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Prolegomena to any future Quantum Gravity

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Abstract

I shall discuss some "conditions of possibility"of a quantum theory of gravity, stressing the need to confront certain fundamental problems in any attempt to quantize the field equations of general relativity, including: the distinction between background-independent, partially background-dependent and background-dependent theories, the choice of dynamical variables for quantization, the definition of probability amplitudes for processes, and consistency between formalism and possibilities of measurement.

1.1 Introduction

Prolegomena means preliminary observations, and my title is meant to recall Kant's celebrated *Prolegomena to Any Future Metaphysics That Can be Claim to be a Science.* My words, like his:

> are not supposed to serve as the exposition of an already-existing science, but to help in the invention of the science itself in the first place.

To use another Kantian phrase, I shall discuss some "conditions of possibility" of a quantum theory of gravity, stressing the need for solutions to certain fundamental problems confronting any attempt to apply some method of quantization to the field equations of general relativity (GR). Not for lack of interest but lack of space-time (S-T), other approaches to quantum gravity (QG) are not discussed here (but see (30)).

1.1.1 Background dependence versus background independence

The first problem is the tension between "method of quantization" and "field equations of GR". The methods of quantization of pre-general-relativistic theories† have been based on the existence of some fixed S-T structure(s), needed both for the development of the formalism and –equally importantly– for its physical interpretation. This S-T structure provides a *fixed kinematical background* for dynamical theories: the equations for particle or fields must be invariant under all automorphisms of the S-T symmetry group. GR theory, on the other hand, is a *background-independent* theory, without any fixed, non-dynamical S-T structures. Its field equations are invariant under all differentiable automorphisms (diffeomorphisms) of the underlying manifold, which has no S-T structure until a solution of the field equations is specified. In a background-independent theory, there is no kinematics independent of the dynamics‡.

1.1.2 The primacy of process

GR and special relativistic quantum field theory (SRQFT) *do* share one fundamental feature that often is not sufficiently stressed: *the primacy of process over state*†. The four-dimensional approach, emphasizing processes in regions of S-T, is basic to both (see, e.g., (27; 20; 6)). Every measurement, classical or quantum, takes a finite time, and thus involves a process. In non-relativistic quantum mechanics (QM), one can sometimes choose a temporal slice of S-T so thin that one can speak

† In particular, non-relativistic quantum mechanics (QM)based on Galilei-Newtonian S-T, special-relativistic quantum field theory based on Minkowski S-T, and quantum field theories in non-flat Riemannian S-Ts. But see (25) for a discussion of topological QFT.

‡ Ashtekar and Lewandowski (2) note that "in interacting [special-relativistic] quantum field theories, there is a delicate relation between quantum kinematics and dynamics: unless the representation of the basic operator algebra is chosen appropriately, typically, the Hamiltonian fails to be well-defined on the Hilbert space;" and go on to suggest that in GR one has the same "problem of choosing the 'correct' kinematical representation" (p. 51). By a "background independent kinematics" for GR they mean a "quantum kinematics for background-independent theories of connections." This phrase obscures the fact that, in a special-relativistic theory, the "background independent kinematics" is dictated by the symmetry group of the background S-T metric; while in GR it only emerges from the field equations after the introduction of some fibration and foliation leads to a canonical division into constraint and evolution equations (see Section 1.6).

† Baez (3) emphasizes that both are included in the category of cobordisms. Two manifolds are cobordant if their union is the complete boundary of a third manifold.

meaningfully of an "instantaneous measurement" of the state of a system; but even in QM this is not always the case. Continuous quantum measurements are often needed. And this is certainly the case for measurements in SRQFT, and in GR (see, e.g. (4; 20)). The breakup of a four-dimensional S-T region into lower-dimensional sub-regions –in particular, into a one parameter family of three-dimensional *hypersurfaces*– raises another aspect of the problem. It breaks up a process into a sequence of instantaneous states. This is useful, perhaps sometimes indispensable, as a calculational tool in both quantum theory and GR. But no fundamental significance should be attached to such breakups, and results so obtained should be examined for their significance from the four-dimensional, process standpoint (see, e.g., (17)). Since much of this paper is concerned with such breakups, it is important to emphasize this problem from the start, as does Smolin in (26):

> [R]elativity theory and quantum theory each... tell us - no, better, they scream at us- that our world is a history of processes. Motion and change are primary. Nothing is, except in a very approximate and temporary sense. How something is, or what its state is, is an illusion. It may be a useful illusion for some purposes, but if we want to think fundamentally we must not lose sight of the essential fact that 'is' is an illusion. So to speak the language of the new physics we must learn a vocabulary in which process is more important than, and prior to, stasis (p. 53).

Perhaps the process viewpoint should be considered obvious in GR, but the use of three-plus-one breakups of ST in canonical approaches to QG (e.g., geometrodynamics and loop QG), and discussions of "the problem of time" based on such a breakup, suggest that it is not. The problem is more severe in the case of quantum theory, where the concepts of *state* and state function and discussions of the "collapse of the state function" still dominate most treatments. But, as Bohr and Feynman emphasized, the ultimate goal of any quantum-mechanical theory is the computation of the *probability amplitude* for some *process* undergone by a system. The initial and final states are just the boundaries of the process, marked off by the system's preparation and the registration of some result, respectively (see (28; 29), which includes references to Bohr and Feynman).

In SRQFT, the primary instrument for computation of probability amplitudes is functional integration (see, e.g., (6)). Niedermaier (18) emphasizes the importance of approaches to general relativity that are:

> centered around a functional integral picture. Arguably the cleanest intuition to 'what quantizing gravity might mean' comes from the functional integral

picture. Transition or scattering amplitudes for nongravitational processes should be affected not only by one geometry solving the gravitational field equations, but by a 'weighted superposition' of 'nearby possible' off-shell geometries. [A]ll known (microscopic) matter is quantized that way, and using an off-shell matter configuration as the source of the Einstein field equations is in general inconsistent, unless the geometry is likewise off-shell" (p. 3).

1.1.3 Measurability analysis

The aim of "measurability analysis", as it was named in (4), is based on "the relation between formalism and observation" (20); its aim is to shed light on the physical implications of any formalism: the possibility of formally defining any physically significant quantity should coincide with the possibility of measuring it in principle; i.e., by means of some idealized measurement procedure that is consistent with that formalism. Non-relativistic QM and special relativistic quantum electrodynamics, have both passed this test ; and its use in QG is discussed in Section 4.

1.1.4 Outline of the article

In QM and SRQFT, the choice of classical variables and of methods to describe processes they undergo played a major role in determining possible forms of the transition to quantized versions of the theory, and sometimes even in the content of the quantized theory†. Section 2 discusses these problems for Maxwell's theory, outlining three classical formalisms and corresponding quantizations. The Wilson loops method, applied to GR, led to the development of a background-independent quantization procedure. Section 3 surveys possible choices of fundamental variables in GR, and Section 4 discusses measurability analysis as a criterion for quantization. The classification of possible types of initial-value problems in GR is discussed in Sections 5 and 6. Section 7 treats various "mini-" and "midi-superspace" as examples of partially background-dependent S-Ts in GR, and the quantization of asymptotically flat S-Ts allowing a separation of kinematics and dynamics at null infinity. There is a brief Conclusion.

