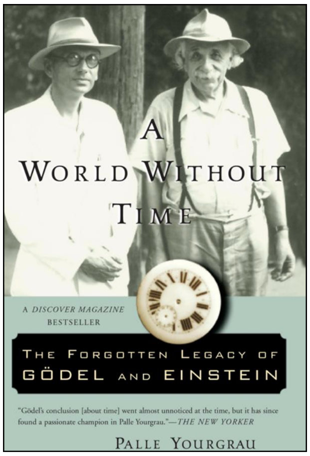

A World Without Time: The Forgotten Legacy of Gödel and Einstein

Reviewed by John Stachel

A World Without Time: The Forgotten Legacy of Gödel and Einstein
Palle Yourgrau
Hardcover, Basic Books, 2004,
US$24.00, 224 pages
ISBN-13: 9780465092932
Paperback, Perseus Publishing, 2006
US$15.00, 224 pages
ISBN-13: 9780465092949

The book's title suggests its three main themes:

1) "A World Without Time": Gödel's argument, based on his interpretation of the theories of relativity (both special and general), for the "unreality" of time. At a generous estimate, no more than forty of the book's 180-odd pages are devoted to this theme (essentially the last part of Chapter 6, and Chapter 7).

2) "Gödel and Einstein": An attempt to draw parallels between the lives and views of its two protagonists. An account of Gödel's life in Vienna (Chapters 3–5) includes a lengthy excursus into his seminal contributions to logic (Chapter 4). The account of the relationship between the two after Gödel's permanent move to the Institute for Advanced Study in 1940 (Chapters 1, 6, and 8; the last also discusses Gödel's final years) includes brief glimpses of Einstein's pre-Princeton years.

3) "The Forgotten Legacy": Yourgrau's polemic against what he sees as the neglect by the analytically-oriented American philosophical establishment of Gödel's significant contributions to metaphysics (the last part of Chapter 8 and Chapter 9). Insofar as Einstein is presumed to share Gödel's "German Bias for Metaphysics" (the title of Chapter 2), he is also portrayed as a victim of this "Conspiracy of Silence" (the title of Chapter 1).

John Stachel is professor of physics at Boston University. His email address is `stachel@buphy.bu.edu`.

The Forgotten Legacy

Yourgrau, himself a philosopher, has been urging recognition of Gödel as "an important philosopher of mathematics and of space and time" (p. 181) for almost two decades. He regards "the dialectic of the formal and the intuitive" as "the leitmotif of Gödel's lifework" (p. 124), seeing both continuity and contrast in this work.

There is continuity in method: "Overarching much of his research in philosophy and logic was the 'Gödel program', the investigation of the limits of formal methods in capturing intuitive concepts" (p. 182; see also pp. 114, 127).

The contrast lies in the conclusions Gödel drew from the existence of these limits: In mathematics, he "concluded from the incompleteness of Hilbert's proof-theoretic system for arithmetic that the Platonic realm of numbers cannot be fully captured by the formal structures of logic. For Gödel, the devices of formal proof are too weak to capture all that is true in the world of numbers, not to say in mathematics as a whole." (p. 136)[1]. But in physics: "When it came to relativistic cosmology, however, he took the opposite tack…[R]elativity is just fine, whereas time in the intuitive sense is an illusion. Relativity…does not capture the essence of intuitive time, because when it comes to time, our intuitions betray us" (pp. 136–137).

[1] *Feferman 2006 points out that: "The incompleteness theorems in and of themselves do not support mathematical Platonism," as Gödel admitted in 1951: "Of course I do not claim that the foregoing considerations amount to a real proof of this view of the nature of mathematics. The most I could assert would be to have disproved the nominalistic view, which considers mathematics to consist solely in syntactic conventions and their consequences" (Gödel 1995, pp. 304–23). Raatikainen 2005 discusses various philosophical interpretations of the incompleteness theorems.*

So far, so good: The argument seems clear enough. Yourgrau explains that "Gödel was at once a mathematical realist," who believed in the reality of "the Platonic realm of numbers", and "a temporal idealist" because "time in the intuitive sense is an illusion." He speaks of "the nonexistence of time in the actual world" (p. 139), presumably because it corresponds to nothing in the realm of Platonic ideas. Yourgrau's *World Without Time* is a world of "real, objective concepts" (p. 171) that does not include time.

