Formulating Emergence in the Physical Sciences

Sebastian De Haro

University of Amsterdam and University of Cambridge

Emergence, Effectiveness, & Equivalence in Physics
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The aim of this talk is to introduce and illustrate a criterion for emergence.

The framework is formal, where by ‘formalization’ I mean a conceptual schema detailed enough that it admits the basic notions of sets and maps.

Formalization is here not a goal, but a tool. The goal is to be able to more easily analyse examples in physics.

The framework will emphasise emergence as being an interpretative, rather than a merely formal, relation between theories.
Aims of this project

I discuss **ontological emergence**: but the *immediate aim* of the project is *not* strongly metaphysical, in the sense of requiring a commitment to a specific metaphysics, and explicating emergence in those terms.

My aim is to clarify what we mean by ‘ontological emergence’ in general: and to give a criterion that is as straightforward as possible.

I aim to give a *minimal* account of ontological emergence, independently of whether we are e.g. Humeans or Aristotelians about causation—further metaphysical details then just adding to the basic picture that I will present. One danger I wish to avoid is to make the conception of ontological emergence depend on specific metaphysical notions.

‘Ontology’ will be here understood in the straightforward sense of ‘the ontology of a scientific theory’, i.e. the domain of application that a theory describes, under a given interpretation. This domain of application is a part of the empirical world, not a matter of language in Quinean fashion.
Outline

1. 20th Century Emergentism
   - The British Emergentists
   - The New Emergentists

2. A Framework for Emergence
   - Scientific theories
   - Being more precise about emergence
   - Ontological emergence

3. Examples
   - Masslessness
   - Ontology
20th Century Emergentism

One well-known definition of emergence, by C. D. Broad:

‘Put in abstract terms the emergent theory asserts that there are certain wholes, composed (say) of constituents, A, B and C in a relation R to each other; that all wholes composed of constituents of the same kind as A, B and C in relations of the same kind as R have certain characteristic properties; that A, B and C are capable of occurring in other kinds of complex where the relation is not of the same kind as R; and that the characteristic properties of the whole $R(A; B; C)$ cannot, even in theory, be deduced from the most complete knowledge of the properties of A, B and C in isolation or in other wholes which are not of the form $R(A; B; C)$.’ (Broad, 1925, 61)

Drawback: does this definition actually apply to anything in physics?

The recent **new wave of emergentism** in physics can be traced back to ‘More is Different’ (1972) by Nobel Prize winner Philip Anderson:

The laws and principles he studies as a condensed matter physicist are emergent, in the sense that they are **entirely different from, but have no lower status than**, those studied by particle physicists.

Anderson accepts reduction but not constructionism: ‘The reductionist hypothesis does not by any means imply a “constructionist” one: The ability to reduce everything to simple fundamental laws does not imply the ability to start from those laws and reconstruct the universe. The more the elementary particle physicists tell us about the nature of the fundamental laws, the less relevance they seem to have to the very real problems of the rest of science.’

Famously opposed by Nobel Prize winner Steven Weinberg: the physics of the very small hold a **privileged position** in a hierarchy of scientific explanation. (Cf. Mainwood (2006)).
Ramifications with science funding

The debate was taken to the US House of Representatives over the funding of the Superconducting Super Collider (SSC):

Anderson: ‘I emphasized the almost complete irrelevance of the results of particle physics not only to real life but to the rest of science, while arguing that they are in no sense more fundamental than what Alan Turing did in founding the computer science, or what Francis Crick and James Watson did in discovering the secret of life.’

Weinberg: ‘One of the members of the [SSC] board argued that we should not give the impression that we think that elementary particle physics is more fundamental than other fields, because it just tended to enrage our friends in other areas of physics. The reason we give the impression that we think that elementary particle physics is more fundamental than other branches of physics is because it is. I do not know how to defend the amounts being spent on particle physics without being frank about this.’
The project was discontinued in 1993...
An informal description of emergence

Emergence is an ubiquitous phenomenon in nature. It is, roughly, the observation that there are “higher-level” phenomena (or entities, or theories), that appear in the macroscopic world, and which are absent from the “lower-level”, or microscopic, world.

More precisely, I will agree with the literature in seeing emergence as a “delicate balance” between:

(A) Dependence, linkage or rootedness.
(B) Independence, autonomy or novelty.


