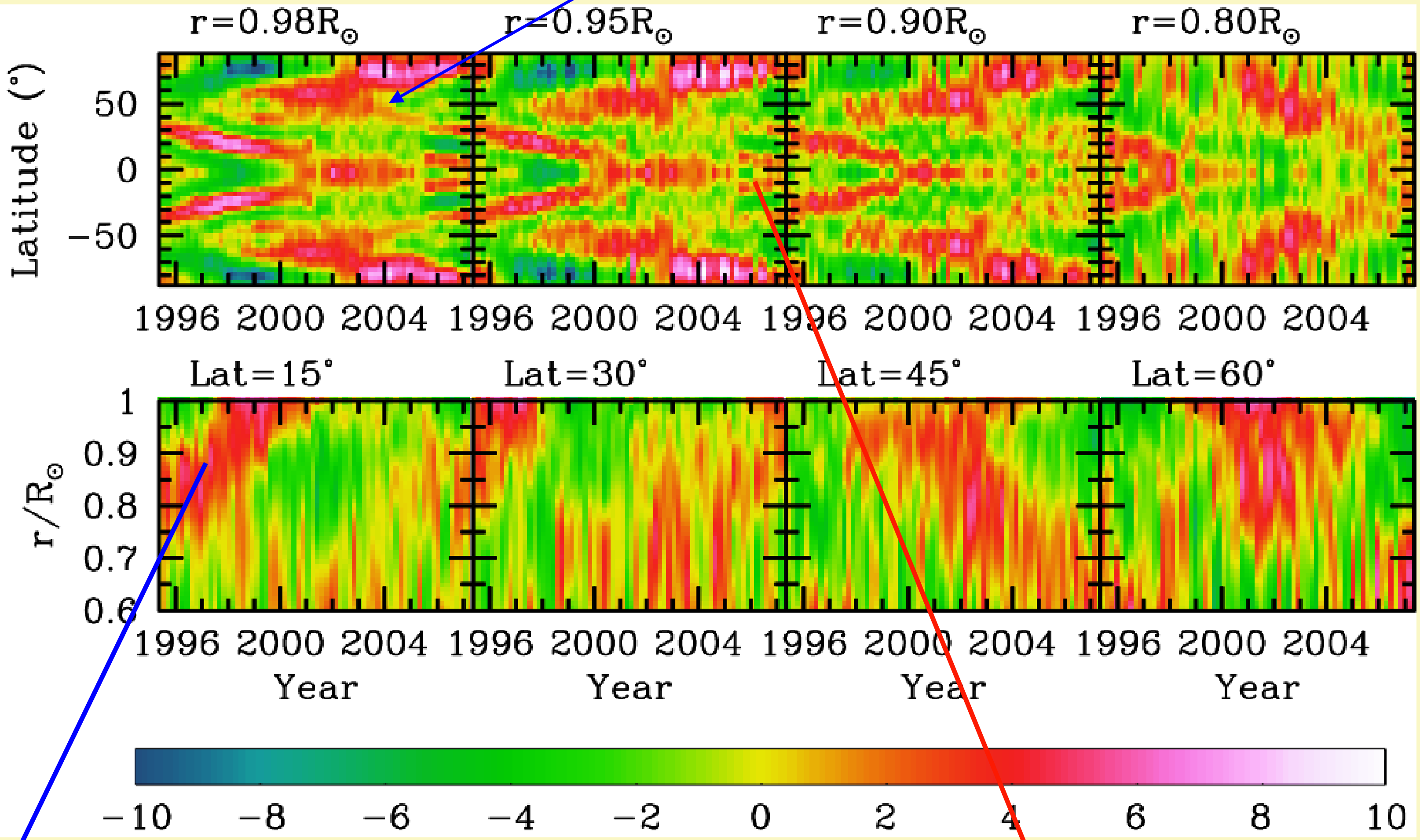




m/s

Solar zonal flows

First sign of cycle 24?

