Abstract: It is well known that Einstein worked to develop a unified field theory that would encompass all of physics including (he hoped) all quantum phenomena. It is not so well known that there was "another Einstein," who from 1916 on was skeptical about the continuum as a foundational element in physics, especially because of the existence of quantum phenomena. This talk will discuss the evidence for the existence of "the other Einstein" and his efforts to find what he called "a purely algebraic physics."
THE OTHER EINSTEIN

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Perimeter Institute, 23 October 2005
Einstein Learns About Science

- As a boy, Einstein was introduced to the best German popular-scientific literature by a young medical student who befriended him and recognized his talent. Among the books was:
Humboldt’s *Kosmos*
Unity of Nature

In keeping with the early 19th century German philosophical tradition, Humboldt stressed the unity of all natural phenomena, an ideal that Einstein soon adopted.

“For Humboldt, humanity's perception of nature's unity and power was as much aesthetic as it was rational, a vision far more in tune with Romanticism than with the dry natural theology of Darwin's mentors”– Peter Bowler
Einstein Beginning and Summing up His Career

"It is a wonderful feeling to realize the unity of a complex of phenomena which, to immediate sensory perception, appear to be totally separate things“ (1901)

• "The real goal of my research has always been the simplification and unification of the system of theoretical physics“ (1932)
Einstein and Unified Field Theory

When Einstein was growing up, the hottest topic in theoretical physics was Maxwell’s theory, which unified the phenomena of electricity, magnetism and optics by means of the theory of the electromagnetic field.
Einstein and UFT (cont’d)

From 1909 until the end of his life (1955) Einstein proposed various field theories to extend this unification:

First to a unification of matter and radiation with the special theory of relativity

And then to a unification of gravitation and electromagnetism as an extension of general relativity (his theory of gravitation)
What Is A Field Theory?

All field theories involve some functions of spatial and temporal coordinates—what kind of functions depends on what physical phenomena the theory is meant to explain.

Consequently all field theories depend on some concepts of space and time, which are assumed to form a continuum; and the functions are assumed to be smooth (i.e., continuous and differentiable) functions of the spatio-temporal coordinates.
What Is A Variable?

Definition Of Variable

“A variable is any quantity that varies. Any attribute, phenomenon or event that can have different values”
Simple Examples

The speed of the car that brought you here— or your speed if you walked
• Your weight
• The number of people in this room
• The length of time I have been talking
• The number of grains of sand that have reached the bottom of an hourglass
There are two main types of variables, continuous and discrete

Types of Variables

Quantitative (continuous)  Qualitative (Discrete)
Simple Example

The **number of people in this room is a discrete variable**

The **total weight of these people is a continuous variable**
Functions: \( Y = F(X) \)

Role Of Variable

- Association
- Independent \(\rightarrow\) Dependent
- Independent \(\leftrightarrow\) Confounding \(\leftrightarrow\) Dependent
- Independent \(\uparrow\) Effect \(\downarrow\) modifier \(\downarrow\) Dependent
Simple Example with a Complex Moral

The temperature (dependent variable) is a continuous function* of the place on the earth (independent spatial variables) and the time (independent temporal variable).

*How do we know how continuous temperature really is— and how do we know whether space and time are really continuous?

Answer: We build theories assuming that they are and then accept and use the theories until some failure of the theory to pass some test tells us we must re-examine our assumptions.
Example (cont’d)

• For example, sand flowing out of a pail can be treated as a continuous flow for many purposes, although we can easily recognize that it is actually composed of discrete grains. Water flowing out of a pail can be treated as a continuous flow for all everyday purposes, even though it took centuries of physics and chemistry to tell us it is actually composed of discrete molecules of $\text{H}_2\text{O}$. 
Some Delicate Problems

It is often said that energy is quantized
This is quite true, given a frequency \( v \)
But as far as we know today, frequency is not quantized: any frequency could occur in some situation
Some Delicate Problems (cont’d)

Let us look a little more closely at the question

Speed $v$, wave length $\lambda$ and frequency $\nu$ of a wave are related by the equation

$$v = \lambda \nu$$

Dimensionally $[L/T] = [L] \times [T]^{-1}$

So if space and time are quantized, so will be wave length and frequency
How Could We Find Out if Space and Time Are Discrete?