† In SRQFT, inequivalent representations of the basic operator algebra are possible.

1.2 Choice of Variables and Initial Value Problems in Classical Electromagnetic Theory

In view of the analogies between electromagnetism (EM) and GR (see Section 3)– the only two classical long-range fields transmitting interactions between their sources– I shall consider some of the issues arising in QG first in the simpler context of EM theory‡. Of course, there are also profound differences between EM and GR– most notably, the former is background dependent and the latter is not. One important similarity is that both theories are formulated with redundant variables. In any gauge-invariant theory, the number of degrees of freedom equals the number of field variables minus twice the number of gauge functions. For Maxwell's theory, the count is four components of the electromagnetic four-potential A (symbols for geometric objects will often be abbreviated by dropping indices) minus two times one gauge function equals two degrees of freedom. For GR, the count is ten components of the pseudo-metric tensor g minus two times four "gauge" diffeomorphism functions, again equals two. There are two distinct analogies between EM and GR. In the first, A is the analogue of g. In the second, it is the analogue of Γ, the inertio-gravitational connection. In comparisons between gauge fields and GR, the second analogy is usually stressed. Maxwell's theory is a $U(1)$ gauge theory, A is the connection one-form, the analogue of the GR connection one-form; and $F = dA$ is the curvature two-form, the analogue of the GR curvature two-form (see Section 3 and 6, for the tetrad formulation of GR).

The first analogy may be developed in two ways. The formulation of EM entirely in terms of the potential four-vector is analogous to the formulation of GR entirely in terms of the pseudo-metric tensor (see Section 3): the field equations of both are second order. This analogy is very close for the linearized field equations: small perturbations $h_{\mu\nu}$ of the metric around the Minkowski metric $\eta_{\mu\nu}$ obey the same equations as special-relativistic, gauge-invariant massless spin-two fields, which are invariant under the gauge transformations $h_{\mu\nu} \rightarrow h_{\mu\nu} + \xi_{\mu,\nu} + \xi_{\nu,\mu}$ where ξ_ν is a vector field†; while A_μ obeys those of a spin-one field, which are invariant under the gauge transformation $A_\kappa \rightarrow A_\kappa + \partial_\kappa \chi$, where χ is a scalar field. The divergence of the left-hand-side of these field equations vanishes identically (in GR this holds for both the exact and

‡ This theory is simplest member of the class of gauge-invariant Yang-Mills theories, with gauge group $U(1)$; most of the following discussion could be modified to include the entire class

† For the important conceptual distinction between the two see Section 7.

linearized equations), so vanishing of the divergence of the right hand side (conservation of energy-momentum in GR, conservation of charge in EM) is an integrability condition. This is no accident: invariance and conservation law are related by Noethers second theorem (see Section 5).

The formulation of GR in terms of pseudo-metric g and independently defined inertio-gravitational connection Γ is analogous to the formulation of EM in terms of a one form A and a second two-form field $G_{[\mu\nu]}$, initially independent of F. The definition of the Christoffel symbols $\{\ \}$ in terms of g and its first derivatives is analogous to the definition of $F = dA$ (see above). The first set of Maxwell equations $dF = 0$ then follows from this definition. Some set of constitutive relations between F and G complete the EM theory. The vacuum relations $F = G$ are analogous to the compatibility conditions $\{\ \} = \Gamma$ in GR. The second set of Maxwell equations: $dG = j$, where j is the charge-current 3-form, are the analogue of the equations $E(\Gamma) = T$ equating the Einstein tensor E to the stress-energy tensor T. This analogy is even closer when GR is also formulated in terms of differential forms (see Section 3). Splitting the theory into three-plus-one form (see Section 6), is the starting point in EM for quantization in terms of Wilson loops, and in QG for the loop quantum gravity (LQG) program (see, e.g., (25)). In some inertial frame in Minkowski space: A splits into the three-vector- and scalar-potentials, $\mathbf{A}$ and ϕ. F and G split into the familiar three vector fields $\mathbf{E}$ and $\mathbf{B}$ and $\mathbf{D}$ and $\mathbf{H}$, respectively; and j splits into the three-current density vector $\mathbf{j}$ and the charge density ρ. In a linear, homogeneous isotropic medium†, the constitutive relations are:

$$\mathbf{D} = \epsilon\mathbf{E} \quad \textbf{and} \quad \mathbf{B} = \mu\mathbf{H},$$

with $\epsilon\mu = (n/c)^2$ ϵ and μ being the dielectric constant and permeability of the medium, and n is its index of refraction and c is the vacuum speed of light. The second order field equations split into one three-scalar and one three-vector evolution equation:

$$\frac{\partial}{\partial t}(div\mathbf{A}) + (del)^2\phi = \rho, \qquad grad\, div\, \mathbf{A} - (del)^2\mathbf{A} - \left(\frac{n}{c}\right)^2 \left(\frac{\partial^2 \mathbf{A}}{\partial t^2}\right) = \mathbf{j}$$

Using the gauge freedom to set $div\mathbf{A} = 0$ initially and $(del)^2\phi = \rho$

† The rest frame of a material medium is a preferred inertial frame. In the case of the vacuum, a similar split may be performed with respect to any inertial frame.

everywhere, the constraint equation then insures that $div\mathbf{A}$ vanishes everywhere, and the evolution equation reduces to the (three-)vectorial wave equation for **A**. By judicious choice of gauge, the two degrees of freedom of the EM field have been isolated and embodied in the divergence-free **A** field, a local quantity the evolution of which proceeds independently of all other field quantities. In GR, this goal has been attained in only a few exceptional cases (see Section 7).

Going over from this second order (Lagrangian) to a first-order (Hamiltonian) formalism, canonical quantization of EM then may take place in either the position-representation; or the unitarily-equivalent momentum-representation, leading to a Fock space representation of the free field. Since the asymptotic in- and out-fields always may be treated as free, this representation is useful for describing scattering experiments. In GR, there is no"natural" analogue of an inertial frame of reference; the closest is an arbitrarily selected foliation (global time) and fibration (relative space) (see Section 6.1). Geometrodynamics attempts to use the (suitably constrained) three-spatial metric (first fundamental form) of a spacelike foliation as position variables, with the second fundamental form as the corresponding velocities (see Section 6.3); but apparently a mathematically rigorous quantization of the theory in this form is impossible (see (2)). LQG takes the Ashtekar three-connection on the hypersurfaces as position variables (see Section 5); but rigorous quantization is based on the introduction of loop variables.

The attempt to better understand LQG inspired a similar approach to quantization of the EM field. The integral $\int_C \mathbf{A}$ around a loop or closed curve C in a hyperplane $t = const$ is gauge-invariant†. It follows from the definition of **E**‡ that $\int_C \mathbf{E} = d[\int_C \mathbf{A}]/dt$; so if the $\int_C \mathbf{A}$ are taken as "position" variables, the latter will be the corresponding "velocities." The momenta conjugate to $\int_C \mathbf{A}$ are $\int\int_S \mathbf{D} \cdot \mathbf{n} dS$, where S is any 2-surface bounded by C§ The relation between **D** (momentum) and **E** (velocity) is determined by the constitutive relations of the medium, the analogue of the mass in particle mechanics, which relates a particle's momentum and velocity.