I will discuss emergence in terms of theories (i.e. I will say that ‘theory X is emergent with respect to theory Y’), but this can equally well be put in terms of ‘entities’, ‘properties’ or ‘behaviour’.
The example I shall give will not be a case of mereology (i.e. “parts and wholes”/microscopic vs. macroscopic physics): while important, I believe it easily can lead to confusion, and we ought to look at easier examples first! Mereology is not essential for emergence (Batterman (2002)).

The kind of emergence I will concentrate on here could be called conceptual: emergence of theories, entities, properties or behaviour as we link different situations or systems through a relation of resemblance (linkage). (I expect that the framework can be applied to mereology as well. But this will not be my focus.)

Discussions of emergence in e.g. the philosophy of mind emphasise causation and-or powers. The new wave of emergence in physics has not focused on these notions, but rather on entities. Perhaps because causation comes in less naturally in modern physics than in Newtonian (‘ball-bearing’) physics.
An Example: Ferromagnetism

Consider an iron bar in a strong external magnetic field, $H$, parallel to its axis. The bar will be almost completely magnetized. Now decrease $H$ to zero, for $T < T_c$: $M$ will decrease, but not to zero. Rather, at zero field it will have a *spontaneous magnetization*, $M_0$. Reversing the field will reverse the magnetization, so $M$ must be an odd function. Thus its graph is of the type (Baxter 1982):

![Graph of $M(H)$ for $T < T_c$.](image)

**Figure**: Graph of $M(H)$ for $T < T_c$. 
An Example: Ferromagnetism

For $T > T_c$, there is no net magnetization when we remove the external magnetic field, and so the graph looks like:

![Graphs of $M(H)$ for: (b) $T > T_c$ and (c) $T = T_c$.](image)

**Figure:** Graphs of $M(H)$ for: (b) $T > T_c$ and (c) $T = T_c$. 
An Example: Ferromagnetism

The spontaneous magnetization is a function of the temperature, and can be defined as:

\[ M_0(T) = \lim_{H \to 0^+} M(H, T) . \]

It is positive for \( T < T_c \) and zero for \( T > T_c \):

Figure: The spontaneous magnetization \( M_0 \) as a function of temperature.
An Example: Ferromagnetism

The behaviour near the critical (Curie) temperature is appropriately described by the critical exponent, $\beta$:

$$M_0(T) \sim (-t)^\beta, \text{ as } t \to 0^-, \quad t := \frac{T - T_c}{T_c}.$$  

Critical exponents can be calculated in a variety of ways (using the Ising model for the ferromagnet, renormalization group techniques, etc.).

In 1949, Onsager gave the exact solution of the 2d Ising model (with $\beta = 1/8$):

$$M_0(T) = (1 - k^2)^{1/8}, \quad k := 1 + t = \frac{T}{T_c}, \quad 0 < k < 1.$$  

The correlation length, $\xi$ (defined by the exponential fall-off of the correlation function between spins), diverges at $T = T_c$:

$$\xi = -\frac{1}{\ln \frac{T}{T_c}} \sim \frac{1}{1 - T/T_c} \to +\infty, \quad \text{as } T \to T_c^-.$$  

The system is macroscopically ordered.
An Example: Ferromagnetism

The iron can be regarded as undergoing a phase transition at $H = 0$, changing suddenly from negative to positive magnetization. In an actual experiment, this discontinuity is smeared out and the phenomenon of *hysteresis* occurs:
An Example: Ferromagnetism

Thus at the critical temperature (and below it) we have the appearance of order (i.e. a macroscopic correlation length, much larger than the distance, $a$, between the individual spins, $\xi \gg a$), and of spontaneous magnetization, i.e. of a non-zero magnetic field $M_0(T)$ of the material when we remove the external magnetic field.

These properties of the ferromagnet at $T = T_c$ are novel, relative to those at $T > T_c$. They fit the informal description of emergence in terms of: (A) linkage between the model at $T = T_c$ and the model at $T > T_c$, (B) novel physics at $T = T_c$. They are also widely regarded as emergent (e.g. Batterman (2002), Castellani (2002), Humphreys (2016)).
An Example: Ferromagnetism

To characterize this *novelty*, some authors (e.g. Batterman) have emphasised the mathematical properties of the model, especially the appearance of discontinuities and infinities.