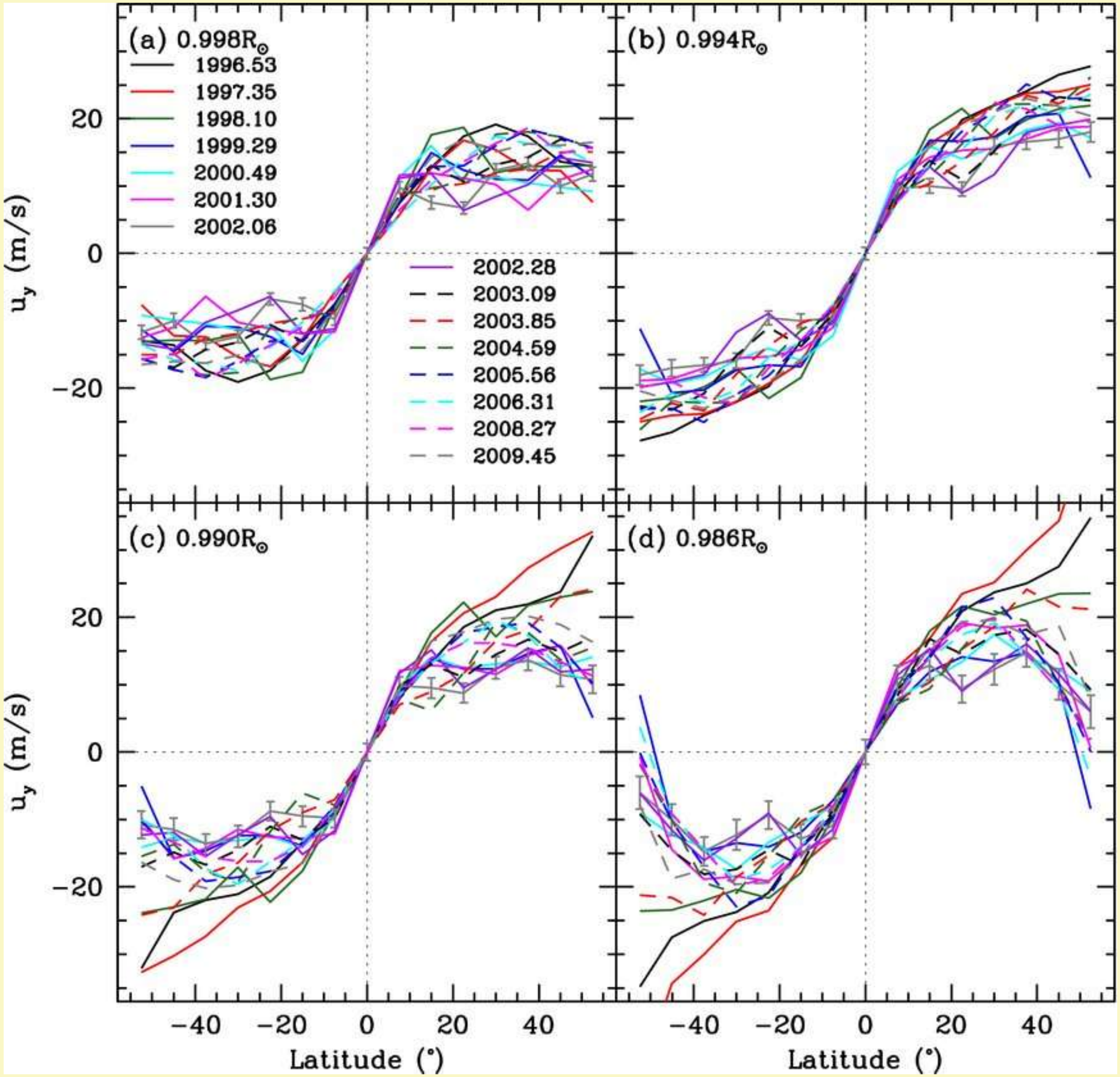


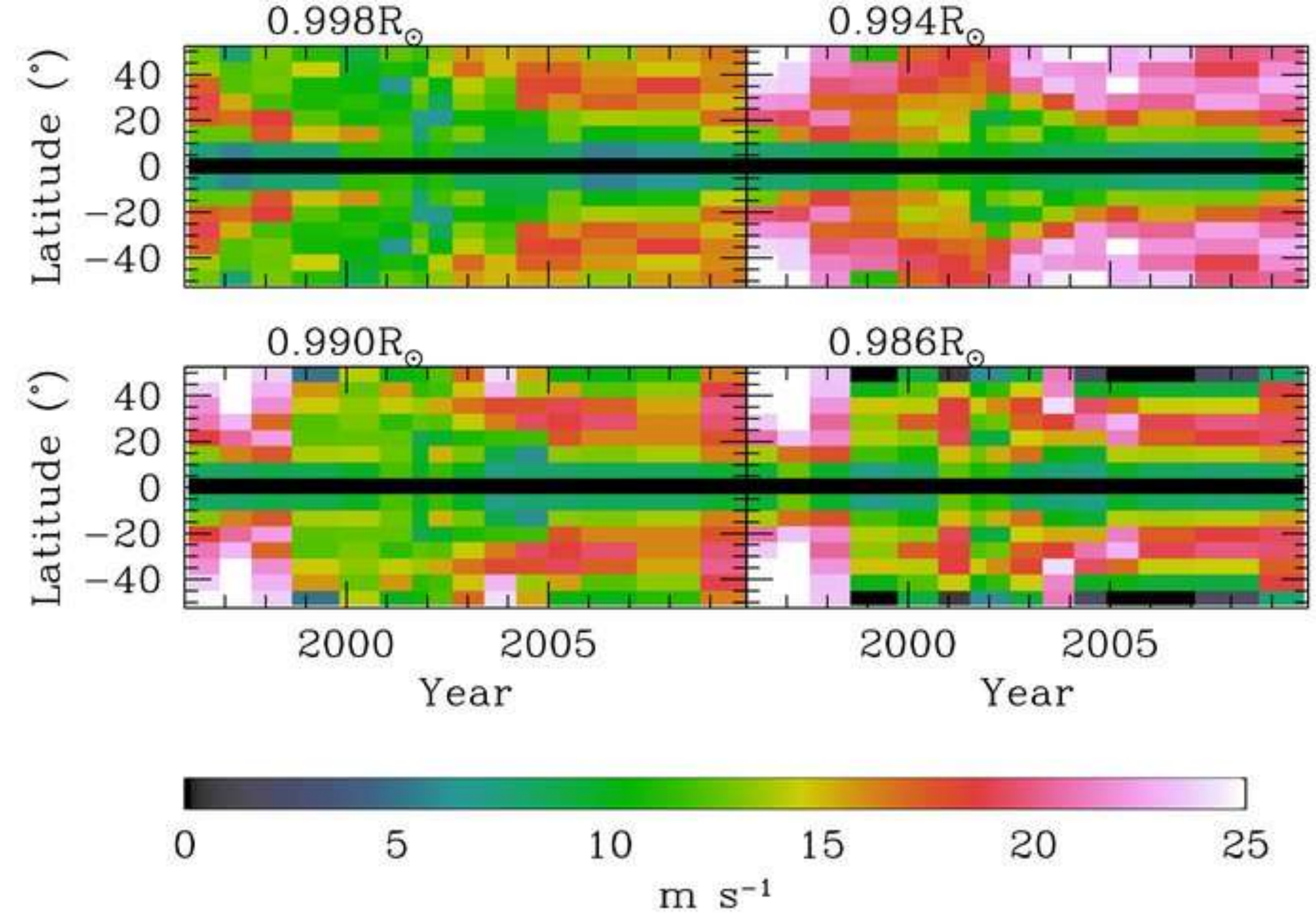
m/s

Rises upwards at about 1 m/s

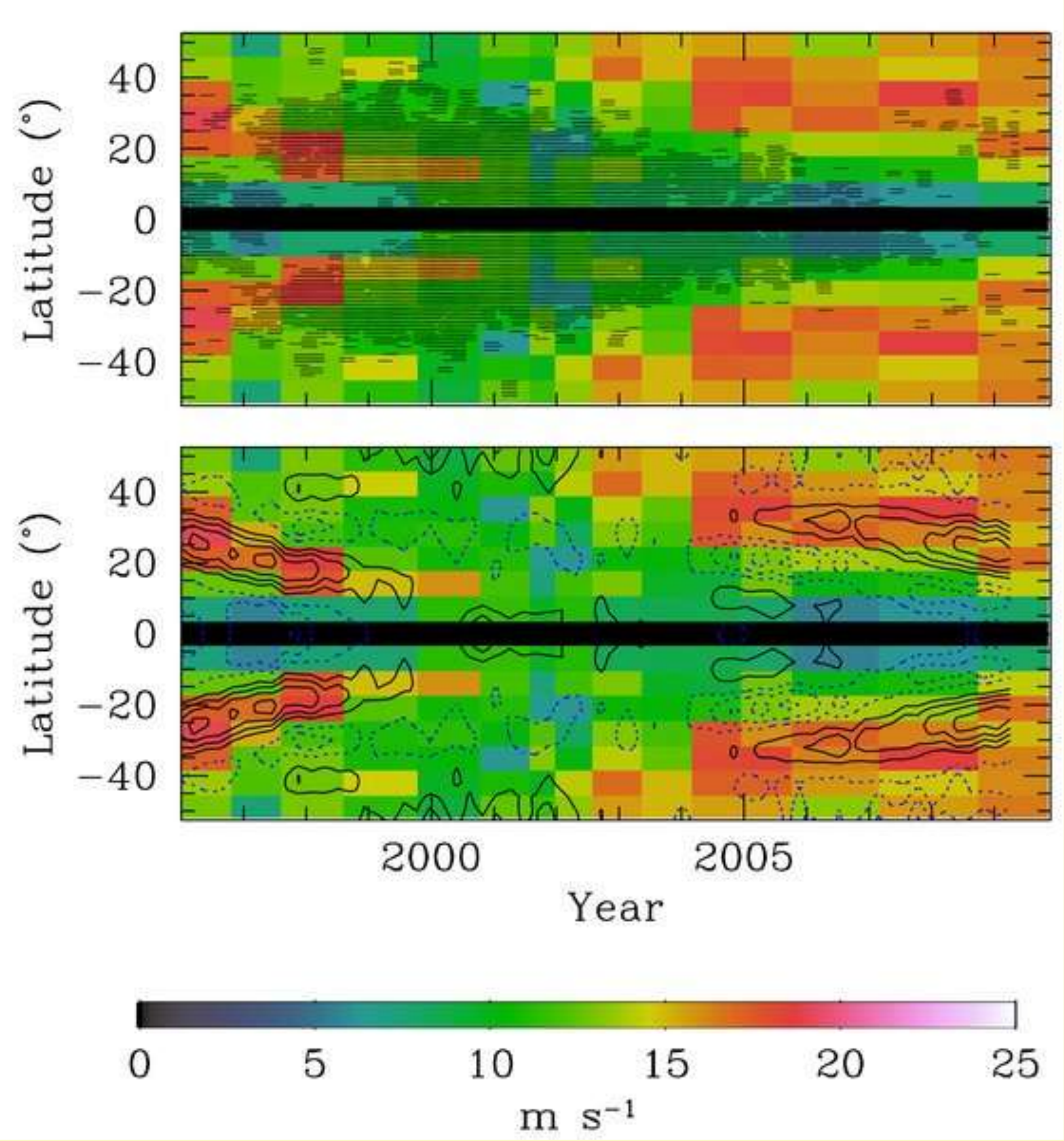
Bifurcation not seen earlier

