BOHR'S DIAPHRAGMS

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ABSTRACT

In his response to EPR, Bohr introduces several ideal experimental arrangements that often are not understood correctly, and his discussion of them is given a positivist reading. Our analysis demonstrates the difference between such a reading and Bohr’s actual position, and also clarifies the meaning of several of Bohr's key physical and philosophical ideas:

The role of the quantum of action in the distinction between classical and quantum systems;

The criterion of measurability for theoretically defined concepts;

The freedom in placement of the "cut" between measuring instrument and measured system;

The non-visualisability of the quantum formalism; and

Bohr's concepts of phenomenon and complementarity.

1. Introduction.

In EPR 1935, Albert Einstein, Boris Podolsky and Nathan Rosen challenged the completeness of the quantum mechanical description of a physical system. Their argument may be summarized as follows:

Let two systems (hereafter I and II) initially be non-interacting, so that the initial wave function of the total system (hereafter III) is the product of separate wave functions for I and II. According to quantum mechanics (QM), if they subsequently interact during a finite time interval, then they become a single composite system III. Even after the interaction has ceased, I and II no longer have individual wave functions, and III is represented by a single joint wave function.

As an example, EPR consider a composite system III such that, after the interaction, the difference in the positions x_1-x_2 and the sum of the momenta p_1+ p_2 of systems I and II are eigenvalues of the joint wave function III.[1]

After the two systems cease to interact, a subsequent sharp measurement of the coordinate x_1 of system I at a certain time[2] thus can be used, together with the information about the joint wave function III, to predict with certainty the coordinate value x_2 of system II at the same time. EPR argue that such a determinate coordinate value x_2 should be taken as an "element of reality" belonging to system II immediately before the measurement made on system I:

> [A]t the time of measurement the two systems no longer interact, [and] no real change can take place in the second system in consequence of anything that may be

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[1] This is possible because the corresponding QM operators are easily seen to commute. For simplicity, here we consider only one spatial dimension of the system. In the next section, the other spatial dimensions are included in the discussion of diaphragms.

[2] Later, we shall return in some detail to the question of why a certain moment of time must be chosen.

done to the first system (EPR 1935, p. 779).

If this argument were accepted, it already would show the incompleteness of the QM description: According to QM, x_2 does not have a determinate value just before the sharp measurement of x_1. But EPR did not base their argument for the incompleteness of QM on this observation. They argue that, instead of measuring the position of system I, one could have chosen to measure another physical quantity, represented by an operator that does not commute with its coordinate operator,[3] in particular, its momentum $p_{1.}$ Measurement of p_1, together with the prior information about the joint wave function, could have been used to predict with certainty a determinate value p_2 of system II's momentum at the same time as that, at which the position measurement of system I was actually made.

From this observation, EPR conclude that system II must have had *both* a determinate position x_2 and momentum p_2 just before any measurement is made on system I (see the quotation given above). But QM precludes the possibility of a description ascribing to a system at a certain time determinate values of a pair of non-commuting variables; in particular, the components of position and momentum in some given direction. Thus, the QM description of a system cannot be complete.

Bohr's response (Bohr 1935) introduces several new ideal experimental arrangements. Although many commentaries on Bohr’s views exist, detailed analyses of his use of these diaphragms are rare, and accurate ones even rarer.[4] In particular, the

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[3] Sometimes, for brevity we shall say that two quantities do or do not commute, but this is always what we really mean.

[4] We have chosen as prime examples of inaccuracy, Beller and Fine 1994, and Beller 1999, Chapter 7 (our criticisms of the book are confined to this chapter), and Fine 2004. One of us ($$$) was a close friend of Mara Beller, whose untimely death he will never cease to mourn. But she and he shared a sense of philosophy of science as a search for clarity through dialogue, and in their correspondence he had explained his criticisms to her.

argument based on these ideal experiments has been given a positivist reading, leading to a positivist interpretation of Bohr’s reply to the EPR paper, and more generally of Bohr’s philosophical viewpoint.[5]

In this paper, we present a reading of these arrangements that we believe coincides substantially with Bohr’s own. In the next section, we shall explain in detail these ideal experimental arrangements based on the use of one or more diaphragms. The third section explains the role of the quantum of action and of the "cut", both crucial to understanding Bohr’s response. In the fourth section, we explain Bohr’s concept of phenomenon, which is closely related to the discussion in the third section, and crucial to the difference between Bohr’s position and a positivist one. Finally, the last section discusses a few difficulties with Bohr’s experimental arrangements. The conclusion summarizes the differences between Bohr’s position and a positivist one.

2. Uses of diaphragm(s) in different experimental arrangements in Bohr’s response.

Bohr introduces several new ideal experimental arrangements using diaphragms. Central to his analysis of all such experiments is the presence of “a [material] support which defines the space frame of reference,” tacitly assumed to be inertial.[6] The coordinates and momenta are defined with respect to this inertial frame, and the instruments for measuring these quantities are located, spatially and temporally, with respect to it. It is also assumed that the material support is so massive that,

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[5] Beller and Fine 1994 speak of "the positivist perspective that Bohr eventually adopted" and "his positivist solution to EPR." See Dickson 2001 and Dickson 2002a for a critique of some aspects of Beller and Fine's account of Bohr's position. Howard 2004 asserts: "Bohr was … in no way a positivist," a position with which we agree.

[6] Bohr refers to "some ultimate measuring instruments, like the scales and clocks which determine the frame of space-time coordination-- on which, in the last resort, even the definitions of momentum and energy

> the momentum exchanged between the particle and the diaphragm will, together with the reaction of the particle on the other bodies [rigidly attached to the support], pass into this common support (Bohr 1935, 697)

without significant effect on its state of motion.