But these mathematical discontinuities and infinities—important as they may be—are not essential for emergence. After all, some of the emergent behaviour might already be there before we reach $T = T_c$.

Thus we would like a characterisation of ‘novel properties and behaviour’ that stays close to Batterman in spirit but, rather than relying on very specific mathematical properties of particular models, such as critical exponents or a divergent correlation length, relies only on the model’s **change of interpretation** at or near the limit (it will be a rather commonsensical characterisation!).
Formulating Emergence
To discuss the emergence of theories, we need a notion of a theory. The main point is to distinguish a bare theory from its interpretation:

A **bare theory**, $T$, is a still uninterpreted, formal structure with a set of rules for forming sentences: an abstract calculus. Typically: a set of axioms or equations (the laws).

To be specific, the framework considers a bare theory as a triple, $T := \langle S, Q, D \rangle$, of state space, quantities, and dynamics (usually with symmetries, as automorphisms on the states and-or the quantities).
An interpretation adds, to the bare theory, a reference: it says what the elements of the bare theory “correspond to” in the world (see Butterfield’s talk). Thus I envisage a referential semantics using interpretation maps on states or quantities, assigning as values (outputs of the map) parts of the empirical world (hunks of reality!). But they are, in general, partial maps, i.e. for some arguments, the map yields no value (output). See Lewis (1970).

Thus we model interpretation as a partial map preserving appropriate structure:

\[ i : T \rightarrow D \]

from the bare theory to a **domain of application** in the world.

The *domain* is regarded as a structured set, containing objects and relations between them.
An example: Newtonian gravitation

Consider Newtonian gravitation. The bare theory is as follows:

\[ F = ma \]
\[ F = -\frac{GMM}{r^2} \hat{r}, \]

A (minimalist) interpretation is given by the map:

\[ F \mapsto \text{gravitational force} \]
\[ m \mapsto \text{mass} \]
\[ a \mapsto \text{body's acceleration}, \]

etc., assuming we already know what ‘gravitational force’, ‘mass’, etc. mean, in the relevant domain of the world (e.g. the solar system).

For a more elaborate interpretation, one needs to spell out the domain more, even in a particular context (e.g. ‘the gravitational force between the earth and the sun, which we measure according to procedure X’).
Comments on interpretation

This conception of a theory is meant to distinguish the formal aspects of a theory from its interpretative aspects, which are mostly non-formal (i.e. conceptual, experimental, etc.).

The domain in the world, $D$, to which the interpretation map maps, is not itself ‘more theory’, but can be seen as an interpreted domain, i.e. a set of objects and relations (parts of the world) mapped to by the theory, characterised by whatever means physicists use: experiments, verbal descriptions, etc.

One might well have a detailed metaphysical account of those objects and relations; but I will not assume this.
There is of course much more to be said on interpretation! About:

1. The relation between interpretations and models, as mediators between theories and phenomena (roughly: distinguishing how we get to an interpretation, from the interpretation that results. Models are embedded in particular interpretations).

2. Different kinds of interpretations, depending on scientific aims.

But I will move on to emergence. I will use Butterfield’s (2011) mnemonic:

\[ T_t = \text{the ‘top theory’} \]
\[ T_b = \text{the ‘bottom theory’} \]

Thus \( T_t \) is the theory that is emergent with respect to \( T_b \).

I will now fill in the conditions for emergence:
(A) Dependence, and (B) Independence.
(A) **Linkage**: a formal relation between the *bare theories*, $T_b$ and $T_t$. Roughly, the idea is that $T_b$ is approximated by $T_t$.

We can model this by a non-injective map:

$$\text{link} : T_b \rightarrow T_t.$$  

The map’s being non-injective embodies the idea of ‘coarse-graining to describe a physical situation’ (but linkage is not restricted to mere coarse-graining!):

Each “microstate” of $T_b$ gets assigned some “macrostate” of $T_t$, according to an approximation, or linkage, that is appropriate to describe a particular physical situation or system. Since the map is non-injective, many microstates may map to the same macrostate.
(A) Characterising linkage

The linkage map, link : $T_b \rightarrow T_t$, can combine three properties:

(i) *A limit in the mathematical sense.* Some parameter of the theory $T_b$ is taken to some special value ($\hbar \rightarrow 0, \ c \rightarrow \infty, \ N \rightarrow \infty$), i.e. there is a sequence in which a continuous or a discrete variable is taken to some value (cf. Fletcher (2016), Landsman (2013)). $T_t$ is the limit theory.