One direct way: Assume they are continuous, calculate the probable outcomes of interactions between particles taking place over smaller and smaller regions of space in smaller and smaller intervals of time and see whether your predictions agree with experiments.

So far, all such high energy experiments are consistent with the continuity hypothesis.

But as we shall see later, these experiments are still very, very, very .... far from reaching regions of space and intervals of time where we have reason to suspect that continuity could break down.
Back to Field Theory

So in field theories, space and time are treated as continuous, independent variables.

The functions describing the field are treated as continuous, dependent variables.
The Continuous Einstein

Because of his life-long work on field theories. Einstein is usually regarded as the advocate par excellence of the effort to solve all the problems of physics based on the space-time continuum. Any discreteness in physics would then have to come from either:

1) particles moving in the continuum, leading to an uncomfortable field-particle dualism (we shall return to this point)

2) conditions that restrict solutions of the continuous field equations to a discrete set
Possible Restrictions

Such restrictions could be based on:

1) overdetermination of the field equations: more equations than independent variables,
2) demanding solutions to the non-linear equations be singularity free
3) imposing boundary conditions on the solutions

The first two possibilities had long been considered by Einstein. The third is the one that was actually used in quantum mechanics.
Example: Standing Waves in a String

A string is clamped at both ends and one attempts to set up a standing wave pattern in. This can only be done for certain discrete vibration frequencies of the string that depend length of and tension in the string.
So Far, What I Have Said About Einstein Is Well Known*--BUT

(*To Those Who Know it Well)
The Other Einstein

There is another Einstein, who had long been skeptical about the adequacy of the space-time continuum itself as a conceptual tool to solve the problems confronting theoretical physics as a result of the existence of the quantum phenomena.
The Other Einstein: Einstein Contra Field Theory,
Science in Context
Einstein to Walter Dällenbach

You have correctly grasped the drawback that the continuum brings. If the molecular view of matter is the correct (appropriate) one, i.e., if a part of the universe is to be represented by a finite number of moving points, then the continuum of the present theory contains too great a manifold of possibilities. I also believe that this “too great” is responsible for the fact that our present means of description miscarry with the quantum theory (November 1916).
Einstein to Dällenbach
(Cont’d)

The problem seems to me [to be] how one can formulate statements about a discontinuum without calling upon a continuum (space-time) as an aid; the latter should be banned from the theory as a supplementary construction not justified by the essence of the problem, which corresponds to nothing ”real”. But we still lack the mathematical structure unfortunately. How much have I already plagued myself in this way!
The *Akademie Olympia*—Did the Plaguing Start Here?
Richard Dedekind

Was sind und was sollen die Zahlen?

von Richard Dedekind.

Erste Auflage, 1888.
Dedekind on Space

- For a great part of the science of space the continuity of its configuration is not even a necessary condition. ... [He gives an example based on a set $M$ of algebraic numbers] The space made up of points of $M$ ... is everywhere discontinuous, but in spite of this discontinuity ... all constructions of Euclid's *Elements* can ... be just as accurately effected as in perfectly continuous space. The discontinuity of this space would not be noticed in Euclid’s science, would not be felt at all.
David Hume

A TREATISE OF
Human Nature:
BEING
An Attempt to introduce the experimen
tal Method of Reasoning
into
MORAL SUBJECTS.

Vol. I
OF THE
UNDERSTANDING.

LONDON:
Printed for James Werns, at the White-Hart, near
Garden’s-Chapel, in Gravelly.
1739.
“Of the Ideas of Space and Time”

The capacity of the mind is not infinite; consequently no idea of extension or duration consists of an infinite number of parts or inferior ideas, but of a finite number, and these are simple and indivisible. ‘Tis therefore possible for space and time to exist conformable to this idea: And if it be possible, ‘tis certain they actually are conformable to it; since their infinite divisibility is utterly impossible and contradictory. … These indivisible parts, being nothing in themselves, are not conceivable when not filled with something real and existent. The ideas of space and time are therefore no separable or distinct ideas, but merely those of the order or manner, in which objects exist.
Bernhard Riemann— "On the hypotheses forming the basis of geometry"

XIII.