A diaphragm is a large screen (idealized as two-dimensional) that is opaque to a beam of particles or radiation impinging upon it, except for the part of the beam that passes through one or more slits in it.[7] The width of the slits is assumed large compared to the (average) wave length of the particles or photons in the incident beam; so that the slit width "may be taken as the uncertainty Δq of the position of the particle relative to the diaphragm, in the direction perpendicular to the slit" (ibid., 697).[8]

One type of diaphragm (hereafter abbreviated D1) has a single slit in it,[9] and "may form part of some more or less complicated experimental arrangement" (ibid., 697). But let us dwell on D1 for a moment. It can be used for measuring one component of the position (q_y) of a particle passing through the slit *relative to the plane of the diaphragm* and to the direction **(y)** perpendicular to the slit. An indeterminateness[10] or uncertainty in position (Δq_y) is associated with this measurement and, in accord with "Heisenberg's general principle $\Delta p \Delta q \sim h$" (ibid., p.697), an uncertainty of the particle's momentum (Δp_y) is associated with it:

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quantities rest…" (Bohr [1939] 1998, 104). Since the discussion is confined to non-relatvistic quantum mechanics, we may confine our consideration of time to the Newtonian absolute time.

[7] We speak of a beam, even though the particles or quanta of radiation may be so spread out in time that (on the average) only one at a time passes through the diaphragm. Quantum mechanics can only treat ensembles of particles produced by some (ideal) device, even if the ensembles are temporal rather than a spatial, and a virtual rather than real (see, e.g., Stachel 1997, Stachel 2006).

[8] We assume that, if the diaphragm has more than one slit, the width of each is small compared to the distance between slits.

[9] The designations of the diaphragms as D1, D2, and of the experimental arrangements as A1, A2, etc., are ours, introduced for clarity in our references to them.

> Obviously the uncertainty Δp is inseparably connected with the possibility of an exchange of momentum between the particle and the diaphragm; and the question of principle interest for our discussion is now to what extent the momentum thus exchanged can be taken into account in the description of the phenomenon to be studied by the experimental arrangement concerned, of which the passage of the particle through the slit [D1] may be considered as the initial stage (ibid, 697).

Bohr considers two such arrangements. In the first (A1), D1 is rigidly fixed to the frame of reference. When a particle passes through such a slit:

> [t]hen the momentum exchanged between the particle and the diaphragm will . . . pass into this common support, and we have thus voluntarily cut ourselves off from any possibility of taking these reactions separately into account in predictions regarding the final outcome of the experiment . . . (ibid., 697)

But, just because the momentum exchange between D1 and the particle is indeterminate, the position component q_y *relative to the support*[11] can be measured.

In the second arrangement (A2), D1 is not rigidly fixed to the support, but connected to it, by a system of springs or the like, that allows D1 to recoil when a particle passes through it, relative to the support. In arrangement (A2), one still has a free choice *after* the particle passes the slit. One can *either*:[12]

1) add an arrangement (A2-1) to the apparatus, allowing measurement of the *Y*-component of the *position* of the slit in D1 relative to the support immediately after the

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[10] Heisenberg's "Unbestimmtheit" is better translated "indeterminateness," but "uncertainty" seems impossible to replace.

[11] From now on, we use "support" and "frame of reference" interchangeably.

[12] "In an arrangement suited for measurements of the momentum of the first diaphragm [i.e., A2], it is further clear that even if we have measured this momentum before the passage of the particle through the slit, we are after this passage still left with a *free choice* whether we wish to know the momentum of the particle or its

particle passes through it; and hence deduce the *y*-component of the position of the particle at that time; or

2) add a different arrangement (A2-2) to the apparatus, allowing measurement of P_y, the *Y*-component of the *momentum* of D1 relative to the support after the passage of the particle through it; and, using conservation of momentum, deduce p_y, the *y*-component of the momentum of the particle, as it passed through the slit in D1.[13]

One actually measures the change in the momentum of D1, and hence in that of the particle. Calculating the total momentum of each presupposes that the *y*-components of their initial momenta have determinate values just before the particle passes through the slit. D1 is usually taken to be at rest initially, so that its initial $P_y = 0$. If it is assumed that the beam of particles was prepared in a sharp momentum state, with the total momentum of each particle in it in the (*x)* direction of propagation of the beam; then the initial $p_y = 0$ and the change in p_y is equal to p_y.

Bohr asserts that in QM sub-arrangements A2-1 and A2-2 are mutually exclusive,[14] and below we shall go into the important reasons for this assertion. But first, a few other points need clarification. First of all, we emphasize that D1 plays a quite *different* role in arrangements A1 and A2. In A1, D1 is rigidly attached to the frame of reference and so

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initial position [i.e., at the moment of passing through D1] relative to the rest of the apparatus" (Bohr 1935, 698).

[13] Note that a measurement of the position of the slit must be made immediately after passage of the particle through it, since the position of the particle does not remain sharp (i.e., position measurements on it made a finite time after passage through the slit would give a spread of results). On the other hand, assuming that particle and diaphragm are not subject to any external forces after passage through the slit, the y-components of their momenta remain constant (i.e., a momentum measurement made a finite time after passage through the slit would give the same sharp result). This point is discussed in greater detail below.

[14] "In the first eventuality, we need only make a second determination of the momentum of the diaphragm [D1], leaving unknown forever its exact position when the particle passed [through it]. In the second eventuality we need only determine its position relative to the space frame with the inevitable loss of the knowledge of the momentum exchanged between the diaphragm and the particle" (Bohr 1935, 698).

forms part of the measuring apparatus. The phenomenon[15] that started with the preparation of the particle in a definite momentum state is complete when the position of the particle at the time of its passage through the slit is registered. A clock, rigidly fixed to the frame of reference, would be needed to actually record this time.[16] In arrangement A1, no further analysis of the behavior of D1 is permitted; or better put: If one were to attempt it, then one would no longer be dealing with the same phenomenon.

Arrangement A2 provides an example of just such an alternative attempt. Here D1 is not part of the measuring apparatus, but is part of another, composite system, consisting of the particle plus D1. Now one can perform a further measurement on D1-- either A2-1 or A2-2-- and use the result to deduce corresponding information about D1's partner in the new composite system: the particle.[17] Bohr emphasizes that, if A2-2 is used, the possibility of also using the output of D1 to produce interference patterns by subsequently passing this output through another diaphragm with two or more slits in it, no longer exists:

> [E]ven the minimum uncertainty of the position of the first diaphragm [D1 associated with the sharp measurement of P_y] . . . will imply the total wiping out of any interference effect . . . to which the presence of more than one slit in the second diaphragm would give rise in case the positions of all apparatus are fixed relative to each other [as in case A1, discussed above]. (Bohr, 1935, 698)

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[15] For Bohr, a phenomenon is a complete process, consisting of an initial preparation of a system, its subsequent interactions, and a final registration of the result of some measurement on it. Any attempt to physically subdivide a phenomenon (for example, by an intermediate measurement) results in a different phenomenon. See the final section for further discussion of Bohr's concept.