† It is a non-local, physically significant quantity. In spaces with non-vanishing first Betti number its periods form the basis of the Aharonov-Bohm effect.

‡ If there are topological complications, the periods of $\int_C (grad\phi)$ may also be needed.

§ (27) gives a Lagrangian density for arbitrary constitutive relations. When evaluated on t= const, the only term in the Lagrangian density containing a time derivative is $(\partial\mathbf{A}/\partial t) \cdot \mathbf{D}$, from which the expression for the momentum follows. If a non-linear constitutive relation is used, the difference between D and E becomes significant.

In a four-dimensional formulation, the "dual momenta" are the integrals $\int\int_S G$ over *any* two-surface S. This suggests the possibility of extending the canonical loop approach to arbitrary spacelike and null initial hypersurfaces. But it is also possible to carry out a Feynman-type quantization of the theory: a classical S-T path of a such loop is an extremal in the class of timelike world tubes S (oriented 2-surfaces with boundaries) bounded by the loop integral $\int_C \mathbf{A}$ on the initial and final hyperplanes . To quantize, one assigns a probability amplitude $\exp iI(S)$ to each such S, where $I(S)$ is the surface action. The total quantum transition amplitude between the initial and final loops is the sum of these amplitudes over all such 2-surfaces¶. More generally, loop integrals of the 1-form A for *all possible types* of closed curves C ought to be considered, leading to a Feynman-type quantization that is based on arbitrary spacelike loops. Using null-loops, null-hypersurface quantization techniques might be applicable (see Section 6).

The position and momentum-space representations of EM theory are unitarily equivalent; but they are not unitarily equivalent to the loop representation. In order to secure unitary equivalence, one must introduce smeared loops†, suggesting that measurement analysis (see the Introduction) might show that ideal measurement of loop variables requires "thickened" four-dimensional regions of S-T around a loop. The implications of measurement analysis for loop quantization of GR also deserve careful investigation (see Section 4).

1.3 Choice of fundamental variables in classical GR

One choice is well known: a pseudo-metric and a symmetric affine connection, and the structures derived from them. Much less explored is the choice of the conformal and projective structures (see, e.g., (12), Section 2.1, Geometries). The two choices are inter-related in a number of ways, only some of which will be discussed here‡.

¶ See (19; 20)

† The loops are "smeared" with a one parameter family of Gaussian functions over the three-space surrounding the loop.

‡ Mathematically, all of these structures are best understood as G-structures of the first and second order; i.e., reductions of the linear frame bundle group $GL(4, R)$ over the S-T manifold with respect to various subgroups (see (23)). The metric and volume structures are first order reductions of the group with respect to the pseudo-orthogonal subgroup $SO(3, 1)$ and unit-determinant subgroup $SL(4, R)$, respectively. The projective structure and the first order prolongation of the volume structure are second order reductions of the frame bundle group. The interrelations between the structures follow from the relations between these subgroups.

1.3.1 Metric and affine connection

The coordinate components of the pseudo-metric§ field $g_{\mu\nu}$ are often taken as the only set of dynamical variables in GR in second order formulations of the theory. The metric tensor plays a dual role physically:

(i) Through the invariant line element ds between two neighboring events $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ it determines the *chrono-geometry* of S-T (the intervals may be space-like, time-like or null), as manifested in the behaviour of ideal rods and clocks. Since ds is not a perfect differential, the proper time between two time-like separated events depends on the path between them.

(ii) Its components also serve as the potentials for the Christoffel symbols, the components of the Levi-Civita connection that determines the *inertio-gravitational field*:

Directly, through its role in the geodesic equation governing the behavior of freely falling particles (metric geodesics are *extremals* of the interval: shortest for space-like, longest for time-like, or zero-length for null curves);

Indirectly, through the role of the Riemann tensor $R_{[\kappa\lambda][\mu\nu]}$ in the equation of geodesic deviation, governing tidal gravitational forces.

According to Einstein's equivalence principle, gravity and inertia are described by a single inertio-gravitational field and a reference frame can always be chosen locally ("free fall"), in which the components of the field vanish. In a four-dimensional formulation of the Newtonian theory as well as in GR, this field is represented by a symmetric linear connection $\Gamma^\kappa_{\mu\nu}$. For this among other reasons, a first order formalism is preferable, taking both pseudo-metric and connection as independent dynamical variables. The connection still describes the inertio-gravitational field through the geodesic equation: affine geodesics, or better affine auto-parallel curves, are the straightest paths in S-T (the connection also determines a preferred affine parameter on these curves. The affine curvature tensor $A^\kappa_{\lambda[\mu\nu]}$, plays a role in the affine equation of geodesic deviation similar to that of the Riemann tensor in the metric equation. The first order field equations can be derived from a Palatini-type variational principle; one set consists of the compatibility conditions between metric and connection, ensuring that the connection is metric: straightest curves coincide with extremals; and the Riemann tensor agrees with the

§ Often I shall simply refer to the metric, the Lorentzian signature being understood.

affine curvature tensor. Introducing a tetrad of basis vectors e_I and dual co-basis of one-forms e^I, the relation between tetrad components of metric, connection and curvature tensor may be expressed in various ways. Recent progress in QG has demonstrated the special significance of the one based on the cartan connection (see, e.g. (24): the chrono-geometry is represented by means of the co-basis of 1-forms: $g = \eta_{IJ} e^I e^J$, where η_{IJ} is the Minkowski metric, and the affine connection and curvature tensor are represented by an $SO(3,1)$ matrix-valued one-form ω^I_J, and two-form $R^I_J = d\omega^I_J + \omega^I_K \wedge \omega^K_J$, respectively (see, e.g., (22) or (31)). Starting from this formulation, Ashtekar put the field equations of GR into a form closely resembling that of Yang-Mills theory by defining the "Ashtekar connection", a three-connection on a spacelike hypersurface that embodies all the information in the four-connection on the hypersurface (see Section 6). Much recent progress in LQG is based on this step.

1.3.2 Projective and Conformal Structures

Neither metric nor connection are irreducible group theoretically (see the earlier note on G-structures): each can be further decomposed: The metric splits into a conformal, causality-determining structure and a volume-determining structure; the connection splits into a projective, parallel path-determining structure, and an affine-parameter-determining structure. Physically, the conformal structure determines the behavior of null wave fronts and the dual null rays. The projective structure determines the preferred ("straightest") paths of force-free monopole particles†. Given a pseudo-Riemannian S-T, the conformal and projective structures determine its metric. Conversely, given conformal and projective structures obeying certain compatibility conditions, the existence of a metric is guaranteed (9). In GR, these compatibility conditions can be derived from a Palatini-type Lagrangian by taking the conformal, projective, volume-determining and affine parameter- determining structures as independent dynamical variables. There are curvature tensors associated with the conformal and the projective structure; the Weyl or conformal curvature tensor plays an important role in defining the structure of null infinity in asymptotically flat S-T's, and the projective curvature tensor plays a similar role in defining timelike infinity. This set of structures is currently being investigated as the possible basis of

† A preferred affine curve, or auto-parallel, curve is parameterized by a preferred affine parameter; a preferred projective path is not so parameterized.

an approach to QG that incorporates the insights of causal set theory (see (30))‡.