Meridional Flows: these are flows from the solar equator to the poles

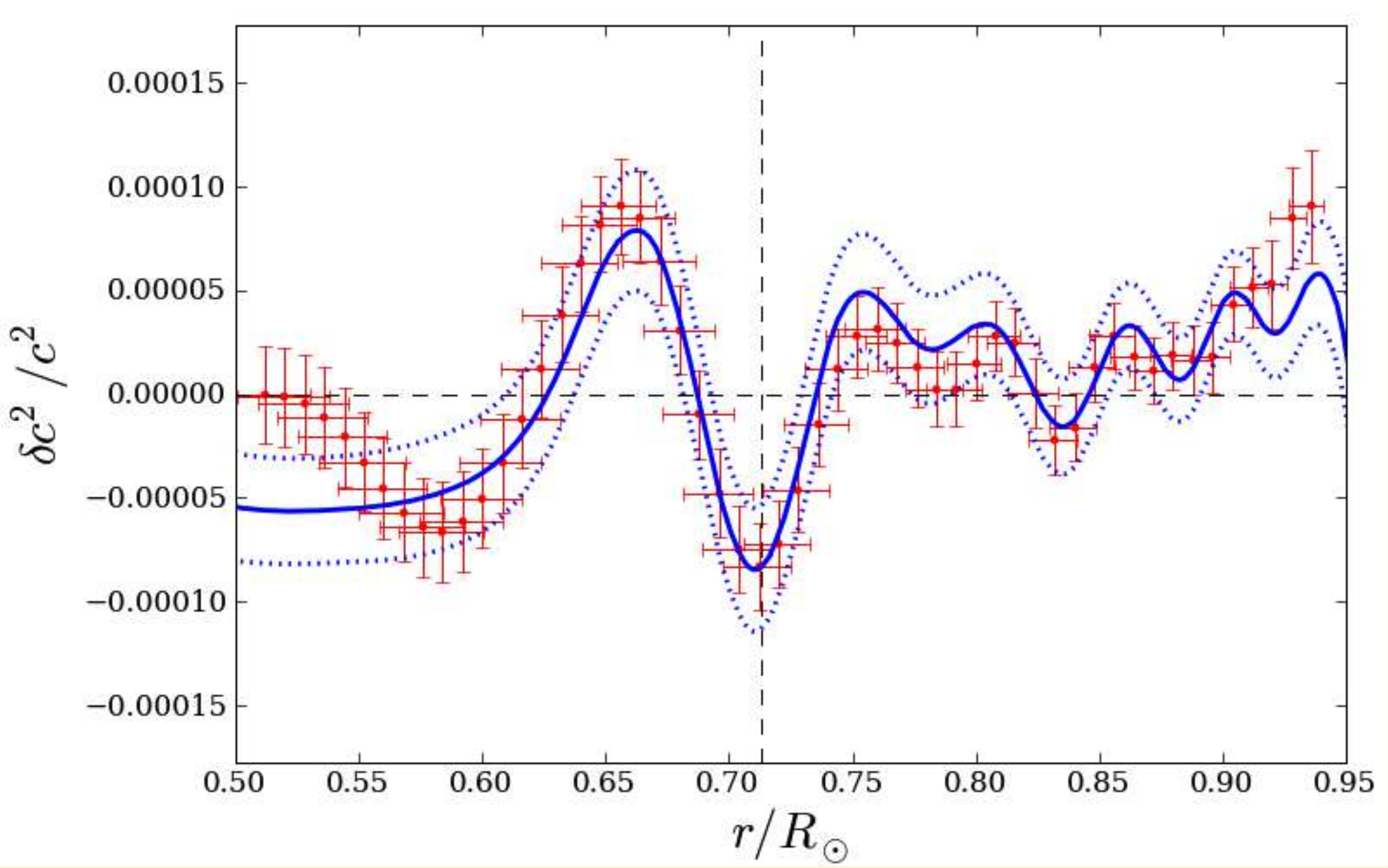




0.998R$_\odot$
0.994R$_\odot$
0.990R$_\odot$
0.986R$_\odot$
Latitude (°)