[16] Elsewhere, Bohr made this point explicitly: "If we want to use the idea of space-time we must have watches and rods which are outside and independent of the object under observation…" (Bohr, 1931 lecture at University of Bristol, cited from Dickson 2002a).

[17] “The principle difference between the two experimental arrangements ... is ... that in the arrangement suited for the control of the momentum of the first diaphragm [i.e., D1 in arrangement A2], this body can no longer be used as a measuring instrument for the same purpose as in the previous case [i.e., D1 in

As far as questions of principle are concerned, arrangement A2 is really quite analogous to the EPR arrangement: Two systems, the particle and diaphragm D1, initially non-interacting before the particle's passage through the slit in D1, interact for the limited time interval it takes the particle to pass through the slit; and remain entangled even though the particle and D1 do not interact physically afterwards. In order to extract information about the particle from D1, a further measurement on it is necessary; and here the alternate arrangements A2-1 and A2-2 enter the story. Although Bohr does not stress the point, the analogy with the two particles of EPR is really quite striking. [18]

With detailed discussion of the one-particle case behind him, Bohr is able to make short work of the two-particle EPR case by means of a third experimental arrangement (A3), utilizing a two-slit diaphragm (D2) that (like the one-slit diaphragm in arrangement A2) is free to move. Bohr describes A3 as:

> a simple experimental arrangement, comprising a rigid diaphragm with two parallel slits [D2], which are very narrow compared to their separation, and through each of which one particle with given initial momentum passes independently of the other. If the momentum of this [mobile] diaphragm is measured accurately before as well as after the passing of the particles, we shall in fact know the sum of the components [of the particle momenta] perpendicular to the slits" (Bohr 1935, 699).

Some further explication may be helpful. The purpose of A3 is to "realize," by means of an ideal experimental arrangement, the wave function initially introduced by EPR. By making measurements of the y-components of the two particles' momenta before passage through

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arrangement A1], but must, as regards its position relative to the rest of the apparatus [i.e., the frame of reference], be treated, like the particle traversing the slit, as an object of investigation..." (Bohr 1935, 698).

[18] Of course, if one does not see the force of this analogy, it may appear that Bohr has never really came to grips with the EPR thought-experiment; we discuss this objection below.

D2, and of the y-component of D2's momentum before and after their passage; and then invoking conservation of momentum, we are able to obtain the y-component of the total momentum of the two particles after passage through the slits. Or to put it more accurately, this y-component is QM-determinate if an apparatus is subsequently introduced to complete the phenomenon by its measurement.

Since they passed through the two slits, the difference in their q_y 's—but not their individual q_y 's, since D2 is mobile—just after passage through the slit is also determinate.[19] But this is just a physical realization of the state described by the EPR wave function, in which both the y-component of their total momentum and the difference between the y-components of their positions are determinate.

A source of possible misunderstanding is to assume that Bohr is treating arrangements A1 and A2 symmetrically; i.e., that, just as A1 can be used only to measure the position q_y of the particle, so A2 can be used only to measure its momentum p_y -- in other words, to assume that Bohr treats A1 and A2 as mutually exclusive, complementary arrangements.[20]

As we have just seen, this is not the case. Arrangement A1 can be used *only* to measure a position component since D1 is part of the measuring apparatus, and thus one

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[19] The requirement that slit widths be small compared to the distance between them assures that the Δq_y's are small compared to the difference between the two q_y's.

[20] Indeed, Beller captions a reproduction of Bohr's drawings: "The rigidly bolted apparatus (*center*) is for position measurement. The apparatus suspended by weak springs (*top*) is for momentum measurement" (Beller 1999, 148); and elaborates in the text: "In addition to the first two-slit diaphragm, we employ a second diaphragm suspended by weak springs or rigidly bolted, depending on whether we intend to measure the position or momentum of the first particle. … For the measurement of P_1 we use the movable diaphragm, so we exclude in principle the possibility of measuring Q_1 …" (Beller 1999, 147). Not only Bohr's critics commit this error. In his defense of Bohr against charges of positivism, Hooker 1972 writes: "[W]hen the diaphragm is not rigidly attached to the coordinate framework, there is no question of applying the concept of position (in the direction perpendicular to [particle] A's trajectory) to the A component's passage through the slit. Hence, there is no question of applying this concept of position to the situation as a whole at all" (223). The same misunderstanding can also be found in Jammer 1974, 95 and 196, and Howard 1994, 211-216. In contrast, Murdoch (Faye and Folse 1993, 308; Murdoch 1987, 168-170) states it correctly.

does not have any choice of how to use it after the particle passes the slit. In order to measure a momentum component, one must have recourse to another arrangement. But, instead of a *complementary* arrangement that would enable *only* a measurement of p_y, Bohr introduces the second arrangement A2, in which D1 itself can be used subsequently *either for* a position *or* a momentum measurement. A2-1 and A2-2 are *truly* complementary to each other; but involve the introduction of *additional* arrangements (such as D2 in the EPR case).[21]

Of course, Bohr 1935 bears some responsibility for the possibility of this misinterpretation. It might *seem* that he regards the two arrangements A1 and A2 as symmetrical; although, if the text is examined carefully, it is readily seen that they are not--and nowhere does he imply that they are. When he does talk about two arrangements, one of which can be used only to measure position and the other only to measure momentum, rather than A1 and A2, he is always referring to the two sub-arrangements A2-1 and A2-2 (otherwise, his "either . . . or" would not be exclusive). Such possibly ambiguous expressions aside, Bohr does clearly and explicitly point out that A2 can be used for *both* kinds of measurements, and that one still has freedom of choice *after* passage of the particle through D1.[22]

Bohr returned to the EPR paper in his account of discussions with Einstein (Bohr [1949]1958), and this later version is often much clearer and more explicit. He writes:

> In fact, after a preliminary measurement of the momentum of the diaphragm, we are in principle offered the choice, when an electron or photon has passed through

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[21] Presumably Bohr wanted to demonstrate that D1 could also be used for a momentum measurement, but could not think of any way of using it *exclusively* for a momentum measurement. At any rate, *we* have not been able to do so.