1.4 The Problem of Quantum Gravity

In the absence of an accepted theory of QG, measurability analysis (see the Introduction) of various classical dynamical variables in GR (see the previous section) may help delimit the choice of a suitable maximal, independent set. Taking into account the quantum of action should then restrict joint measurability to compatible subsets, which could serve as a basis for quantization. The formal representation of such ideal measurements will require introduction of further, non-dynamical structures on the S-T manifold, such as tetrads, bivector fields, congruences of subspaces, etc, which are then given a physical interpretation in the measurement context (see, e.g., (21) and Sections 5 and 6 below). This question is closely related to that of initial value problems: possible choices of initial data and their evolution along congruences of subspaces (see Section 6). Measurability analysis in GR could be carried out at three levels: metric, connection and curvature (see the previous section):

The pseudo-metric tensor: Measurements of spatial or temporal intervals along some curve; or similar integrals of spatial two-areas and three volumes†, or of spatio-temporal four-volumes– or integrals of other similar quantities– could provide information about various aspects of the metric tensor. In a sense, all measurements ultimately reduce to such measurements‡. The Introduction and Section 2 present arguments suggesting that four-dimensional process measurements are fundamental, measurements of apparently lower-dimensional regions actually being measurements of specialized processes approximating such lower-dimensional regions. Because of its fundamental importance, this question deserves further investigation.

The affine connection: While the inertio-gravitational connection is not a tensor, an appropriately chosen physical frame of reference can be used to define a second, relative inertial connection; and the dif-

‡ A Lagrangian based on the volume-defining and causal structures is cubic in the conformal dynamical variables.

† This is especially important in view of the claim that quantized values of spatial two-areas and three-volumes are measurable (see, e.g., (2; 22)); for critical comments on this claim, see (17)). Possible measurability of all two-surface integrals of the curvature two-forms, and not just over spatial two-surfaces, should be investigated.

‡ Kuhlmann 2006 notes, in the context of SRQFT: "[S]pace-time localizations can specify or encode all other physical properties."

ference between the inertio-gravitational and the relative inertial connection, like the difference between any two connections is a tensor. So a frame-dependent gravitational tensor can be defined, and might be measurable for example, by deviations of time-like preferred affine inertio-gravitational curves from the preferred purely inertial curves defined with respect to such a frame. Fluctuations around a classical connection, also being tensors, the mean value of classical or quantum fluctuations might also be measurable.

Structures abstracted from the affine connection: Measurement analysis of "smeared" loop integrals of connection one-forms over S-T loops –both spatial and non-spatial– should be done in connection with canonical and non-canonicals formulations of LQG. The possibility of similar measurements on the preferred paths of a projective structure, with results that depend only on that structure, should also be studied .

The Riemann or affine curvature tensor: DeWitt (6), and Bergmann and Smith (4) studied the measurability of the components of the linearized Riemann tensor with respect to an inertial frame of reference, and drew some tentative conclusions about the exact theory. Arguing that, in gauge theories, only gauge-invariant quantities should be subject to the commutation rules, they concluded that measurement analysis should be carried out exclusively at the level of the Riemann tensor. However, this conclusion neglects three important factors:

(i) It follows from the compatibility of chrono-geometry and inertio-gravitational field in GR that measurements of the former can be interpreted in terms of the latter. As noted, the interval ds between two neighboring events is gauge invariant, as is its integral along any closed world line. Indeed, all methods of measuring components of the Riemann tensor ultimately depend on measurement of such intervals, either space-like or time-like, which agree (up to a linear transformation) with the corresponding affine parameters on geodesics.

(ii) Introduction of additional geometrical structures on the S-T manifold to model macroscopic preparation and registration devices produces additional gauge-invariant quantities relative to these structures (see (21)).

(iii) While a geometric object may not be gauge-invariant, some non-local integral of it may be. The electromagnetic four-potential, for example, is not gauge invariant, but its loop integrals are (see Section 2). Similarly, at the connection level, the holonomies of the set of connection one-forms play an important role in LQG. (see, e.g., (2; 22)).

In both EM and GR, one would like to have a method of loop quantization that does not depend on singling out a family of spacelike hypersurfaces. The various "problems of time" said to arise in the canonical quantization of GR seem to be artifacts of the canonical technique rather than genuine physical problems†. The next section discusses some non-canonical possibilities.

Some tensor abstracted from the Riemann tensor, such as the Weyl or conformal curvature tensor. For example, measurability analysis of the Newman-Penrose formalism, based on the use of invariants constructed from the components of the Weyl tensor with respect to a null tetrad (see, e.g., Stephani et al 2003, Chapter 7), might suggest new candidate dynamical variables for quantization.

1.5 The Nature of Initial Value Problems in General Relativity

Any initial value problem for a set of hyperbolic‡ partial differential equations on an n-dimensional manifold consists of two parts:

(i) Specification of a set of initial data on some submanifolds of dimension $d \leq (n-1)$ just sufficient to determine a unique solution; and

(ii) Construction of that solution, by showing how the field equations determine the evolution of the initial data along some $(n-d)$-dimensional congruence of subspaces.

The problems can be classified in terms of the *value of d*, the nature of the *initial submanifolds*, characteristic or non-characteristic, and the nature of the $(n-d)$-dimensional *congruence of subspaces*. In GR, there are essentially only two possibilities for d:

† That is, problems that arise from the attempt to attach physical meaning to some global time coordinate introduced in the canonical formalism, the role of which in the formal-ism is purely as an ordering parameter with no physical significance (see (21; 20)). The real problem of time is the role in QG of the local or proper time, which is a measurable quantity classically.

‡ Initial value problems are well posed (i.e., have a unique solution that is stable under small perturbation of the initial data) only for hyperbolic systems. It is the choice of Lorentz signature for the pseudo-metric tensor that makes the Einstein equations hyperbolic; or rather, because of their diffeomorphism invariance (see Section 5.1), only with the choice of an appropriate coordinate condition (e.g. harmonic coordinates) does the system of equations become hyperbolic.

- $d = 3$: Initial hypersurface(s), with evolution along a vector field (three-plus-one problems).
- $d = 2$: Two dimensional initial surfaces with evolution along a congruence of two-dimensional subspaces (two-plus-two problems).

Below we discuss the possible nature of the initial submanifolds and of the congruence of subspaces.

1.5.1 Constraints Due to Invariance Under a Function Group

If a system of m partial differential equations for m functions is derived from a Lagrangian invariant up to a divergence under some transformation group depending on q functions of the q independent variables ($q \leq m$), then by Noethers second theorem (see, e.g., (32)) there will be q identities between the m equations. Hence, q of the m functions are redundant when initial data is specified on a (non-characteristic) $(q-1)$-dimensional hypersurface, and the set of m field equations splits into q constraint equations, which need only be satisfied initially, and $(m-q)$ evolution equations. As a consequence of the identities, if the latter are satisfied everywhere, the former will also be.