How are we to square Yourgrau's words on pp. 136–139 with his account thirty pages later of Gödel's views on time and intuition? "Time, for example, in relation to being, Gödel considered one of the basic concepts [of metaphysics], but he believed that in the attempt to discover what is fundamental about our thinking about time we can receive no assistance from physics, which, he argued, combines concepts without analyzing them. Instead, we must reconstruct the original nature of our thinking…For this, he turned not to Einstein but to Husserl and phenomenology…Gödel saw phenomenology as an attempt to reconstruct our original use of basic ideas…on what we meant in the first place by our most fundamental acts of thought…[B]oth Gödel and Husserl (in his later period) were conceptual realists" (pp. 170–171).

It seems to follow from these quotations that, for Gödel, time *is* a basic metaphysical concept, one of "the fundamental concepts that underlie reality," about the nature of which "we can receive no help from physics."[2] Instead, one must use self-reflection to grasp this "real, objective concept".

Remember, the Yourgrau of pp. 136–139, also expounding Gödel, had assured us of "the nonexistence of time in the actual world." The disparity between the two Yourgraus left this reader unable to answer a basic question raised by the book: What is the Yourgrau-Gödelian concept of time, which must be grasped by self-reflection but is not based on an intuition of time that is illusory? In the final section of the review, I shall return to the question of what relativistic physics (*pace* the Yourgrau of pp. 170–171) *does* tell us about the nature of time and what is perhaps best left forgotten in Gödel's *Forgotten Legacy*.

The neglect of Gödel's philosophical views by analytic philosophers is mainly due to his affiliation with their *bête noir*: The metaphysical tradition associated with Plato, Leibniz, and the later Husserl, to name some of Gödel's favorites. "Concepts have an objective existence" Gödel wrote in a notebook entry on "My Philosophical Viewpoint" (quoted on p. 104), and his "conceptual realism" is more or less the same as what other philosophers call "objective idealism".[3]

Indeed, while having problems with the "Kantian philosophy, which is strong in epistemology but weak in ontology (weak that is for [conceptual–JS] realists like Gödel, Frege, and Husserl)" (p. 175), Gödel was an admirer of Hegel (see pp. 157, 182), and the method used in "the Gödel program" in logic has interesting parallels with Hegel's dialectical method of subverting a philosophical system from within.[4] Starting from the system's own premises, one demonstrates its inability to reach its own goals by exposing some contradiction between premises and goals. These contradictions are then "sublated"[5] by synthesis in some higher, more advanced system.

Hilbert's formalist program started from some set of axioms and syntactic rules of deduction with the goal of proving the completeness and consistency of arithmetic. Gödel subverted the program from within: using a newly-developed formal technique (Gödel numbering), he proved the impossibility of reaching this goal. One might even say that he did so by "sublating" the syntactic concept of provability within a formal system in the semantic concept of truth in some model of that system (see the sidebar "Gödel's Theorems"). Yourgrau writes of "Gödel's dialectical dance with intuitive and formal time in the theory of relativity (p. 128)"; similarly there is a dialectical dance with semantics and syntax in his logic.