[22] See the citation in Note 12.

> the slit, either to repeat the momentum measurement or to control the position of the diaphragm, and, thus, to make predictions pertaining to alternative subsequent observations. (Bohr [1949] 1958, 57)

Later, he repeats this point (Bohr [1949]1958, 60).[23] It is clear that, for Bohr in 1949 just as in 1935, the movable diaphragm can be used for either a momentum-component or a position-component measurement, and that one has free choice of which to measure *after* passage of the particle through the diaphragm.

This point is important for Bohr's reply to the EPR paper. Freedom of choice *after* passage of the particle is closer to EPR’s alternative measurement requirement (i.e., one could decide to make either a position or a momentum measurement) than would be such freedom *before* passage of the particle. More importantly, assuming that A1 and A2 are symmetrical is simply wrong physics. Both in the case of classical mechanics (CM) and QM, in arrangement A1, one can measure only a position component with D1; while in both CM and QM, in arrangement A2, one can make a position and a momentum component measurement with D1. In this case, the "only" difference between CM and QM is that, according to the former, we can make *both* measurements using one experimental arrangement; while, according to the latter, we must make a choice between either A2-1 or A2-2, the possibility of making the other being thereby excluded.

3. The role of quantum of action, the idea of the cut, and an implicit use of entanglement.

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[23] After presenting her misinterpretation of Bohr's argument (see note 20), Beller goes on to cite these lines of Bohr (Beller 1999, 147), suggesting that: "Two different, even incompatible, answers are concurrently present in Bohr's response to EPR" (Beller 1999, 146). There are indeed "two different, even incompatible

Now we come to the crucial question: Why this difference between CM and QM? Is it merely that a measurement of one quantity so disturbs the measured system that it is unusable for a second measurement of a different quantity? Indeed, even in CM, a classical-mechanical disturbance during a measurement is often inevitable, so that subsequent measurement of a second quantity gives a different result than if that quantity had been measured first. But such a classical disturbance can be "controlled" in the sense that: either the effect of the disturbance can be made negligible by some appropriate modification of the first measurement procedure; or, with the help of physical theories, the quantitative effect of this disturbance can be calculated and subtracted from the second measured result to obtain the answer that would have been obtained if the first measurement had not been made. So, classical, "controllable" disturbances cannot be the reason for this difference.

According to Bohr, what is unique to the case of QM is that the disturbance is "uncontrollable" in principle. Indeed, he uses the word "uncontrollable" again and again throughout his writings. According to Bohr, it is this "uncontrollable" nature of the quantum exchange between the measured system and the measuring instrument that makes them into a whole that cannot be further subdivided.

The reason that the quantum exchange is uncontrollable is not a question of the "accuracy of measurements," but lies in the existence of Planck's quantum of action *h* (Bohr 1963, 5). *Because of the existence of the quantum of action*, the exchange between system and measuring instrument becomes uncontrollable.[24] [25] The quantum of action

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answers" in her text, but only one of them is Bohr's. See Dickson 2002b for further comments on Beller's treatment of Bohr.

[24] "[T]he *finite interaction between object and measuring agencies* conditioned by the very existence of the quantum of action entails– because of the impossibility of controlling the reaction of the object on the

cannot be explained by or derived from a classical or quasi-ordinary disturbance picture. Rather, the existence of *h* is responsible for, is revealed in, and is presupposed by, the nature and limits of possible measurements in QM. Similarly, the indeterminacy relations and other QM characteristics, such as "the necessity of a final renunciation of the classical ideal of causality and a radical revision of our attitude towards the problem of physical reality," also arise from the existence of the quantum of action (Bohr 1935, 697). For Bohr, the existence and efficacy of the quantum of action is the most fundamental feature of QM, which cannot be reduced to any other empirical facts. Bohr states that one must treat

> the quantum of action as an element evading customary explanation. . . similar to the role of the velocity of light in relativity theory as a maximal speed of signals" (Bohr 1963, 12).

In the case of Bohr's diaphragms, the uncontrollable, "finite" interaction is therefore the result of the existence of the quantum of action. Applying this general principle to his experimental arrangements, Bohr argues:

> The principle difference between the two experimental arrangements under consideration is, however, that in the arrangement suited for the control of the momentum of the first diaphragm [i.e., the second arrangement A2], this body can no longer be used as a measuring instrument for the same purpose as in the previous case, but must, as regards its position relative to the rest of the apparatus, be treated, like the particle traversing the slit, as a system of [i.e., under] investigation, in the

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measuring instruments if these are to serve their purpose– the necessity of a final renunciation of the classical ideal of causality..." (Bohr 1935, 697).

[25] In a highly problematic presentation of the EPR debate (we only point out a few problems closely related to our topic), Fine 2004 keeps criticizing Bohr's use of "the language of disturbance." However, he fails to make clear the essential distinction Bohr makes between controllable classical disturbances and uncontrollable quantum disturbances; and hence doesn't explain what makes quantum disturbances uncontrollable (see Section 4).

> sense that the quantum-mechanical uncertainty relations regarding its position and momentum must be taken explicitly into account. In fact, even if we knew the position of the diaphragm relative to the space frame [of reference] before the first measurement of its momentum, and even though its position after the last measurement can be accurately fixed, we lose, on account of the uncontrollable displacement of the diaphragm during each collision process with the test bodies, the knowledge of its position when the particle passed through the slit. The whole arrangement is therefore obviously unsuited to study the same kind of phenomena as in the previous case. (Bohr, 1935, 698)

Taken in isolation, the last sentence could be taken to mean that the second arrangement can be used only to study momentum, but not position. However, continuing with Bohr's own words (parts of which were quoted above in the discussion of D2), one sees that the different phenomena Bohr is talking about are the presence of interference effects, or the lack of them, if a diaphragm with several slits (D2) and a photographic plate are placed behind D1. In short, Bohr is arguing here that the first diaphragm D1 in the second arrangement A2 has to be treated as part of the quantum system, and hence the uncertainty relations applied to D1.