(ii) *Comparing different physical situations or systems.* One may link a given physical situation, or system, to another that resembles it.

For example, in the quantum-to-classical transition, one compares situations with very different-valued actions (*in addition* to taking the limit $\hbar \rightarrow 0$).

(iii) *Mathematical approximations* (whether good or poor). One compares expressions mathematically: perhaps numerically, or in terms of some parameter(s) of approximation. I will not consider this in this talk.
(A) Example of linkage

As an example of linkage, consider the link between special relativity and classical mechanics (cf. Malament (1986), Fletcher (2016)).

Classical mechanics can be obtained from special relativity as: a limit in which we take the speed of light to infinity, \( c \to \infty \), compared to all the other speeds in the theory. We have:

\[
T_b = \text{special relativity} \\
T_t = \text{classical mechanics ,}
\]

and the linkage is:

\[
\text{link (} T_b \text{)} = \lim_{c \to \infty} T_b = T_t .
\]

For each formula of \( T_b \) we get a formula of \( T_t \). But the map is not injective: the information about \( c \) is “washed out”: different formulas in \( T_b \) give the same formula of \( T_t \). Also, we use (ii).
(B) Novelty of reference

‘An *approximation* is an inexact description of a target system.
An *idealization* is a real or fictitious, idealizing system, distinct from the target system, whose properties provide an inexact description of the target system.’ (Norton 2012, p. 209).

Norton (2012) summarises the difference as an answer to the question: *Do the terms involve novel reference?*
Yes, in and only in a case of idealisation.

**PROPOSAL:** define ontological novelty as novel reference.
*This restricts the linkage map to be an idealisation:* cf. condition (ii), i.e. linkage entails comparing different physical situations or systems.

This is a *far from trivial* requirement on the map link. For some limits ascribe contradictory properties to the limit system, so that a limit system does not exist (Norton (2016)).
(B) Novelty of reference

Thus we restrict to cases in which there IS a physical system that is described by the top theory, $T_t$. The requirement is that $T_t$ has an interpretation, mapping to a domain of application, $D_t$:

$$i_b : T_b \rightarrow D_b$$

$$i_t : T_t \rightarrow D_t$$

Accordingly, we have two cases: the domains of application are the same, or different:

(a) $D_b = D_t$
(b) $D_b \neq D_t$.

(Condition (a) can be weakened to $D_b \subseteq D_t$, so that (b) is: $D_b \nsubseteq D_t$.)
(a) **Same domains**, i.e. $D_b = D_t$. The elements and their relations are the same between the two domains.

There can be **no ontological emergence**, because there is no novelty “in the world”, i.e. there is a single domain. The theories describe the same items (they may do so in different ways, e.g. using different laws: in this sense there is the possibility of *epistemic emergence*).

Since I am interested in ontological emergence, I will leave this case aside.

(b) **Different domains**, i.e. $D_b \neq D_t$: at least some elements/relations differ. The more the domains differ, the more significant the novelty.

I PROPOSE this as a criterion for **ontological emergence**: novelty of reference, expressed as a difference between the domains related by the linkage map.
Ontological emergence

There is **ontological emergence** when conditions (A: Linkage) and (B: Novelty) are met, i.e. when the bare theories are related by a linkage map, and the domains of their interpretations are distinct, i.e. $D_t \neq D_b$:

$$
\begin{align*}
T_t & \; \xrightarrow{i_t} \; D_t \\
\text{link} & \; \uparrow \\
T_b & \; \xrightarrow{i_b} \; D_b
\end{align*}
$$

**Figure:** The linkage and interpretation maps’ “failure to mesh”.

Thus **interpretation and linkage fail to mesh** ("commute"):

$$i_t \circ \text{link} \neq i_b. \tag{1}$$

In case (i) of limits: $T_t := \lim_{x \to 0} T_b(x)$. Rewrite the criterion for emergence, Eq. (1), as:

$$D_t := i_t(T_t) \neq D_b|_{x=0} = i_b(T_b)_{x=0}.$$
I will take an example from classical physics: the emergence of masslessness (a theory of massless point particles) with respect to a theory of massive particles.