Gödel and Einstein

Both Gödel and Einstein are described in over-the-top superlatives: Gödel is "the greatest logician of all time, a beacon in the intellectual landscape of the last thousand years" (p. 1). Einstein is "the most famous scientist of all time" (p. 2), "the greatest scientist since Newton" (p. 31). "Together with another German-speaking theorist, Werner Heisenberg, they were the authors of the three most fundamental scientific results of the century. Each man's discovery, moreover, established a profound and disturbing *limitation*" (p. 2). Even limiting oneself to limitations, one might well argue, for

[2] *On p. 105, Yourgrau cites Gödel's list of "the fundamental concepts that underlie reality," which includes "time".*

[3] *Terminological confusion abounds here since different philosophers attach opposing senses to the terms "real" and "realism," and "ideal" and "idealism". For advocates of "conceptual realism", the adjective "ideal" is pejorative: It implies that the noun it modifies does not have an objective conceptual counterpart.*

[4] *The similarity is in method, not motivation. Gödel's original intent was to contribute to Hilbert's program, and only years later did he realize that he had subverted it (see, e.g., Feferman 2006).*

[5] *"Sublation" is the best English equivalent for Hegel's "das Aufheben", which means simultaneously to preserve, destroy, and raise to a higher level (see the entry "Sublation" in Inwood 1992, pp. 283–285).*

example, that Bell's theorem beats Heisenberg's uncertainty principle hands down.[6] But are such claims necessary? Isn't a sober statement of the results and their profound implications sufficient?

All is not rosy in Yourgrau's picture of Einstein: "After he arrived at the institute [for Advanced Study in 1933]…never again would he enjoy the intellectual camaraderie that had formed a cloak against all the ugliness that beset his years in Berlin" (p. 148). There is no mention of Walter Mayer, Peter Bergmann, Valentine Bargmann, Nathan Rosen, Leopold Infeld, Bruria Kaufman, Ernst Straus, all of them Einstein's scientific collaborators in Princeton; he remained close to many of them, both intellectually and personally, long after their collaborations ended. Nor is there mention of visits or longer stays at the institute by such scientific colleagues as Niels Bohr, Abraham Pais, Wolfgang Pauli, H. P. Robertson; nor of his close contact with fellow-expatriates such as the historian Erich Kahler and his wife Lily, the writer Hermann Broch; the philosopher Paul Oppenheim and his wife Gaby; art historian Erwin Panofsky and Princeton librarian Johanna Fantova; not to mention various romantic liaisons, such as that with Margarita Konenkova, a Russian woman recently accused of being a spy. Nor was he isolated at the institute: Batterson 2006 describes the important role Einstein played in its affairs from its formative years until his retirement.

Yourgrau's picture of Gödel as a social isolate in Princeton, with few friends except Einstein, and of his tragic descent into paranoia and death by self-starvation, is duly accurate. But to say "together they remained isolated and alone" at the institute (p. 4) is simply to overlook the profound difference between the personalities of the two.

Reliability of the Book

In contrast to his earlier book on the topic (Yourgrau 1999), "intended primarily for philosophers… this one [is] accessible to normal readers" (p. vii). Presumably, he means non-scholars, i.e., that the book is intended for a popular audience.[7] The writer of such a book has a particularly great responsibility, because its readers often take the author's word for factual and technical assertions not substantiated in the text. So if anything, popular books should be held to even higher standards of sobriety and accuracy than books addressed to other experts, capable of forming independent judgments on such matters. This book often falls short of such standards. I have already given some examples of lack of sobriety and, unfortunately, it is not hard to find examples of inaccuracy.

Contradictory assertions occur within a few pages: "Further separating Einstein from Gödel was the fact that Einstein never fully resolved his native suspicion of mathematics. …[T]he physicist remained forever wary of being led by the nose by mathematicians" (p. 15). "Einstein and Gödel, in turn, each in his own way, approached the world mathematically. For both, mathematics was a window onto ultimate reality, not, as for many of their scientific colleagues, a mere tool for intellectual bookkeeping." (p. 17).

Sometimes one of the two statements is so downright silly that it can only be ascribed to carelessness: On p. 44, Yourgrau speaks of "*rational numbers as infinite sequences of natural numbers*, and irrational numbers as infinite sequences of rational numbers [my emphasis–JS]." Three pages later he describes "irrational numbers [as] those that cannot be expressed as *ratios of two natural numbers* [my emphasis–JS]"—correctly implying that all positive rationals can be so defined. Yourgrau's comment on Einstein: "Never too concerned with consistency—unlike his logician companion [Gödel]" (p. 14) applies to many passages in this book!