His argument involves several interesting points: First, it is wrong to assume that the role of an object as part of the measuring instrument or the measured system is uniquely fixed. In the arrangement A1, diaphragm D1 is part of the measuring instrument; while in A2, the same diaphragm D1 is part of the quantum system and subject to measurement by other instruments in other possible experimental arrangements. He gives a clear physical criterion for when an entity may be treated as a measuring instrument and when it must be considered part of the quantum system: It depends on whether the "quantum-mechanical uncertainty relations"-- or more generally, the effects of the quantum of action-- may be overlooked or must be applied to the treatment of the entity in question.

> In fact it is an obvious consequence of the above argumentation that in each experimental arrangement and measuring procedure we have only a free choice of this place [where the discrimination is made between object and measuring apparatus] within a region where the quantum-mechanical description of the process concerned is effectively equivalent with the classical description (Bohr 1935, 702).

The measuring apparatus as a whole must be sufficiently complex to record an irreversible change as a result of its interaction with the quantum system;[26] but this does not imply anything about the size of an entity that forms a part of the total measuring apparatus: even a "small" micro-system can be included as a measuring instrument if, in the phenomenon under investigation, one does not have to take quantum effects into account when dealing with this micro-system. Conversely, if such quantum effects have to be taken into account, then an object that is usually taken as a measuring instrument—even one that is macroscopic in size and mass, as is the case with D1[27]—has to be considered part of the quantum system.[28]

Second, Bohr argues that, after passage of the particle, one still has freedom of choice to determine either a position or a momentum component of the particle by

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[26] "Every atomic phenomenon is closed in the sense that its observation is based on registrations obtained by means of suitable amplification devices with irreversible functioning such as, for example, permanent marks on a photographic plate, caused by the penetration of electrons into the emulsion" (Bohr [1949]1958, 73).

[27] See Bohr's drawing of it.

[28] Hence, Beller and Fine are not quite accurate, when they state: "for him [Bohr] the measuring device must-in principle-be 'heavy' and classical" (Beller and Fine 1994, 27). Fine 2004 makes a similar error, claiming that the picture of Bohr's diaphragms is "a tiny object banging into a big apparatus."

measuring either the position or momentum component of the diaphragm. In "state" language, this implies that a measurement on the diaphragm immediately determines the state of the particle, which may have separated completely from the diaphragm. In more contemporary terminology, after its passage through the diaphragm, particle and diaphragm are *entangled*.[29] As we have seen, the concept of quantum entanglement is exactly what is at issue for EPR. Although Bohr does not use the word entanglement, he was nevertheless well aware of the concept, which is implicit in his definition of a phenomenon (see the next section).[30] Indeed, as has been seen, he takes this concept for granted and utilizes it in his response. One may criticize Bohr for not expressing this awareness more explicitly and answering the EPR challenge more straightforwardly and in more detail; but one cannot justly claim that he was unaware of it.

Beller and Fine challenge the analogy between the Bohr's example of particle-diaphragm interaction and EPR's example of particle-particle interaction, claiming that

> Bohr’s "wholeness" cannot extend to composite systems consisting of ` micro-systems, who [sic] might be said to lose their individuality when their state functions become entangled (Beller and Fine 1994, 27).

Their argument seems to be: In the former case, one of the entangled systems—the diaphragm—is a macro-system, and Bohr’s wholeness criterion is applied to the relation between macro-system and micro-system; while the EPR paper involves a micro-subsystem--micro-subsystem entanglement within a composite micro-system. They conclude that Bohr did not have the conceptual tools to handle the EPR problem.

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[29] The term entanglement was introduced in Schrödinger 1935.

[30] See also ~~~, Chapter V.

But, as discussed above, the “macro and micro” distinction is not the same as Bohr’s "measuring instrument and system" distinction. The mobile macro-diaphragm is part of a composite quantum system. And the key quantum phenomenon that EPR challenge is entanglement: it does not really matter whether it is entanglement between two micro-systems or between a micro-system and a macro-system.

4. Bohr's concepts of phenomenon and complementarity.

However, a question still remains. Did Bohr adopt a “quantum disturbance view” after all? That is, did he believe that, although-- in contrast to the classical disturbance view-- the source of “uncontrollable” disturbances is quantum-mechanical, a system may still have a determinate position *before* a sharp position measurement has been arranged even though QM does not attribute one to it?[31] Did he maintain that, in spite of this, QM is not incomplete because the existence of the quantum of action makes knowledge of this position inaccessible to human beings, and what counts for physics is only what is measurable by man? If Bohr did espouse this viewpoint, then his answer to EPR would indeed be positivistic.

But Bohr’s view is quite different. For him, a quantum system does not have a position unless it is part of a process defining one. Position is a (system-instrument) holistic concept. Separated from such a process, talk about the position of a quantum (macro- or micro-) system makes no sense. Absolutely crucial to an understanding of Bohr's approach is a clear understanding of his concept of a quantum phenomenon; from it follows the role of complementarity in the "proper study of quantum phenomena." Bohr states:

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[31] Rather than Bohr's view, this is Heisenberg's (see the last section). Talk of a single "Copenhagen interpretation" has led to frequent conflation of their two differing viewpoints.

> I advocated the application of the word phenomenon exclusively to refer to the observations obtained under specified circumstances, including an account of the whole experimental arrangement (Bohr [1949] 1958, 64).

To expatiate on Bohr, a phenomenon consists of a total process, which includes preparation of a system, subsequent interactions, and final registration of some quantity associated with the system. The word "measurement" is often used in connection with the acts of preparation and registration.

> The essential lesson of the analysis of measurements in quantum theory is thus the emphasis on the necessity, in the account of the phenomena, of taking the whole experimental arrangement into consideration, in complete conformity with the fact that all unambiguous interpretation of the quantum mechanical formalism involves the fixation of the external conditions, defining the initial state of the atomic system concerned and the character of the possible predictions as regards subsequent observable properties of that system. Any measurement in quantum theory can in fact only refer either to a fixation of the initial state or to the test of such predictions, and it is first the combination of measurements of both kinds which constitutes a well-defined phenomenon (Bohr [1939] 1998, 101).[32]

These conditions, including an account of the properties and manipulation of ideal measuring instruments involved in preparation and registration, constitute in fact the only basis for the definition of the concepts used to describe the phenomenon.