It illustrates, in a simple setting, a widespread phenomenon in physics: the existence of a massless, or scale-invariant, regime in a massive theory (e.g. via renormalization group flow, phase transitions, etc.).

The massless regime is characterised by its different symmetries and geometrical properties.
Example of ontological emergence: masslessness

Take $T_b$ = the theory for a classical point particle in Minkowski space:

$$\ddot{x}^{\mu} = 0 \quad \text{(geodesic equation)} \quad (2)$$

$$p^2 = -m^2 c^2 \quad \text{(momentum four-vector)} \quad (3)$$

In the massive case, $m \neq 0$, Eq. (3) is the condition that the velocity four-vector is timelike. If $m = 0$, it lies on the light-cone. An example of how the interpretations differ:

If $m \neq 0$, the two equations get the familiar interpretation. For example:

$x^{\mu}(\tau) \mapsto \text{‘the position of a massive particle, with mass } m \text{, moving freely in Minkowski space, as a function of the particle’s proper time’}.$

If $m = 0$, the interpretation changes. For example:

$x^{\mu}(\sigma) \mapsto \text{‘the position of a massless particle moving at the speed of light in Minkowski space, as a function of the affine parameter of its worldline’}.$
Example of ontological emergence: masslessness

The **bottom theory** is:

\[ T_b(m) = \text{‘the theory (Eqs. (2)-(3)) for the particle of mass } m \text{’}. \quad (4) \]

The **linkage map** is the limit \( m \to 0 \), yielding a **massless particle** theory:

\[ T_t = \lim_{m \to 0} T_b(m) = \text{‘the theory (Eqs. (2)-(3)) for the particle of mass } 0 \text{’}. \quad (5) \]

The **interpretation** of the bottom and top theories (simplified!):

\[
\begin{align*}
D_b & := i_b(T_b) = \{\text{a free, massive point particle of mass } m\} \quad (6) \\
D_t & := i_t(T_t) = \{\text{a free, massless point particle}\} \quad (7)
\end{align*}
\]

The interpretation of the top theory, Eq. (7), is **different** from the one we get by setting \( m = 0 \) in the bottom theory’s interpretation, Eq. (6):

\[
D_b|_{m=0} = i_b(T_b(m))|_{m=0} \overset{\text{Eq. (6)}}{=} \{\text{a free, massive point particle of mass } 0\} \\
\neq D_t \overset{\text{Eq. (7)}}{=} \{\text{a free, massless point particle}\}
\]
Example of ontological emergence: masslessness

The underlying point is that (as I will next argue):

\{a free, massless point particle\} ≠ \{a free, massive point particle of mass 0\}

In a diagram:

\[ T_t = \text{Eq. (13)}(m = 0) \quad \xrightarrow{i_t} \quad D_t = \text{massless particle} \]

\[ T_b(m) = \text{Eq. (12)} \quad \xrightarrow{i_b} \quad D_b = \text{massive particle of mass } m \]

(≠ massive particle of mass 0)

**Figure:** Emergence of the massless particle, as the lack of commutativity between the linkage relation \((m \to 0)\) and the interpretation: \(i_t \circ \text{link} \neq i_b\).
Arguing that: \( \{ \text{a free, massless point particle} \} \neq \{ \text{a free, massive point particle of mass 0} \} \)

(i) *Symmetries of the solutions.* The models of the massive and massless particle theories have different symmetries. Massive particle solutions have \(O(3)\) symmetry, while the symmetry of the massless particles is \(E(2) \cong ISO(2)\). \((E(2)\) is not a subgroup of \(O(3)\), hence the *novelty*).