Confusion even creeps into one of the best parts of the book: the account in Chapter 4 of Gödel's results in logic. Yourgrau's definition of ω-consistency (p. 67) is actually the definition of ω-incompleteness[8]. Conflating the two concepts is particularly unfortunate at this point, since the discussion concerns precisely Gödel's proof that ω-consistency implies ω-incompleteness.

The book also has its share of historical blunders. I cite just two related examples, the Schwarzschild and deSitter solutions of the Einstein equations, treated on pp. 116–117: "When Karl Schwarzschild …discovered in 1916 that if a star began an extreme gravitational collapse into itself, its mass would eventually reach a critical point after which space-time would be so severely curved that nothing inside (what is now known as) the 'event horizon', including light, would be able to escape, Einstein dismissed the 'Schwarzschild singularity' as a mathematical anomaly with no physical significance" (p. 116).

[6] *For a discussion of Bell's theorem "that there is an upper limit to the correlation of distant events, if one just assumes the validity of the principle of local causes" (Peres 1993, p. 160), and the profound significance of its violation by quantum phenomena, see ibid, Chapter 6.*

[7] *But even scholarly readers, let alone "normal" ones, will often find it rough going: "The physicist's prophetic idea of describing a physical system by locating it in a logical framework in various dimensions of physical significance would have not only a profound effect on the future of quantum mechanics but on the Bible of the Schlick circle" (p. 38). That the physicist is Boltzmann and the Bible is Wittgenstein's Tractatus is clear from the context, but otherwise I can't make sense of the sentence, perhaps because I am not "normal".*

[8] *I thank Martin Davis for pointing this out to me (personal communication, December 25, 2005). Davis 2001 includes an excellent chapter on Gödel's contributions to logic.*

> ### What Did Einstein Know and When Did He Know It?
>
> Yourgrau is not alone in propagating the myth that Einstein was taken by surprise when presented with Gödel's results. Stephen Hawking states: "It was therefore a great shock to Einstein when, in 1949, Kurt Gödel…discovered a solution that represented a universe full of rotating matter, with closed time-like curves through every point" (Hawking 2002, p. 90).
>
> Actually in 1914, almost as soon as Einstein realized the need to introduce a non-flat, dynamical space-time metrical structure, and well before he arrived at the final form of his field equations, he worried in print about the problem of closed time-like world-lines. Since his words seem little known, perhaps it is worthwhile to present here what Einstein wrote then:
>
> "I shall now raise an even deeper-reaching question of fundamental significance, which I am not able to answer. In the ordinary [i.e., special–JS] theory of relativity, every line that can describe the motion of a material point, i.e., every line consisting only of time-like elements, is necessarily non-closed, for such a line never contains elements for which dx_4 vanishes. An analogous statement cannot be claimed for the theory developed here. Therefore a priori a point motion is conceivable, for which the four-dimensional path of the point would be an almost a closed one. In this case *one and the same* material point could be present in an arbitrarily small space-time region in *several seemingly mutually independent exemplars*. This runs counter to my physical imagination most vividly. However, I am not able to demonstrate that the theory developed here excludes the occurrence of such paths" (I have modified the translation of Einstein 1914, p. 1079, given in Einstein 1997, pp. 77–78).
>
> —*J. S.*

What Schwarzschild actually did soon before his untimely death in 1916 was to find the unique spherically-symmetric solution to the vacuum Einstein field equations (i.e., outside any source) and show that the solution is static in this region. No discussion of gravitational collapse of a spherically symmetric source beyond the Schwarzschild radius was published until the late 1930s, and the interpretation of this radius as an event horizon came even later (see below) [see for example Stachel 1995].

> Later in 1917, the Dutch Astronomer Wellem [sic!] de Sitter proposed a cosmological model for general relativity in which the universe was not static, as Einstein believed it to be, but rather expanding (p. 117).