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[32] ### Bohr' concept of "phenomenon" is quite similar to Feynman's concept of "process." Bohr's attitude to the usual state-vector formalism is also close to Feynman's, and Feynman's emphasis on calculation of the probability amplitude for a process as the fundamental task of quantum mechanics fits well with Bohr's approach.

Bohr's understanding of phenomena, together with his understanding of the nature of quantum disturbance, his idea of (the limited arbitrariness of) the cut, and his implicit reference to what is now called entanglement, are crucial to an appreciation of Bohr’s central argument in response to EPR. According to Bohr, a sharp measurement of system I exerts “*an influence on the very conditions which* … constitute an inherent element of the description of any phenomenon” involving system II (Bohr 1935, 700).

Fine 2004 completely misreads Bohr’s crucial argument (also see Beller and Fine 1994): First, this conditional disturbance becomes an “informational disturbance,” and the notion of phenomenon (he uses the term “relational property”) becomes merely a linguistic and semantic one. Second, as seen earlier in this section and the last, the reason why a measurement on system I is a condition of the phenomenon involving system II, is what is now known as “entanglement” or *non-separability* of the two systems, to which Bohr implicitly refers; but, according to Fine, Bohr has to appeal to *non-locality*. In addition to this conflation, Fine also conflates quantum non-separability or entanglement with the kind of harmless classical non-locality shown in the example he offers: “becoming the ‘best’ when your only competitor – who might be miles away – fails.”

With the concept of phenomenon so understood, it follows then that any attempt to subdivide a phenomenon physically (for example, by an intermediate measurement) results in a different phenomenon:

> [I]n the phenomena concerned we are not dealing with an incomplete description characterized by the arbitrary picking out of different elements of physical reality at the cost of sacrificing other such elements, but with a rational discrimination between essentially different experimental arrangements and procedures which

> are suited either for an unambiguous use of the idea of space location, or for a legitimate application of the conservation theorem of momentum … Indeed we have in each experimental arrangement suited for the study of proper quantum phenomena not merely to do with an ignorance of the value of certain physical quantities, but with the impossibility of defining these quantities in an unambiguous way (Bohr 1935, 699).

Fine 2004 quotes a related passage by Bohr. Commenting on the double slit experiment, he says:

> If we only imagine the possibility that without disturbing the phenomena we determine through which hole the electron passes, we would truly find ourselves in irrational territory…".

What Bohr is arguing against is the idea that we can physically subdivide a phenomenon without altering it (i.e., without replacing the original phenomenon with new ones), asserting that this would indeed land us "in irrational territory." But Fine calls this "the language of disturbance" and says "Bohr defends locality and regards the very contemplation of non-locality as 'irrational' and 'completely incomprehensible'....". Fine's interpretation of the whole passage again conflates a form of non-separability (with two slits open, the electron cannot be said to go through either) with non-locality (if the electron *did* go through one of the two slits, it *would* be influenced by whether the other slit is open or not). As mentioned above, he then uses Bohr's apparent denial of non-locality to argue that Bohr cannot endorse a relational view, which is based upon non-locality. But he is wrong, for the relational view is actually based upon non-separability.

To be clear, Bohr did not argue that *all* physical properties have to be defined by way of phenomena. For example, he never challenged the idea that certain physical properties of a quantum system, such as the mass, charge, and spin of the elementary particles, were intrinsic and independent of any phenomenon (or process). This fact poses yet another challenge to Fine's metaphysical reading of Bohr that attempts to universalize (Fine's misunderstanding of) his concept of phenomenon and turns it into a universal positivist ontology that covers all physical properties.

It is sometimes claimed that Bohr's views on these matters only developed after, and in response to, EPR. Of course, some evolution of his views is only natural. Yet, Bohr [1932] 1998, contains very similar ideas:[33]

> [T]he unambiguous application of such fundamental concepts as space and time is essentially limited on account of the finite interaction between object and the measuring tools, which, as a consequence of the existence of the elementary quantum [of action], is involved in any measurement. …[W]e must remember that this interaction cannot be taken fully into account in the description of the phenomena, since the very definition of the space time-frame implies the negligence of the reaction of the objects on the measuring instruments. … Inversely, every application of conservation theorems … involves an essential renunciation as regards the pursuance in space and time of the individual atomic particles. …

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[33] Hooker 1972 observes: "[T]here is no suggestion, that I can detect, that EPR did alter Bohr's conception of quantum theory. … It may … be true that, as Bohr himself seems to allow, Einstein's penetrating criticisms of quantum theory served to crystallize the elements of the doctrine of complementarity, giving impetus to a more precise development and to the broadening of their scope" (149).

> [T]he viewpoint of "complementarity" in the description of atomic phenomena is forced upon us by the existence of the quantum of action … [S]pace time co-ordination and dynamical conservation laws may be considered *as two complementary aspects of ordinary causality* which in this field exclude one another to a certain extent, although neither of them has lost its intrinsic validity. (Bohr [1932] 1998, 53-55 passim).

As for "visualizability," if we understand it, as did Bohr, as the kind of picture that CM offers—a causal description of a process in terms of momentum and energy that also provides information about a system's position in space and time, all of which are determinate—then, we have to say that no classical visualization or picturing of quantum phenomena is possible. QM can only offer such pictures in a complementary fashion. That is, we can either have position as a function of time, i.e., a well-defined spatio-temporal description, defined by the specification of a corresponding experimental arrangement allowing a continuous monitoring of position, at the sacrifice of any application of classical concepts of causality; or we can have well defined momentum and energy, with their conservation laws obeyed as demanded by classical causality, but only by the complete sacrifice of a spatio-temporal description of this process. No single QM picture can unite both descriptions in a single process, and in this sense the quantum world cannot be visualized. This also explains why Bohr avoided Heisenberg's early disturbance language, especially in Bohr's later writings. He argues that it is misleading to formulate Heisenberg's indeterminacy relations

> by such a statement as: "the position and momentum of a particle cannot simultaneously be measured with arbitrary accuracy." According to such a