The ten homogeneous (empty space) Einstein equations for the ten components of the pseudo-metric field as functions of four coordinates are invariant under the four-function diffeomorphism group. Hence, there are four (contracted Bianchi) identities between them. In the Cauchy or three-plus-one initial value problem on a spacelike hypersurface (see (10)), the ten field equations split into four constraints and six evolution equations. The ten components of the pseudo-metric provide a very redundant description of the field, which as noted earlier has only two degrees of freedom per S-T point. Isolation of these "true" degrees of freedom" of the field is a highly non-trivial problem. One approach is to find some kinematical structure, such that they may be identified with components of the metric tensor in a coordinate system adapted to this structure (see, e.g., the discussion in Section 6 of the conformal two-structure). Apart from some simple models- (see Section 7) their complete isolation has not been achieved; but the program is still being pursued, especially using the Feynman approach (see, e.g., (18)). Quantization of the theory has been attempted both after and before isolation of the true observables. In quantization methods before isolation, as in loop quantum gravity, superfluous degrees of freedom are first quantized and then eliminated via the quantized constraints (see, e.g., (2)).

Classical GR initial value problems can serve to determine various ways of defining complete (but generally redundant) sets of dynamical variables. Each problem requires introduction of some non-dynamical structures for the definition of such a set, which suggests the need to develop corresponding measurement procedures. The results also provide important clues about possible choices of variables in QG. These questions have been extensively studied for canonical quantization. One can use initial value formulations as a method of defining ensembles of classical particle trajectories, based on specification of half the maximal classical initial data set at an initial (or final) time. The analogy between the probability of some outcome of a process for such an ensemble and the corresponding Feynman probability amplitude (see, e.g., Stachel 2005b) suggests a similar approach to field theories. In Section 2, this possibility was discussed for the loop formulation of electromagnetic theory. The possibility of a direct Feynman-type formulation of QG has been suggested (see, e.g., (6; 18)); and it has been investigated for connection formulations of the theory, in particular for the Ashtekar loop variables. Reisenberger and Rovelli (20) maintain that: "Spin foam models are the path-integral counterparts to loop-quantized canonical theories"†. These canonical methods of carrying out the transition from classical to quantum theory are based on Cauchy or spacelike hypersurface initial value problems (see Section 6.1). Another possible starting point for canonical quantization is the null-hypersurface initial value problem (see Section 6.1). Whether analogous canonical methods could be based on two-plus-two initial value problems (see Section 6.2) remains to be studied.

1.5.2 Non-Dynamical Structures and Differential Concomitants

GR is a *covariant* or *diffeomorphism-invariant* theory, this invariance being defined as invariance under the group of active point diffeomorphisms of the underlying manifold†. It is also *generally covariant*, meaning there are no additional intrinsic, non-dynamical background S-T structures in the theory. Such non-dynamical structures as fibrations and foliations of the manifold, subsequently introduced in order to formulate initial value

† See (3) for the analogy between spin foams in GR and processes in quantum theory: both are examples of cobordisms.

† It is trivially true that all physical results are independent of passive changes of the coordinate system.

problems for the dynamical variables should be introduced by means of geometrical, coordinate-independent, definitions. In particular, evolution of the dynamical variables should not involve the introduction of a preferred "global time" coordinate‡. The dynamical fields include the pseudo-metric and inertio-gravitational connection, and any structures abstracted from them (see Section 3), so any differential operator introduced to describe their evolution should be independent of metric and connection§. In other words these operators should be differential concomitants of the dynamical variables and any non-dynamical structures introduced¶. The ones most commonly used are the *Lie derivatives* $\mathcal{L}_v\Phi$ of geometric objects Φ with respect to a vector field v, and the *exterior derivatives* $d\omega$ of p-forms ω‖ (see, e.g., (31), Chapter 2). Various combinations and generalizations of both, such as the Schouten-Nijenhuis and Frlicher-Nijenhuis brackets, have been -or could be- used in the formulation of various initial value problems.

1.6 Congruences of Subspaces and Initial-Value Problems in GR

Initial value problems in GR involve:

(i) a) choice of initial submanifold(s) and of complementary congruence(s) of subspaces†, and b) choice of differential concomitant(s) to describe the evolution of the initial submanifold(s) along the congruence of complementary subspaces;

(ii) a) choice of a set of dynamical variables, usually related to the pseudo-metric and the affine connection, and their split-up by projection onto the initial submanifold(s) and the complementary subspace(s), and b) choice of differential concomitants to describe their evolution;

‡ Subsequent introduction of a coordinate system adapted to some geometrical structure is often useful for calculations. But coordinate-dependent descriptions of an initial value problem implicitly introduce these structures. But doing tacitly what should be done explicitly often creates confusion.

§ If the conformal and projective structures are taken as primary dynamical variables, the operators should be independent of these structures.

¶ A differential concomitant of a set of geometric objects is a geometric object formed from algebraic combinations of the objects in the set and their partial derivatives.

‖ Or, equivalently, the "curl" of a totally antisymmetric covariant tensor and the "divergence" of its dual contravariant tensor density.

† "Complementary" in the sense that the total tangent space at any point can be decomposed into the sum of the tangent spaces of the initial sub-manifold and of the complementary subspace.

(iii) a break-up of the field equations into constraint equations on the initial submanifold(s) and evolution equations along the congruence(s) of complementary subspaces.

The non-dynamical steps 1a), 1b) and 2a) will be discussed in this subsection, the dynamical ones 2b) and 3) in the next.

As discussed above, in GR there are only two basic choices for step 1a): three-plus-one or two-plus-two splits†. But two further choices are possible: a congruence of subspaces may be holonomic or non-holonomic; and some submanifold(s) may or may not be *null*.

In the three-plus-one case, a sufficiently smooth vector field is always holonomic (curve-forming); but in the two-plus-two case, the tangent spaces at each point of the congruence of two-dimensional subspaces may not fit together holonomically to form submanifolds.

In any theory involving a pseudo-metric (or just a conformal structure), the initial submanifold(s) or the complementary subspace(s) may be *null*, i.e., tangent to the null cone. A null tangent space of dimension p always includes a unique null direction, so the space splits naturally into $(p-1)$- and 1-dimensional subspaces. The choice of the $(p-1)$-dimensional subspace is not-unique but it is always spacelike.

A non-null tangent space of dimension p in a pseudo-metric space of dimension n has a unique *orthogonal* tangent space of dimension $(n-p)$; so there are orthogonal projection operators onto the p- and $(n-p)$-dimensional subspaces. The evolution of initial data on a spacelike p-dimensional submanifold is most simply described along a set of $(n-p)$-orthonormal vectors spanning the orthogonal congruence of subspaces (or some invariant combination of them (see the next subsection). Otherwise, *lapse and shift functions* must be introduced (see Sections 6.1 and 6.2), which relate the congruence of subspaces actually used to the orthonormal congruence.

By definition, null vectors are self-orthogonal, so construction of an orthonormal subspace fails for null surface-elements. And since there is no orthonormal, the null-initial value problem is rather different (see the next subsections). A similar analysis of two-plus-two null versus non-null initial value problems has not made, but one would expect similar results.

† Various sub-cases of each arise from possible further breakups, and I shall mention a few of them below.