De Sitter's original interpretation of his solution was similar to that of the Schwarzschild solution: a static model with a singularity. This interpretation was accepted by Einstein and others, and debate raged over the interpretation of both of these presumed singularities. It was not until 1922–23, when Lanczos found a singularity-free but non-static form of the de Sitter metric, that it began to be interpreted as an expanding universe [see for example Kerszberg 1989]. In 1924, Eddington similarly found out how to remove the Schwarzschild singularity, but this did not become common knowledge among relativists until Finkelstein rediscovered it in 1959.

A World Without Time?

Yourgrau's views on the impact of relativity theory on the concept of time often clash directly with Einstein's. Following Gödel, Yourgrau identifies the concept of "time" with that of "global simultaneity" ("simultaneity, and thus time"). They proceed to reject the reality of time because there exist cosmological models (such as the Gödel universe), in which no such concept of cosmic or global time can be defined.

Einstein, on the other hand, in his *Autobiographical Notes* (in Schilpp 1949, the same volume as Gödel's article), lists "the insights of a definitive nature[9] that physics owes to the *special* theory of relativity [my emphasis–JS]." He gives pride of place to the insight:

> There is no such thing as simultaneity of distant events (*Es gibt keine Gleichzeitigkeit distanter Ereignisse*) (translation from Einstein 1979).

If this is Einstein's view of special relativity, Yourgrau's assertion is surely wrong that "the father of relativity was shocked" (p. 7) by Gödel's demonstration that there are cosmological models in *general* relativity, for which *no* global definition of distant simultaneity is even possible. Indeed, Einstein took this so much for granted that he does not even mention it in his comments on Gödel (Einstein 1949).

Since the exclusive identification of the concept of time with that of global simultaneity is the crux of Gödel's argument for the unreality of time, let us pause for further discussion of this point. Surely, we all have some intuitive concept of time. Does it embrace the concept of a unique cosmic or *global* time, marching forward in lock step throughout the entire universe? The only intuitive concept of time that *I* have is a purely *local* one, associated with *my* progress through the universe. And I seriously doubt that, without a good deal of education, anyone has an "intuition" that the march of his or her local time must coincide with the march of everyone else's local times, let alone the march of time on the sun, planets, and other stars—or even that such marches must exist. Ask a young child,

[9] *I take his characterization of this insight as "definitive" to imply that it holds for general relativity as well.*

just learning to handle the concept of time, what time it is on the sun![10]

This subjective concept of individual, local time has been objectified and incorporated in relativity theory—both special and general—as the concept of the proper time along any time-like world line. If Einstein wasn't shocked by the absence of a global time, was he shocked by Gödel's demonstration that there are models of general relativity containing *closed* time-like world lines? No: Einstein says, "The problem…disturbed me already at the time of the development of the general theory of relativity, without my having succeeded in clarifying it (Einstein 1949)."[11] He ends his reply to Gödel on a skeptical note: "It will be interesting to weigh whether these [solutions] are not to be excluded on physical grounds."[12]

Another conflict: Yourgrau writes, "Relativity had rendered time, the most elusive of beings [sic!], manageable and docile by transforming it into a fourth dimension of space, or rather of relativistic space-time. …the four-dimensional universe of space-time that he himself [i.e., Einstein–JS] had conjured into being."

Einstein writes, "It is a widespread error that the special theory of relativity is supposed to have… first discovered or, at any rate, newly introduced the four-dimensionality of the physical continuum. This, of course, is not the case. Classical mechanics too, is based on the four-dimensional continuum of space and time" (Einstein 1979, p. 55).

Lest Einstein is thought to be overmodest, I shall quote one sentence from Lagrange's 1797 *Mécanique analytique*: "Mechanics may be regarded as a four-dimensional geometry, and mechanical analysis [i.e., analytical mechanics] as an extension of geometrical analysis."