> formulation it would appear as though we had to do with some arbitrary renunciation of the measurement of either the one or the other of the two well-defined attributes of the object, which would not preclude the possibility of a future theory taking both attributes into account on the lines of the classical physics. From the above considerations it should be clear that the whole situation in atomic physics deprives of all meaning such inherent attributes as the idealization of classical physics would ascribe to the object. (Bohr, 1937, 292-3)

To restate what was said above, concepts such as position are not applicable to an "isolated" quantum system, i.e., to one treated in isolation from an appropriate process, including some preparation. This position might seem closer to the one held by Pascual Jordan, who claimed that a position measurement creates a position. But Bohr shies away from this kind of language, in fact criticizing this “creation” view:

> The unaccustomed features of the situation with which we are confronted in quantum theory necessitate the greatest caution as regards all questions of terminology. Speaking, as is often done, of disturbing a phenomenon by observation, or even of creating physical attributes to objects by measuring processes, is, in fact, liable to be confusing, since all such sentences imply a departure from basic conventions of language which, even though it sometimes may be practical for the sake of brevity, can never be unambiguous.[34] It is certainly far more in accord with the structure and interpretation of the quantum mechanical symbolism, as well as with elementary epistemological principles, to reserve the word "phenomenon" for the comprehension of the effects observed under the given

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[34] Fine 2004 quotes the passage up to here, but claims that Bohr’s (justified) refusal to use either “disturbing” or “creating” shows the latter’s difficulty in framing concepts that can be used to derail the EPR challenge.

> experimental conditions.
>
> These conditions, which include an account of the properties and manipulation of all measuring instruments concerned, constitute in fact the only basis for the definition of the concepts by which the phenomenon is described (Bohr [1939] 1998, 104).

Fifteen years later, he repeated and amplified the same thought:

> In this context, one sometimes speaks of "disturbance of phenomena by observation" or "creation of physical attributes to atomic object by measurements." Such phrases, however, are apt to cause confusion, since words like phenomena and observation, just as attributes and measurements, are here used in a way incompatible with common language and practical definition. On the lines of objective description, it is indeed more appropriate to use the word phenomenon to refer only to observations obtained under circumstances whose description includes an account of the whole experimental arrangement. In such terminology, the observational problem in quantum physics is deprived of any special intricacy and we are, moreover, directly reminded that every atomic phenomenon is closed in the sense that its observation is based on registrations obtained by means of suitable amplification devices with irreversible functioning such as, for example, permanent marks on a photographic plate, caused by the penetration of electrons into the emulsion. (Bohr 1958, 73)

In summary, we can now see that Bohr's use of diaphragms is not the result of a positivist tendency, as has sometimes been claimed. He does not believe that theoretical concepts like position and momentum are too abstract, and that such theoretical talk should be eliminated by reducing all such concepts to sensations or empirical data, or by defining them in terms of an “ordinary” experimental set-up. Rather, for Bohr, there is no problem in using such theoretical concepts in either CM or QM. But in the latter case, we must not use them without specifying a phenomenon (in Bohr's sense), because attributes such as position and momentum inherent in classical systems, which may be treated as isolated, are not inherent in quantum systems, which cannot be treated in isolation. In short, the reason Bohr appeals to experimental arrangements, such as diaphragms, is not metaphysical or semantic but *physical*; the challenge to physics posed by the existence of the quantum of action.

5. A few final issues.

Having explained and defended Bohr’s experimental arrangements, we shall conclude by discussing three further issues raised by his diaphragms and by clarifying his general attitude to the role of experience. First, one might ask: How it is possible that the position of a macro system such as a diaphragm is indeterminate? At first sight, this might seem counter-intuitive;[35] but the point is that "position" here means "position with respect to a fixed inertial frame of reference,"[36] which is associated physically with a massive system[37] that is quite independent of the diaphragm. If the diaphragm is rigidly attached to the frame, then its position is indeed fixed in the above sense. But when it is capable of

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[35] When one recollects that the closest classical analogue to a quantum system is a canonical ensemble (see, e.g., Stachel 2006), then it seems less counterintuitive, since even for a rigid macroscopic body there will be a distribution of positions associated with an ensemble having a sharp momentum, and vice versa.

[36] See Dickson 2004 for an extensive discussion of "the role that reference frames play in the definition of physical concepts in quantum theory."

[37] It must be massive for two reasons: it must be so massive that absorbing uncontrollable recoil energy and momentum do not appreciably effect its own energy and momentum; and so massive that, when comparing it to other inertial frames (in such comparisons it is treated as a system subject to the uncertainty relations),

moving relative to the inertial frame, if it is treated as a quantum system its position relative to the frame is subject to effects of the uncertainty relations. Even if its mass is macroscopic, if it is small compared to the mass(es) of the object(s) defining the frame of reference, these effects need not be negligible.

Moreover, it should be remembered that Bohr's experimental arrangements are part of thought experiments, involving idealized apparatus. As he himself explicitly points out, the obvious impossibility of actually carrying out these thought experiments

> does clearly not affect the theoretical argument, since the procedures in question are essentially equivalent with atomic processes, like the Compton effect, where a corresponding application of the conservation theorem of momentum is well established (Bohr 1935, 698, footnote).

Bohr and Rosenfeld 1933 make a related point about any theory: The need to assure a complete accord between quantities *defined* as observable in a theory and what is measurable *in principle*. This is important, not because the definition of a quantity is based on its measurability (operationalism); but to assure that theory and (ideal) measurement are consistent. Bergmann and Smith 1982 call this criterion "measurability analysis." [38] "Measurability in principle" implies that the measuring devices introduced need not be realistic, but their construction must be consistent with the theory to which they are being applied. In other words, far from implying a positivist or operationalist approach, ideal measuring instruments are themselves theoretical tools. For example, in order to show the measurability in principle of the vacuum electric and magnetic field components, Bohr and

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attributing a position and velocity to it does not fall foul of the uncertainty relations: $\Delta V = (\Delta P)/M$, so for sufficiently large M, even a considerable ΔP causes no appreciable change in its velocity.

[38] They emphasize this criterion is quite distinct from the so-called "measurement problem" in quantum mechanics, which arises when the state function is interpreted ontologically.