1.6.1 Vector Fields and Three-Plus-One Initial Value Problems

In the Cauchy problem, the use of a unit vector field n normal to the initial hypersurface leads to the simplest formulation of the Cauchy problem. Lie derivatives w.r.t. this field $\mathcal{L}_n\Phi$ are the natural choice of differential concomitants acting on the chosen dynamical variables Φ in order to define their "velocities" in the Lagrangian and their "momenta" in the Hamiltonian formulation of the initial-value problem. Their evolution in the unit normal direction can then be computed using higher order Lie derivatives. If $\mathcal{L}_v$ with respect to another vector field v is used, the relation between v and n must be specified in terms of the lapse function ρ and the shift vector σ, with

$$v = \rho n + \sigma$$

There is a major difficulty associated with the Cauchy problem for the Einstein equations. The initial data on a space-like hypersurface, basically the first and second fundamental forms of the hypersurface, are highly redundant and subject to four constraint equations (see Section 5), which would have to be solved in terms of a pair of freely specifiable initial "positions" and "velocities", of the "true observables"; their evolution would then be uniquely determined by the evolution equations. Only in a few highly idealized cases, notably for cylindrical gravitational waves (see Section 7), has this program been carried out using only locally-defined quantities. In general, on a spacelike hypersurface, quantities expressing the degrees of freedom and the equations governing their evolution are highly non-local and can only be specified implicity; for example,in terms of the conformal two structure (see (7)).

Things are rather better for null hypersurface and two-plus-two initial value problems. By definition, no amount of initial data on a characteristic hypersurface of a set of hyperbolic partial differential equations suffices to determine a unique solution. In GR, the characteristics are the null hypersurfaces, and data must be specified on a pair of intersecting null hypersurfaces in order to determine a unique solution in the S-T region to the future of both (see, e.g., (7)). There is a sort of "two-for-one" tradeoff between the initial data needed on a single Cauchy hypersurface and such a pair of null hypersurfaces. While "position" and "velocity" variables must be given on a spacelike hypersurface, only "position" variables need be given on the two null hypersurfaces. Various approaches to null hypersurface quantization have been tried. For example, one of the two null hypersurfaces may be chosen as future or past null infinity $\Im^{\pm}$ (read "scri-plus" or "scri-minus"; for their use in asymptotic quantization, see Section 7) and combined with another finite null hypersurface (11). As noted above, a null hypersurface is naturally fibrated by a null vector field, and the initial data can be freely specified in a rather "natural" way on the family of transvecting spacelike two surfaces: The projection of the pseudo-metric tensor onto a null hypersurface is a degenerate three-metric of rank two, which provides a metric for these two-surfaces. Due to the halving of initial data (discussed above), only two quantitities per point of each initial null hypersurface (the "positions") need be specified, leading to considerable simplification of the constraint problem; the price paid is the need to specify initial data on two intersecting null hypersurfaces. One way to get these hypersurfaces is to start from a spacelike two-surface and drag it along two independent congruences of null directions, resulting in two families of spacelike two-surfaces, one on each of the two null hypersurfaces. The initial data can be specified on both families of two-surfaces, generating a double-null initial value problem. But the same data could also be specified on the initial spacelike two-surface, together with all of its Lie derivatives with respect to the two congruences of null vectors. This remark provides a natural transition to two-plus-two initial value problems.

1.6.2 Simple Bivector Fields and Two-Plus-Two Initial Value Problems

In the two-plus-two case, one starts from a space-like two-manifold, on which appropriate initial data may be specified freely (see (7)); the evolution of the data takes places along a congruence of time-like two surfaces that is either orthonormal to the initial submanifold, or is related to the orthonormal subspace element by generalizations of the lapse and shift functions. The congruence is holonomic, and a pair of commuting vector fields† spanning it may be chosen, and evolution off the initial two-manifold studied using Lie derivatives w.r.t. the two vector fields They may be chosen either as one time-like and one space-like vector, which leads to results closely related to those of the usual Cauchy prob-

† They are chosen to commute, so that all results are independent of the order, in which dragging along one or the other vector field takes place.

lem‡; or more naturally as two null vectors, which, as noted above, leads to results closely related to the double-null initial value problem. It is also possible entirely to avoid such a breakup of the two-surfaces by defining a differential concomitant that depends on the metric of the two-surface elements.

1.6.3 Dynamical Decomposition of Metric and Connection

A p-dimensional submanifold in an n-dimensional manifold can be rigged at each point with a complementary $(n-p)$-dimensional subspace normal to it†. Every co- or contra-variant vector at a point of the surface can be uniquely decomposed into tangential and normal components; and hence any tensor can be similarly decomposed.

Metric: The concept of "normal subspace" may now be identified with "orthogonal subspace"‡, the metric tensor **g** splits into just two orthogonal components§:

$$g = {}'g +'' g \qquad {}'g \cdot'' g = 0$$

where $'g$ refers to the p-dimensional submanifold, and $''g$ refers to the $(n-p)$-dimensional orthogonal rigging subspace. The properties of these subspaces, including whether they fit together holonomically to form submanifolds, can all be expressed in terms of $'g$, $''g$ and their covariant derivatives; and all non-null initial value problems can be formulated in terms of such a decomposition of the metric. It is most convenient to express 'g in covariant form, in order to extract the two dynamical variables from it, and express "g in contravariant form, in order to use it in forming the differential concomitant describing the evolution of the dynamical variables. Note that "g is the pseudo-rotationally invariant combination of any set of pseudo-orthonormal basis vectors spanning the timelike subspace, and one may form a similarly invariant combination of their Lie derivatives¶. In view of the importance of the analysis of

‡ If one drags the spacelike-two surface first with the spacelike vector field, one gets an initial spacelike hypersurface.

† The word *normal* here is used without any metrical connotation. *Transvecting* would be a better word, but I follow the terminology of Weyl.

‡ This identification excludes the case of null submanifolds.

§ Here again, I avoid the use of indices where their absence is not confusing.

¶ The simple multivector formed by taking the antisymmetric exterior product of the basis vectors is also invariant under a pseudo-rotation of the basis, and the exterior product of their Lie derivatives is also invariant and may also be used.

the affine connection and curvature tensors in terms of one- and two-forms, respectively, it is important in carrying out the analysis at the metric level, to include representations based on tetrad vector fields and the dual co-vector bases, spanning the p-dimensional initial surface and the $(n-p)$-dimensional rigging space by corresponding numbers of basis vectors.

Connection: An n-dimensional affine connection can be similarly decomposed into four parts with respect to a p-dimensional submanifold and complementary "normal" $(n-p)$-dimensional subspace (see the earlier note). Consider an infinitesimal parallel displacement using the n-connection in a direction tangential to the submanifold. The four parts are:

(i) *The surface or* (t,t) *affine connection*: The p-connection on the submanifold that takes a tangential (t) vector into the tangential (t) component of the parallel-displaced vector.

(ii) *The longitudinal or* (t,n) *curvature* †: The mapping taking a tangential (t) vector into the infinitesimal normal (n) component of its parallel-displaced vector.

(iii) *The* (n,n) *torsion*‡: The linear mapping taking a normal (n) vector into the infinitesimal normal (n) component of its parallel-displaced vector.

(iv) *The transverse or* (n,t) *curvature*: The linear mapping that taking a normal (n) vector into the infinitesimal tangential (t) component of its parallel-displaced vector.