What about Yourgrau's claim that Einstein's accomplishment was "transforming [time] into a fourth dimension of space"? In a review of Emile Meyerson's book *La déduction relativiste*, Einstein praises the book for "rightly insist[ing] on the error of many expositions of relativity which refer to the 'spatialization of time'. Time and space are fused in one and the same continuum, but the continuum is not isotropic. The element of spatial distance and the element of duration remain distinct in nature…The tendency he denounces, although often latent, is nonetheless real and profound in the mind of the physicist, as is unequivocally shown by the extravagances of the popularizers and even of many scientists in their expositions of relativity" (Einstein 1928).

Gödel's Theorems

Young Gödel startled the symbolic logic community in the early 1930s by proving two metatheorems about the incompleteness—or better the incompletability—of any formal logical system based on a set of axioms strong enough to include ordinary arithmetic. A consistent axiomatic formal system is syntactically complete if, for every closed well-formed formula (sentence), either the formula or its negation can be proved from the axioms. Gödel constructed a well-formed formula that is not deducible from the axioms but that nevertheless can be seen to be true in the standard model of the formal system. Indeed, when interpreted semantically in the model, the sentence corresponding to the formula asserts precisely its own unprovability; so if it could be proved, the system would be inconsistent! If one attempts to complete the system by adding a finite number of such true but unprovable formulas to the list of axioms, then still other well-formed formulas will exist in the new system that have the same property. One could also add the negation of the unprovable formula to the axioms, resulting in an axiomatic system that would correspond to a valid statement in some nonstandard model. So "incompletability" seems more appropriate than "incompleteness" as a characterization of the situation.

—*J. S.*

Gödel and Einstein on Time

I shall devote this rest of this review to my own account of Gödel's and Einstein's views on time and to why I agree with John Earman's claim—which so horrifies Yourgrau—that the philosophers' neglect of Gödel's views is "benign" (p. 178).

Gödel 1949a offers two arguments based on relativity theory for "the unreality of change". Both are based on the premise that "change becomes possible only through the lapse of time," by which he means "an objective lapse of time". He explains that this "means (or at least is equivalent to the fact) that reality consists of an infinity of layers of 'now' which come into existence successively" (pp. 557, 558).

Gödel comes down hard on the side of endurance in the old debate between two views of temporal change: endurance versus perdurance:

> An object is said to endure just in case it exists at more than one time. …Objects perdure by having different temporal parts at different times with

[10] *I find more attractive Thomas Sattig's thesis that there is no conflict between the viewpoint of one-dimensional "ordinary time" and of "four-dimensional spacetime": "I find it overwhelmingly plausible that all facts about ordinary time logically supervene on facts about spacetime; what goes on in spacetime fully determines what goes on in ordinary time" (Sattig 2006, p. 1). His treatment covers only Minkowski spacetime, but I believe it could be extended to general relativity.*

[11] *The sidebar "What Did Einstein Know and When Did He Know It?" shows that Einstein discussed this possibility in 1914.*

[12] *Similar skepticism is common in the relativity literature; see for example, Hawking and Ellis 1973, p. 170. Ignoring Einstein's comment, Yourgrau regards "Hawking's attempt to neutralize the Gödel universe" as "show[ing] how dangerous it is to break the conspiracy of silence that has shrouded the Gödel-Einstein connection" (p. 8).*

> no part being present at more than one time. Perdurance implies that two hypersurfaces [in space-time] …do not share enduring objects but rather harbor different parts of the same four-dimensional object (Wüthrich 2003, p. 1).

From the perdurance viewpoint, process is primary. The spatial and temporal aspects of a process—its many possible "heres" and "nows"—are just different "perspectival" effects of "viewing" the same process from different spatio-temporal reference frames.

Yourgrau opts for endurance without even mentioning, let alone discussing, the opposing viewpoint. At least Gödel presents an argument against the relative, perdurance view of time, but one based on a particularly ill-chosen analogy: "A lapse of time, however, which is not a lapse in some definite way seems to me as absurd as a colored object which has no definite color" (Gödel 1949a, p. 558, footnote 5).