Rosenfeld 1933 invoke massive charged test bodies while totally neglecting their atomic constitution and the quantization of their charge, insisting that:

> In our discussion of the limitations of the measurability of field quantities these difficulties play no role, since for this purpose the atomistic structure of matter is not an essential issue. It is true that the measurement of fields requires the use of material charged test bodies, but their unambiguous application as measuring instruments depends exactly on the extent to which we can treat their response to the fields as well as their influence as field sources on the basis of classical electrodynamics (Bohr and Rosenfeld 1933 [1979], 360).

This discussion concerns special-relativistic quantum fields but, suitably modified, their comments apply to non-relativistic quantum mechanics. We might mention for example, that Bohr 1935 assumes the possibility of completely rigid diaphragms and a rigid reference support, and of a completely rigid attachment of the one to the other-- all impossible to realize in practice or special relativistic theory; but quite consistent theoretical constructs within Newtonian classical mechanics.

Second, as pointed out several times, Bohr's arrangements require that, if the position measurement on one particle is to be used to define the corresponding position coordinate of the other particle, the position measurement be made immediately after the particle's passage through the two-slit diaphragm, which defines the difference in the x-coordinates of the two particles, because this difference is not a conserved quantity. This is indeed a difficulty, but it is not fair to hold Bohr responsible.[39] It was EPR who introduced the wave function depending on the sum of momenta and difference of positions; and Bohr was just showing the consequences of preparing a system with this wave function. An experimentally feasible arrangement of the EPR type was first proposed by Bohm 1951, who replaced these two quantities with two conserved spins.[40]

Third, there is a real complication, not stressed by Bohr, in applying Bohr's diaphragms or some arrangement of this sort to the EPR case. He states:

> If the momentum of this [double slit] diaphragm is measured accurately before as well as after the passing of the particles, we shall in fact know the sum of the components perpendicular to the slit of the momenta of the two escaping particles, as well as the difference of their initial positional coordinates in the same direction. … In this arrangement, it is therefore clear that a subsequent single measurement either of the position or of the momentum of one of the particles will automatically determine the position or momentum, respectively, of the other particle with any desired accuracy… (Bohr 1935, 700).

Bohr does not work out the relation between the determinate quantities. If one does so, an interesting point arises. To illustrate it, we present the details for the momentum case, After the two particles pass through the mobile double slit diaphragm and its recoil momentum has been determined, the two particles are entangled: Their total momentum is determinate, as is the distance between them. Suppose particle 1 is subsequently passed through a mobile single slit diaphragm initially at rest, then the momenta of both the diaphragm and of particle 1 after its passage must be measured. The interesting point, which is not mentioned by Bohr, is the need to measure the latter quantity. From this data and conservation of momentum, one can now determine the momenta of both particles

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[39] See Beller and Fine 1994, 14-15.

[40] One of us ($$$) is grateful to Dr. Tilman Sauer of the Einstein Papers for informing him that "I found a version of the EPR incompleteness argument, apparently very late: late 1954 perhaps, on the bottom half of

after they passed through the double slit diaphragm and *before* one of them entered the single slit diaphragm.

The calculation goes as follows:

Single slit diaphragm: momentum before passage = 0, after passage = P_D, both known

Particle 1: momentum before passage = p_1, unknown; after passage = $p_{1'}$; this is the quantity needed, but not mentioned in Bohr 1935.

Particle 2: momentum before = p_2, unknown

Conservation of momentum after passage of particle 1through the single slit diaphragm:

$$p_1 = P_{\mathrm{D}} + p_{1'}$$

Sum of momentum of two particles, known from measurement of momentum of double slit diaphragm:

$$P = p_1 + p_2$$

which is conserved and conserved until particle one passes through the single slit diaphragm, which decouples (disentangles) the two particles.

It follows that:

$$p_2 = P - p_1 = P - (P_{\mathrm{D}} + p_{1'})$$

To recapitulate, *two* measurements *after* passage of particle 1 through the single slit diaphragm are needed to determine the values of p_1 and p_2 *before* passage through the diaphragm.

Finally, a general comment is in order here. We have already defended Bohr's analysis of the EPR experiment from its characterization as positivist. But some of his more general comments on the role of measurement and experiment might lead people to

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one of his U[nified] F[ield] T[heory] calculations. In it he discusses the argument in terms of spin observables" (private communication).

read him in a positivist way:

> The extent to which an unambiguous meaning can be attributed to such an expression as "physical reality" cannot of course be deduced from *a priori* philosophical conceptions, but-- as the authors of the article cited themselves emphasize-- must be founded on a direct appeal to experiments and measurements. (Bohr 1935, 696)

But, as Bohr indicates, in fact it was EPR, the "realists," who introduced this point. EPR claim:

> The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. (Einstein et al. 1935, 777)

Moreover, the view Bohr and Einstein share here is *not* that *only* what is measurable exists, the idea that "a physical property of a given system has reality only when it is actually measured," the position Beller and Fine accuse Bohr of holding (Beller and Fine 1994, 9-10, 27 and 29). Rather, his (and Einstein's) view is that what exists must be measurable, or more accurately, must have measurable consequences. The key point here is that talk of experiments, measurements, instruments, operations, observations or experiences in general does *not necessarily* imply a positivist position. The issue, rather, is whether experiments and the like are assigned an absolutely privileged position; whether everything in a theory has to be reducible directly to experiences in order to be accepted as real. For Bohr, as well as for EPR, being measurable or "experiment-able" is a sufficient but not a necessary condition for the existence of something or the meaningfulness of the corresponding concept. It may be capable of direct exhibition through experiments, or more generally in experience; but it may only be capable of indirect exhibition. For example, a theoretical "posit" may be related to other elements of a theory in such a way that it is only the organic whole that is directly connected with observation and experiment, priority being given to the existence of the "theoretical" entity (or entities) behind the measurement, and not necessarily to the measurement. The "theoretical" entity is *manifested* in the experiment, but is *not defined or determined* by it. Such a "tempered" emphasis on experience does not, therefore, imply positivism. One can say that Bohr and Einstein are both empirical in their approach to physical reality, but they are not empiricists.

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