One gets a similar decomposition of the matrix of connection one-forms by using covectors. These decompositions of metric and connection can be used to investigate $(3+1)$ and $(2+2)$ decompositions of the first order form of the field equations and of the compatibility conditions between metric and affine connection (see Sections 3 and 6), and in first order formulations of initial value problems. If the n-connection is metric, then "normal" has the additional meaning of "orthogonal" (see discussion above). The (t,t) surface affine connection is (uniquely)compatible with the surface metric; the (t,n) (n,t) curvatures are equivalent; and the (n,n) torsion reduces to an infinitesimal rotation. On a hypersurface $(p=n-1)$, the torsion vanishes, and the (t,n)

† The use of "curvature" here is a reminder of its meaning in the Frenet-Serret formulas for a curve, and has nothing to do with the Riemannian or affine curvature tensors.

‡ Note this use of "torsion" has nothing to do with an asymmetry in the connection. All connections considered in this paper are symmetric.

and (n,t) curvatures are equivalent to the second fundamental form of the hypersurface.

The Ashtekar connection combines the (t,t) and (n,t) curvatures into a single three-connection. Extension of the Ashtekar variables, or some generalization of them, to null hypersurfaces is currently under investigation†. In the two-plus-two decomposition, there is a pair of second fundamental forms and the (n,n) rotation is non-vanishing. For a formulation of the two-plus-two initial value problem when the metric and connection are treated as independent before imposition of the field equations. Whether some analogue of the Ashtekar variables can be usefully introduced in this case remains to be studied.

1.7 Background Space-Time Symmetry Groups

The isometries of a four-dimensional pseudo-Riemannian manifold are characterized by two integers: the dimension $m \leq 10$ of its isometry group (or group of automorphisms or motions) and the dimension $o \leq min(4,m)$ of this group's highest-dimensional orbits (see, e.g., (31; 13)). There are two extreme cases:

The maximal symmetry group: $(m = 10, o = 4)$. Minkowski S-T is the unique Ricci-flat S-T in this group. Its isometry group is the Poincar or inhomogeneous Lorentz group, acting transitively on the entire S-T manifold. Special-relativistic field theories involving field equations that are invariant under this symmetry group and are the most important example of background-dependent theories (see Introduction). At the other extreme is

The class of generic metrics: $(m = 0, o = 0)$. These S-T's have no nontrivial isometries. The class of all solutions to a set of covariant field equations (see Section 5.2) will include a subclass -by far the largest- of generic metrics‡. Covariant theories not involving any background S-T structures, such as GR, are called generally covariant, background-independent theories (see Section 5.2).

† For a review of some results of a generalization based on null hypersurfaces. For null Ashtekar variables, see DInverno et al 2006.

‡ This global, active diffeomorphism group should not be confused with the groupoid of passive, local coordinate transformations. Nor must the trivial freedom to carry out active diffeomorphisms acting on all structures on the manifold, including whatever fixed background metric field (such as the Minkowski metric) may be present, be confused with the existence of a subgroup of such diffeomeorphisms that constitutes the isometry group of this background metric.

1.7.1 Non-Maximal Symmetry Groups and Partially-Fixed Backgrounds

Covariant theories not involving any background S-T structures, such as GR, are called generally covariant, background-independent theories (see Section 5.2). We shall say a theory is a *partially-fixed background theory* if the metric solutions to a background-independent theory are further required to preserve some fixed, non-maximal isometry group. These solutions belong to some class between the two extremes discussed above. Although the overwhelming majority of solutions to the Einstein equations must be generic, no generic solution is known. Only the imposition of a partially-fixed background isometry group enables construction of explicit solutions (see, e.g., (31)). The background-dependent isometry group determines a portion of the pseudo-metric tensor field non-dynamically, and the remaining, unrestricted portion obeys a reduced set of dynamical field equations. For each isometry group one must determine how much dynamical freedom remains. Considerable work has been done on the quantization of two classes of such solutions:

(i) The "*mini-superspace*" cosmological solutions, in which the isometry group imposed is so large that only functions of one parameter (the "time") are subject to dynamical equations. Quantization here resembles that of a system of particles rather than fields, and does not seem likely to shed too much light on the generic case.

(ii) The "*midi-superspace*" solutions, notably the cylindrical wave metrics (see (5)), for which sufficient freedom remains to include both degrees of free of the gravitational field. In an appropriately adapted coordinate system, they can be isolated and represented by a pair of "scalar"fields obeying non-linear, coupled scalar wave equations in two-dimensional flat S-T. In addition to static and stationary fields, the solutions include gravitational radiation fields having both states of polarization. Their quantization can be carried out as if they were two-dimensional fields. But, of course, the remaining portions of the metric must be constructed and diffeomorphism invariance of all results carefully examined, as well as possible implications for the generic case. Niedermaier in (18) summarizes the work done on Feynman path quantization of such models.

Marugan and Montejo (1998) discuss quantization of gravitational plane waves, and Stephani et al. in (31) discuss solutions to the Einstein equations having groups of motions with null and non-null orbits, so it should be possible to study the quantization of such metrics in a systematic way.

1.7.2 Small Perturbations and the Return of Diffeomorphism Invariance

While the fiber manifold consisting of all four-metrics over a base manifold is itself a manifold, the space of all four-geometries is not†. It is a *stratified manifold*, partitioned into slices; each of which consists of all geometries having the same isometry group. But, unless it is restricted to lie within some isometry group, the smallest perturbation of a geometry with nontrivial isometry group takes the resulting geometry into the generic slice of the stratified manifold. This observation is often neglected; in particular, when perturbation-theoretic quantization techniques developed for special relativistic field theories are applied to perturbations of the Minkowski solution in GR. Infinitesimal diffeomorphisms of such perturbations cannot be treated as pure gauge transformations on the fixed background Minkowski S-T, but modify the entire causal and inertio-gravitational structure (see, e.g., (8), Chapter 21). This is the fundamental reason for the problems that arise in formally applying special relativistic quantization techniques to such perturbations.

1.7.3 Asymptotic symmetries

An important class of solutions to the field equations, while lacking global symmetries, has a group of asymptotic symmetries as infinity is approached along null directions, which permits their asymptotic quantization (see (15), Section VI, and (1)). Imposition of certain conditions on the behavior of the Weyl tensor in the future or past null limit allows conformal compactification of this class of S-Ts by adjoining boundary null hypersurfaces, $\Im^{\pm}$, to the S-T manifold. Both $\Im^{\pm}$ have a symmetry

† The space of all metrics divides into equivalence classes under the diffeomorphism group, suitably restricted for each subclass of metrics having a common isometry group. Each equivalence class corresponding to a single four-geometry, or physical S-T. The quotient space (see (4)) of the space of all metrics by the (suitably restricted) diffeomorphism group is a four-dimensional superspace (for three-dimensional superspace see Fischer 1970), which is a stratified manifold.

group that is independent of particular dynamical solutions to the field equations in this class. Thus, on $\Im^{\pm}$ there is a separation of kinematics and dynamics, and a more or less conventional quantization based on this asymptotic symmetry group can be carried out. "More or less" because the asymptotic symmetry group, the Bondi-Metzner-Sachs (BMS) group, is not a finite-parameter Lie group like the Poincar group usually used to introduce gravitons in the linear approximation, but includes four so-called "supertranslation," functions that depend on two "angular" variables. Nevertheless, asymptotic gravitons with two states of polarization may be defined as representations of the BMS group, no matter how strong the interior gravitational field (1).