There is an objective process, on which everyone can agree: The physical composition of the light rays falling on the eye of the subject, both from some object of perception and its surroundings. But the perceived color of that object—and color is nothing but a perception—is a "perspectival" effect, depending on the conditions of illumination of the object, the contrast with its surroundings, and the properties of the eyes and nervous system of the subject (ask a color-blind and a normal-sighted person whether all objects have a definite color!). So if one accepts Gödel's analogy, which we are under no obligation to do, it argues against rather than for his case.

While the debate between endurance and perdurance views arose long before relativity theory and endures—or perdures—to this day, relativity certainly has changed the terms of the debate. This brings us finally to Gödel's two arguments for "the unreality of change" based on relativity. The first is based on the special theory: "The very starting point of special relativity theory consists in the discovery of a new and very astonishing property of time, namely the relativity of simultaneity, which to a large extent implies that of succession." Gödel immediately qualifies this in footnote 2, p. 557, noting that although there is no longer "a complete linear ordering of all point events," "[t]here exists an absolute partial ordering." And I would add that, as Robb realized as early as 1914, this causal ordering is all that is needed for physics.

Gödel omits mention of the central point about simultaneity that Einstein emphasized from 1905 on: Any characterization of the simultaneity of distant events involves a convention or stipulation; so that there can be no right or wrong of the matter, only a question of the merits and drawbacks of the convention adopted. This would present a grave problem for the objectivity of physics if the nature of any physical process depended on the convention adopted, but it is easily seen that no physical result does. Indeed it is possible to formulate all the results of the special theory without adopting *any* simultaneity convention (see, e.g., the delightful exposition of his K-calculus in Bondi 1964). So the relativity of simultaneity is not the addition of "a new and very astonishing feature of time;" rather, it amounts to the *removal* from the concept of time in physics of an old, hitherto accepted feature: absolute simultaneity.

The most important new feature of time to emerge from the special theory of relativity is that the local or proper time between two events (discussed in the previous section), as measured for example by an ideal clock traveling between the two events, depends on the history of the clock, i.e., its path through space-time. We are quite familiar with the similar dependence of spatial distance on path: The reading of a pedometer worn by someone walking from one place to another depends on the path taken. The most important thing that special relativity has taught us about time is that clocks are a lot more like pedometers than assumed in pre-relativistic kinematics. Put in mathematical terms, it had long been a commonplace that the spatial differential $d\sigma$ between two neighboring points is not a perfect differential. But it had been assumed in Galilei-Newtonian kinematics that the temporal differential dt between two events is a perfect differential, which integrates to the absolute time $t_2 - t_1$ between the two events. Consequently, it was not so important to distinguish between the local time, as measured by a clock transported along some path, and the global time, stipulated to be equal to the absolute time: they always agreed.

Even before the advent of special relativity, a few careful analysts of the foundations of kinematics, notably Henri Poincaré and James Thomson (brother of William Thomson, Lord Kelvin), realized that the introduction of the concepts of distant simultaneity and global time always involves a definitional element, even if the definition using the absolute time seemed entirely unproblematic at the time.

In special-relativistic kinematics, the differential $d\tau$ of proper time is not a perfect differential but depends on the path in space-time between two events. Of course, there is still a big difference between space and time: The straightest path in space is the shortest distance between two points, while the straightest time-like path in space-time is the longest time interval between two events. This is the essence of the twin paradox.

Appropriately modified, these results of special relativity still hold in the general theory: Again, the proper time interval between two events depends on the time-like path and is a local maximum for any time-like geodesic path between them (assuming one exists). This is the truly revolutionary new feature of time that emerges from the two theories of relativity. And perhaps it is worth emphasizing that this path-dependent proper time is *absolute*; that is, its value does not depend on the reference frame in which it is calculated.