40
20
0
−20
−40
2000
2005
Year
0
5
10
15
20
25
m s^{-1}


Latitude (°)
40
20
0
−20
−40
Latitude (°)
40
20
0
−20
−40
2000
2005
Year
0
5
10
15
20
25
m s⁻¹

Solar variability: Changes in solar structure

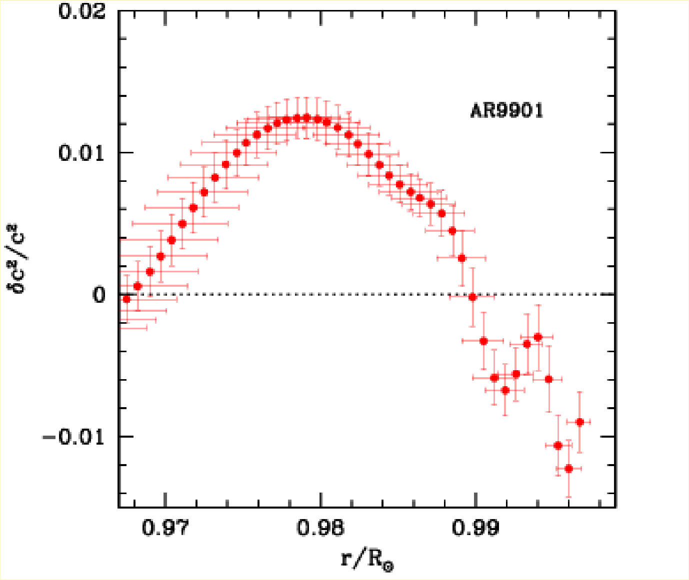


Baldner & Basu (2008)

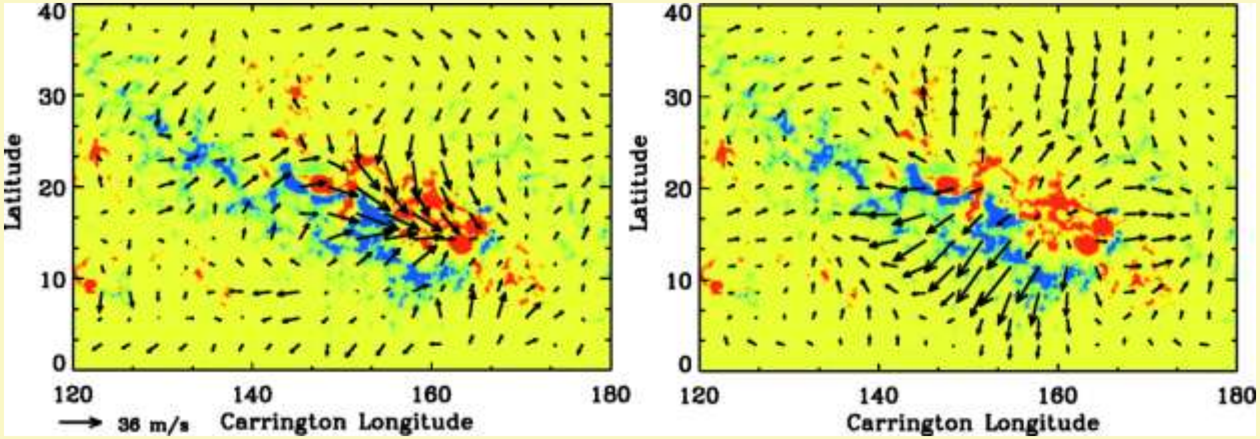
The Future of Helioseismology:
The study of active regions