 Feder die Hypothesen, welche der Geometrie zu Grunde liegen.


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Continuous vs Discrete Manifolds

The question of the validity of the postulates of geometry in the indefinitely small is involved in the question concerning the ultimate basis of relations of size in space. ...While in a discrete manifold the principle of metric relations is implicit in the concept of the manifold, it must come from somewhere else in the case of a continuous manifold. Either then the actual things forming the foundations of space must constitute a discrete manifold, or else the basis of metrical relations must be sought for outside that actuality, in colligating forces that operate on it.
Back to 1916 and the Letter to Dällenbach

Einstein discussed the other side of the particle-field dualism—get rid of fields and just have particles:

“Yet I see difficulties of principle here too. The electrons (as points) would be the \textit{ultimate entities} in such a system (building blocks). \textit{Are there indeed such building blocks? Why are they all of equal magnitude?}

Is it satisfactory to say: God in his wisdom made them all equally big, each like every other, because He wanted it that way; if it had pleased him, he could also have created them different. \ldots Further, the \textit{old question of the vacuum}! But these considerations must pale before the overwhelming fact: \textit{the continuum is more ample than the things to be described.}
1923—First Published Discussion of the Issue

- The essential element of the previous theoretical development lies in the circumstance that they work with differential equations ... in a four-dimensional spatio-temporal continuum. In view of the existing difficulties, one has despaired of the possibility of describing the actual processes by means of differential equations. Even beyond that, the possibility of a complete extension of the law of causality on the basis of the four-dimensional space-time continuum has been doubted. All of these doubts are epistemologically permissible and quite understandable in view of the existing profound difficulties. Before we seriously consider such extreme possibilities, however, we have to test whether one must conclude from the previous facts and arguments that it is really impossible to succeed with partial differential equations.
Old Friends: Einstein & Langevin in 1923
One does not have the right today to maintain that the foundation [of physics] must consist of a field theory in the sense of Maxwell. The other possibility leads in my opinion to a renunciation of the space-time continuum, and to a purely algebraic physics.
Letter to Langevin (cont’d)

Logically this is quite possible (the system is described by a number of integers; “time” is only a possible viewpoint, from which the other “observables” can be considered— an observable logically coordinated to all the others. Such a theory doesn’t have to be based upon the probability concept. For the present, however, instinct rebels against such a theory.
1936: Back in Print-- “Physics and Reality”

In view of the molecular structure of all events in the small, the introduction of a space-time continuum may be considered contrary to nature. Perhaps the success of [matrix mechanics] points to a purely algebraical method of description of nature to the elimination of continuous functions from physics. Then, however, we must also give up, on principle, the utilization of the space-time continuum. It is not impossible that human ingenuity will some day find methods that will make it possible to proceed along this path. Meanwhile, however, this project resembles the attempt to breathe in an airless space.
1954: Einstein to Michele Besso

I consider it entirely possible that physics cannot be based upon the field concept, that is on continuous structures. Then nothing will remain of my whole castle in the air, including the theory of gravitation, but also nothing of the rest of contemporary physics.
1954: Einstein to David Bohm

I must confess that I was not able to find a way to explain the atomistic character of nature. My opinion is that if the objective description through the field as an elementary concept is not possible, then one has to find a possibility to avoid the continuum (together with space and time) altogether. But I have not the slightest idea what kind of elementary concepts could be used in such a theory.
1954: Einstein to H. S. Joachim