Once one realizes that the temporal interval between two events is path dependent, the role of a global time coordinate t is diminished considerably. As emphasized above, even when it can be defined, this definition always involves some convention or stipulation. Moreover, even if a global time can be defined, the proper time $\tau = \int d\tau$ between two events—the only physically significant time—occurring at global times t_1 and t_2 is not independent of the path in between them; and so *cannot* equal $t_2 - t_1$—or indeed *any* function of the two. In fact, in general relativity, there is usually *no* path for which $\tau = t_2 - t_1$.

Even when global times *can* be defined, as in special relativity, it is not always advantageous to do so, for there exist time-like fibrations of Minkowski space-time that are not hypersurface-orthogonal. Hence there is no way for a family of observers traveling along these world lines to synchronize their clocks so that their proper times coincide with any global time. The simplest example is a family of observers in uniform rotational motion relative to some inertial frame. The realization that these observers could not synchronize their proper times played an important role in Einstein's transition from special to general relativity, since it helped to liberate him from the preconception that a coordinate system always must have an immediate physical significance (see Stachel 1980).

The existence of solutions to the Einstein field equations, for which no global time can be defined, such as those found by Gödel (1949b), is certainly interesting. However, their existence does not decisively alter the relativistic concept of time, which as seen above in Einstein's comments on Meyerson, is basically local. The philosophical moral I draw from this discussion is that process is primary and absolute, while its division into spatial states evolving over time is secondary and always relative to the choice of some frame of reference, local or global. Translated into the language of relativity theory, space-time takes precedence over space and time.

Gödel's second argument against the reality of change is based on general relativity, which brought about a much more profound physical revolution than the special theory, the effects of which are still being felt in theoretical physics to this day. Special relativity brought about a change in the metrical structure of space-time, the stage on which all the dramas of physics are enacted. But, while it replaced Galilei-Newtonian space-time with Minkowski space-time, this is still a *fixed* background space-time; so special relativity is still a theory with a kinematics that is independent of all dynamical processes. In general relativity, *all* space-time structures—chrono-geometrical as well as inertio-gravitational—are dynamical fields, interacting via the Einstein equations with all other physical entities: fields and particles. It is a background-independent theory: the general-relativistic stage becomes part of the play, and there is no kinematics prior to and independent of dynamics.

Einstein soon realized that space-times exist, for which no global time can be defined for topological reasons (see above). In general their existence has no bearing on the concept of local (proper) time along a time-like world line. However, he also realized that space-times exist with closed or nearly-closed time-like world lines, and their existence *does* have a bearing on the *local* concept. It provides an extreme example of the fact, mentioned above, that the proper time between two events depends on the path between them. In the case of a (nearly) closed time-like world line, one of the possible values is (almost) zero, and the other is some large number.

Similar paradoxical-seeming results can be formulated for spatial intervals in spatially closed but unbounded universes: In such a universe, by going straight ahead along a spatial geodesic it is possible to return to one's starting point. Einstein's original 1917 static cosmological model, being the topological product of a spacelike three-sphere and a timelike line (see, e.g., Hawking and Ellis 1973), is of this type. So, even restricting ourselves to geodesic paths, the spatial distance between two points is both zero and a positive number. I don't know if anyone has actually used this observation as an argument against the reality of space, but it would be fair to say that arguments like Gödel's against "the reality of time" can be matched by similar arguments against "the reality of space".

The real question is: What is the physical significance of such models? Every physical theory that we know has two properties:

1) There are physical phenomena that fall outside its scope, i.e., that cannot be modeled by the theory (it is not a "theory of everything").

2) There are "unphysical" models of the theory, which do not correspond to any physical phenomena. The class of all models must be restricted by some additional criteria, such as boundary conditions, not inherent in the theory, in order to fit some limited range of physical phenomena.

The smaller the number of phenomena in class 1), and smaller the number of models in class 2), the more we value a theory. But there is no reason to believe that general relativity is an exception to this rule. To use the existence of a class of models with closed time-like world lines as an argument against the concept of time, without a shred of evidence that such models apply to any physical phenomena, is an example of that fetishism of mathematics, to which some Platonists are so prone.

I thank Martin Davis for several critical comments on this review.

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