(ii) *Timelike vs. null geodesics.* In the limit \(m \rightarrow 0\), the massive equation, \(p^2 = -m^2 c^2\), of course reproduces the massless one, \(p^2 = 0\). And thus a timelike geodesic converges to a null geodesic. But their interpretations contain important differences.
Getting massless particles from massive particles

**Figure:** (a) Two particles with constant speeds, $v_1/c = \tan \theta_1$ and $v_2/c = \tan \theta_2$, colliding at $t = t_c$. (b) Massless limit, $v_1 = v_2 = c$: the particles do not collide at any finite time. As $m \to 0$, the lines in (a) are continuously deformed to the lines in (b). But the *crossing point* is pushed to infinity!
Getting massless particles from massive particles

In the theory’s domain there is a question about events: 
*will two particles coming from the same direction collide?*

The two theories answer this question differently.

As long as $m \neq 0$, they always collide within a finite time.

For $m = 0$, they will never collide within a finite time.

So, $i_b$ ascribes a property to these particles (‘the particles will collide in finite time’) that *contradicts* the properties of the solutions of $T_t$, which we find in $D_t$ (two massless particles coming from the same direction will never collide).

This is a case of a limit of a bare theory whose interpretation ascribes contradictory properties to the limit system: namely, ‘a free massive particle of mass $m$’, i.e. $i_b(T_b(m))|_{m=0} = \emptyset$ (in special relativity, of course! Cf. Norton (2016)). There is no limit system described by $i_b$. To get the limit system, we need to construct $i_t$. 
Figure: Emergence of the massless particle, as the lack of commutativity between the linkage relation \((m \to 0)\) and the interpretation: \(i_t \circ \text{link} \neq i_b\).
I construe ‘ontology’ here in the straightforward sense of ‘the ontology of a scientific theory’. Crucially, I take this to be more than mere semantics, since the theories are subject to the requirement of empirical adequacy: they describe the same target system (even if sometimes inexacty).

(1) The domains that I have been describing are not to be (naively, and wrongly!) identified with the world as it is in itself—whatever that might be taken to mean.

(2) There is an interesting interpretative project of:
(a) studying how the entities postulated by our theories can be, even if: 
(b) we have not yet decided whether and how they exist, i.e. we have not yet decided how the idealised systems characterise the target system.
We can address the question, (a), of the properties of those entities, according to the theory, before we ask about (b), actual existence.
Realist vs. empiricist ontologies

Intensional semantics appropriately models the interpretative practices of both realists and empiricists. Namely, realists and constructive empiricists can agree about the interpretation of a theory or model, i.e. about its basic ontology (‘the picture of the world drawn by the theory’, van Frassen 1980, p. 57), even though they have different degrees of belief in the entities that the ontology of the theory postulates.

My position here resembles what A. Fine (1984) has called the ‘core position’ that realists and non-realists share: both accept the results of scientific investigations as, in some sense, ‘true’, even if they give a different analysis of the notion of truth.

Thus realists may disagree amongst themselves about the right metaphysical construal of those entities. Think, for example, of Quine’s (1960) referential indeterminacy: the linguist, upon hearing the native say ‘gavagai’ while pointing at a rabbit, might for simplicity translate it as ‘rabbit’—while still being at a loss whether the objects to which this term applies are rabbits, or stages, i.e. brief temporal parts of rabbits, or mereological fusions of spatial parts of rabbits.
Summary and discussion

1. I have illustrated the idea of ontological emergence as a *non-commuting diagram* between linkage and interpretation maps.

2. The interpretation of the massless particle theory is different from the interpretation of the massive particle theory ‘with the mass set to zero’.

3. The account is metaphysically weak: I have focused on the basic question of when ontological emergence obtains.

4. The framework is easily applied in other examples (forthcoming).
Come to Amsterdam for a workshop on emergence! https://www.d-iep.org.
Thank you!
Aristotelian vs. Quinean projects

The idea here is that working out the ontology of scientific theories, the way they are interconnected, and their logical structure, is different from explicating how the elements of such an ontology exist in our world.

It corresponds to the contrast between a (neo-)Aristotelian metaphysical project (of enquiry into how things are) vs. a Quinean project of strict enquiry into what exists, in a narrow sense of the word (as in: ‘to be is to be the value of a bound variable’).

The former project permissively allows for things, and categories, to appear in our ontology, that we might one day come to reject as literal parts of our world. Those things are, in some sense: even if they do not exist in the literal sense in which the theory would say they do. Thus my position is closer to the ‘jungle landscapes and coral reefs’ of the neo-Aristotelian project than to the desert ecosystems that Quine’s first-order logic suggests.