1.8 Conclusion

This paper has discussed only a few possible approaches to quantization of the field equations of GR. In spite of its emphasis on background-independent techniques, it is rather conservative, ignoring such promising avenues of research as causal set theory, causal dynamic triangulation, twistor theory; and attempts to derive S-T structures as emerging from radically different underlying entities, such as the symmetries of coherent states in quantum information theory; such theories are reviewed elsewhere in this volume. It is by no means certain that any of the conservative approaches will lead to a fruitful fusion of quantum theory and GR -indeed, it is even probable that they will not. But until some approach has been developed leading to a consensus in the QG community, every approach deserves to be explored to its limits, if only to draw lessons from the limited successes and ultimate failure of each such attempt, for the formulation of better alternative approaches.

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References

[1] A. Ashtekar, Asymptotic Quantization Based on the 1984 Naples Lectures (Naples: Bibliopolis) (1987).
[2] A. Ashtekar, J. Lewandowski, "Background Independent Quantum Gravity: A Status Report," (2004) ¡arXiv:gr-qc/0404018 v1¿

[3] J. Baez, Quantum Quandaries: A Category-Theoreti8c Perspective, in D.P. Rickles, S. French and J. Saatsi, eds., *Structural Foundations of QG*, Oxford University Press (2006).

[4] Bergmann, Peter G. and Gerritt Smith (1982). Measurability Analysis for the Linearized Gravitational Field, GR and Gravitation 14: 1131-1166.

[5] Bick, Jir (2000), The role of exact solutions of Einsteins equations in the development of GR, ¡arXiv:gr-qc/0004016 v1¿.

[6] B. S. DeWitt, The Quantization of Geometry, in Lou Witten (ed), Gravitation: An Introduction to Current Research. New York: J. Wiley and Sons, pp. 266-381 (1962). "The Global Approach to SRQFT", 2 vols. Oxford (2003): Clarendon Press.

[7] D'Inverno, R. A. and John Stachel (1978), "Conformal Two Structure as the Gravitational Degrees of Freedom in GR," Journal of Mathematical Physics 19: 2447-2460.

[8] Doughty, Noel A (1990), Lagrangian Interaction/ An Introduction to Relativistic Symmetry in Electrodynamics and Gravitation. (Reading, MA/Addison-Wesley).

[9] J. Ehlers, F. A. E. Pirani, A. Schild, "The Geometry of Free Fall and Light Propagation," in GR/Papers in Honor of J. L. Synge, L. O'Raifeartaigh, ed., Oxford: Clarendon Press, pp. 63-85 (1972).

[10] H. Friedrich, A. Rendall, "The Cauchy Problem for the Einstein Equations," (2000), ¡arXiv;gr-qc/0002074 v1¿.

[11] S. Frittelli, C. Kozameh, E. Newman, C. Rovelli, R. Tate, "On the Quantization of the Null-Surface Formulation of GR, (1997), ¡arXiv:gr-qc/96102010 v1¿.

[12] Goenner, Hubert (2005), On the History of Unified Field Theories, http://www.livingreviews.org/lrr-2004-2

[13] Hall, G. S. (2004), Symmetries and Curvature Structure in General Relativity (Singapore: World Scientific).

[14] Klarfeld, Joseph and Arthur Komar (1971), "Commutation Relations at Null Infinity for Interacting Massless Fields," Physical Review D: 987-994.

[15] Komar, Arthur (1973), "The General Relativistic Quantization Program, in Contemporary Research in the Foundations and Philosophy of Quantum Theory, C. A. Hooker, ed. (Dordrecht/Boston: D. Reidel), pp. 305-327..

[16] Kuhlmann, Manfred (2006), "Quantum Field Theory, " in Stanford Encyclopedia of Philosophy, http://plato.stanford.edu/entries/quantum-field-theory/¿

[17] H. Nicolai, K. Peeters, "Loop and Spin Foam Gravity: A Brief Guide for Beginners," (2006), ¡arXiv: hep-th/0601129 v2¿.

[18] M. Niedermaier, "The asymptotic safety scenario in quantum gravity - an introduction",(2006), arXiv:gr-qc/9412035 v1

[19] Reisenberger, Michael (1994), World Sheet Formulations of Gauge Theories and Gravitation, ¡arXiv:gr-qc/9412035 v1¿.

[20] M. Reisenberger, C. Rovelli, Sum over surfaces form of loop QG", Physical Review D 56: 3490-3508 (1997), gr-qc/9612035; S-T states and covariant quantum theory, ¡arXiv:gr-qc/0111016 v2¿, Physical Review D 65: 125016 (2002).

[21] C. Rovelli, , "What is observable in classical and quantum gravity?," Classical and Quantum Gravity 8: 297-316 (1991).

[22] C. Rovelli, *Quantum Gravity*, Cambridge University Press (2004).

[23] I. Sanchez-Rodriguez, "Intersection of G-Structures of first or second order," Proceedingsof the Eigth International conference on Differential Geometry and Its Applications, O. Kowalski, D. Krupka and J. Slovak, eds. (Opava: Silesian University), pp. 135-140 (2001).

[24] R.W. Sharpe, Differential Geometry: Cartan's Generalization of Klein's Erlangen Program (New York: Springer-Verlag) (1997).

[25] L. Smolin, "An Invitation to Loop Quantum Gravity," (2004), ¡arXiv:hep-th/0408048 v3¿.

[26] L. Smolin, *Three Roads to Quantum Gravity*, New York: Basic Books(2001).

[27] J. Stachel, "The Generally Covariant Form of Maxwell's Equations," in J. C. Maxwel, lthe Sesquicentennial Symposium, M. S. Berger, ed. (Amsterdam/New York/Oxford: North Holland)(1984).

[28] J. Stachel, "Feynman Paths and Quantum Entanglement: Is There Any More to the Mystery?," in Potentiality, Entanglement and Passion-at-a-Distance/ Quantum Mechanical Studies for Abner Shimony, vol 2, Robert S. Cohen et al, eds. (Dordrecht/Boston/London: Kluwer Academic), pp. 245-256 (1997).

[29] J. Stachel, "Quantum Logic," in The Philosophy of Science/An Encyclopedia, vol. 2, Sahotra Sarkar and Jessica Pfeffer, eds. (New Yor/Abington: Routledge), pp. 633-644 (2005).

[30] J. Stachel, "Structure, Individuality and QG," in D.P. Rickles, S. French and J. Saatsi, eds., Structural Foundations of QG (Oxford University Press), pp. 53-82 (2006).

[31] H. Stephani, K. Dietrich, M. MacCallum, C. Hoenselaers, E. Herlt, *Exact Solutions to Einstein's Field Equations*, 2nd ed., Cambridge University Press (2003).

[32] S. Winitzki, Topics in Advanced GR $http://www.theorie.physik.uni-muenchen.de/\tilde{}serge/T7/GR_course.pdf$. (2006).

[33] Wuthrich, Christian (2006), "To Quantize or Not to Quantize: Fact and Folklore in Quantum Gravity," Philosophy of Science, to appear.

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† Additional references are in a longer version of this paper, available from JS