We are beginning to be able to probe the structure of active regions

From Basu et al. 2004

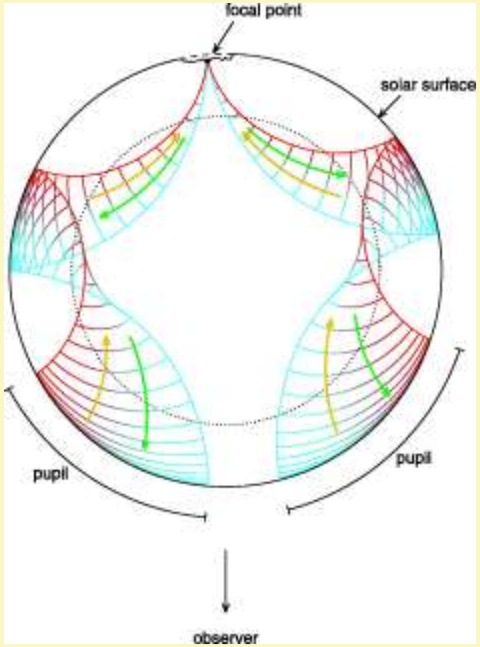


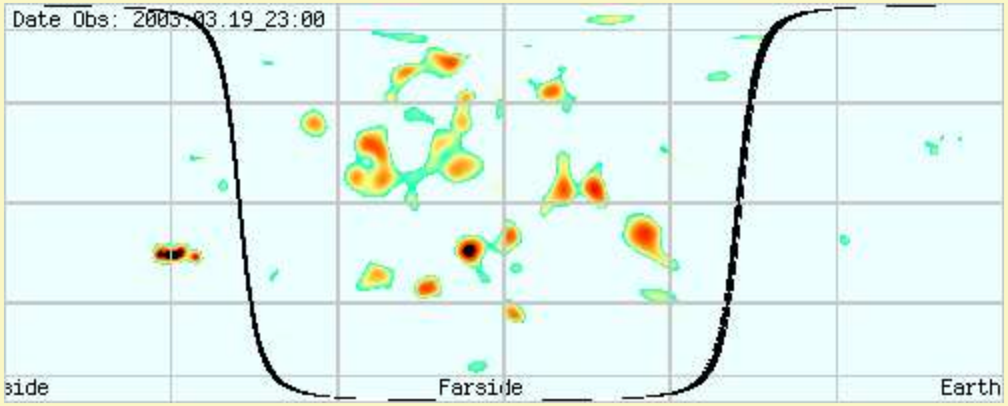
Sub-surface flows appear to converge towards active regions



From Zhao & Kosovichev, 2004

And the far-side of the Sun of course!





Courtesy C. Lindsey & D. Braun

Resources for global helioseismology

① Nitty-gritty of pulsation equations, properties of pulsations etc.: "Lecture Notes on Stellar Oscillations" by J. Christensen-Dalsgaard (http://astro.phys.au.dk/~jcd/oscilnotes/)

② Review on inversions for solar rotation: Howe, R., "Solar Interior Rotation and its Variation" in Living reviews in Solar Physics, (http://solarphysics.livingreviews.org/Articles/lrsp-2009-1/)

③ How we determine various details of solar structure, details on how good solar models are and the abundance problem: Basu & Antia, "Helioseismology and solar abundances," Physics Reports (http://adsabs.harvard.edu/abs/2008PhR...457..217B)

Resources for local helioseismology

- Details of different helioseismic techniques: Birch & Gizon, "Local Helioseismology", Living Reviews in Solar Physics (http://solarphysics.livingreviews.org/Articles/lrsp-2005-6/)