The alternative continuum-discontinuum seems to me to be a real alternative; i.e., there is here no compromise.... Physics up to now is naturally in essence a continuum physics, in spite of the use of the material point, which looks like a discontinuous conceptual element, and has no more right of existence in a field description. Its strength lies in the fact that it posits parts that exist quasi-independently, beside one another. Upon this rests the fact that there are reasonable laws, that is rules which can be formulated and tested for individual parts. Its weakness lies in the fact that it has not been possible up to now to see how that atomistic aspect including quantum relations can result as a consequence. ..
Einstein to Joachim (cont’d)

An algebraic theory of physics is affected with just the inverted advantages and weaknesses; aside from the fact that no one has been able to propose a possible logical schema for such a theory. It would be especially difficult to derive something like a spatio-temporal quasi-order from such a schema. I cannot imagine how the axiomatic framework of such a physics would appear, and I don’t like it when one talks about it in dark apostrophes. But I hold it entirely possible that the development will lead there; for it seems that the state of any finite spatially limited system may be fully characterized by a finite number of numbers.
Einstein’s Last Words on the Subject—

Final words of Appendix Two to the Fifth Edition

One can give good reasons why reality cannot at all be represented by a continuous field. From the quantum phenomena it appears to follow with certainty that a finite system of finite energy can be completely described by a finite set of numbers (quantum numbers). This does not seem to be in accordance with a continuum theory, and must lead on to an attempt to find a purely algebraic theory for the description of reality. But nobody knows how to obtain the basis of such a theory (1954).
Where do we stand today?

The search for a theory of quantum gravity has led many physicists to search for ways to go beyond the continuum. Indeed, by setting a limit to how much energy can be crammed into a given volume before a black hole is formed, general relativity seems to set a lower limit to the validity of the concept of space-time region.
Thomas Thiemann (2001)

The concept of a smooth space-time should not have any meaning in a quantum theory of the gravitational field where probing distances beyond the Planck length must result in black hole creation which then evaporate in Planck time, that is, spacetime should be fundamentally discrete.
**Planck Units—Key to the Problems?**

\[ \ell_p = \sqrt{\frac{\hbar G}{c^3}}. \]

<table>
<thead>
<tr>
<th>PLANCK LENGTH</th>
<th>PLANCK TIME</th>
<th>PLANCK AREA</th>
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<tr>
<td>10^{-33} cm</td>
<td>10^{-43} sec (roughly the time light travels the Planck length)</td>
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**Planck Units:**
- Planck length, Planck time,
- Planck energy, Planck mass, and Planck temperature.

Each is determined by a unique combination of three constants, Planck’s constant, gravitational constant, and the velocity of light.
Thomas Thiemann (cont’d)

But clearly smooth diffeomorphisms have no room in such a discrete spacetime. The fundamental symmetry is probably something else, maybe a combinatorial one, that looks like a diffeomorphism group at large scales.
Most physicists believe that in any final theory of quantum gravity, space-time itself will be quantized and grainy in nature. .... So the smallest possible volume in four-dimensional space-time, the Planck volume, is $10^{-42}$ cubic centimetre seconds.
Rafael Sorkin & Causal set theory

Its aim is to construct a quantum theory of causal sets based on two features of classical general relativity that it takes as fundamental:

1) the causal structure, which is replaced by a discrete causal set; and
2) The four-volume element, which is replaced by the quantum of process.
Fay Dowker

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Lee Smolin and Carlo Rovelli
Quanta of volume and area
Quanta of Space or of Space-Time?

Whether one should be looking for quanta of space or quanta of space-time seems to be one essential point of difference between the canonical loop quantum gravity approach and the covariant causal set approach.

To my great surprise, I recently learned from an essay by John Stachel, that the ideas explored here are very similar to the ones that Einstein was exploring near the end of his career. Apparently, these ideas remained obscure because Einstein did not have the mathematical tools to carry them to fruition, and discussed them only orally or in letters.
Clock and Category (cont’d)

I was able to resist the urge to retitle this paper "On some ideas of Professor Einstein," but given the unconventional nature of my proposal, I am happy for the moral support. Einstein, after all, was often ahead of his time.
Thank You!
Thank you for coming to EinsteinFest