Multi-Resolution Inference: An Engineering (Engineered?) Foundation for Statistical Inference

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Liu, Keli and Xiao-Li Meng (2014). "A Fruitful Resolution To Simpson's Paradox via Multi-Resolution Inference." *The American Statistician*, 68: 17-29.

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What Does Big Data Mean For You?

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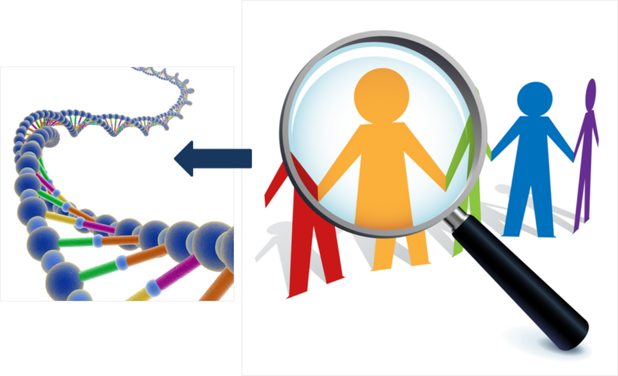
We see you and others more clearly.



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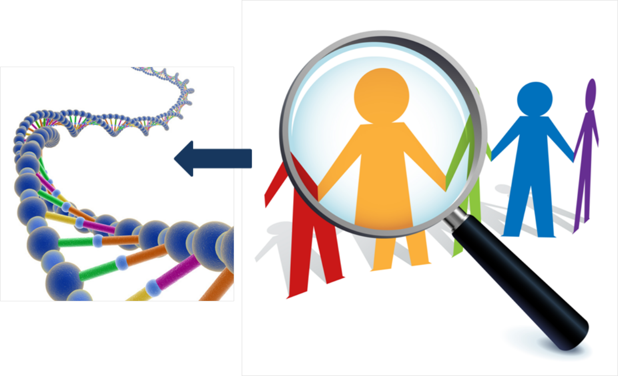

- individualized medicine

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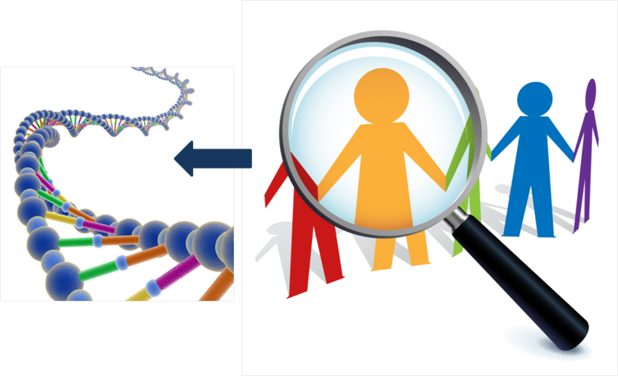

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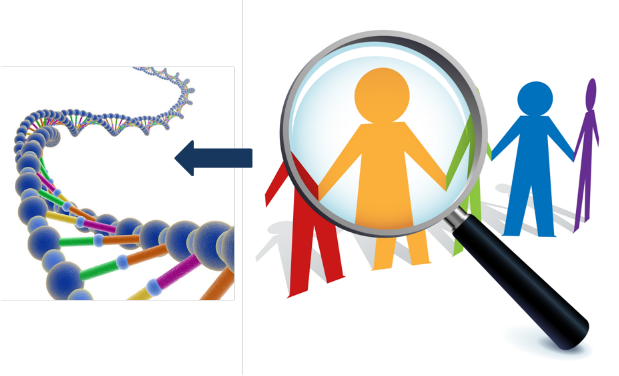

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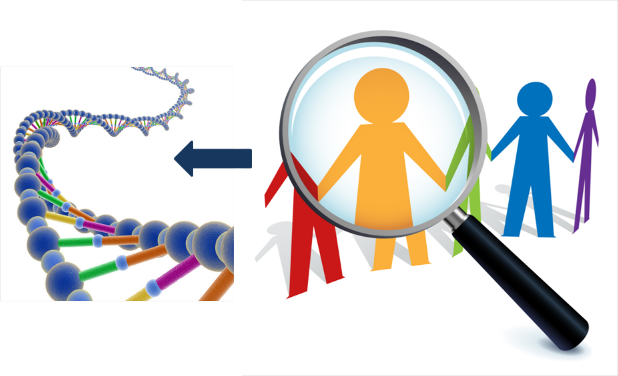

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The Gift and Curse of Big Data

Gift: Treatment for you based only on data from people like you.

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Gift: Treatment for you based only on data from people like you.

Curse: No one is perfectly like you.

What Does This Mean for Inference?

No two different objects are alike in all respects but one. It is obviously true; but it has no bearing on induction, where we deal with objects which we well know are, like all existing things, alike in numberless respects and unlike in numberless other respects.

Charles Peirce

The Statistically Payneful Question of Individualized Medicine

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3. **Inferential Question**: Given information constraints, how do we best approximate the ideal decision making process by an achievable one?

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Decompose aggregate signal into signals at varying resolutions.

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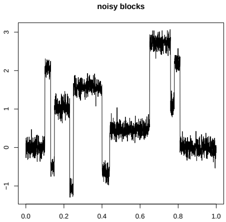

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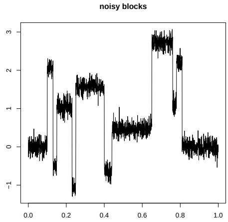

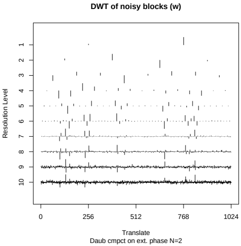

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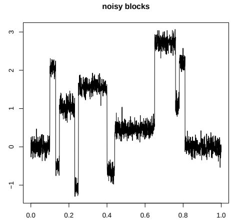

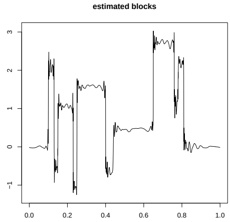

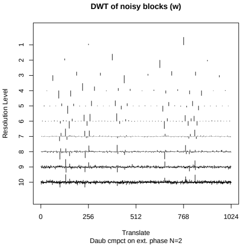

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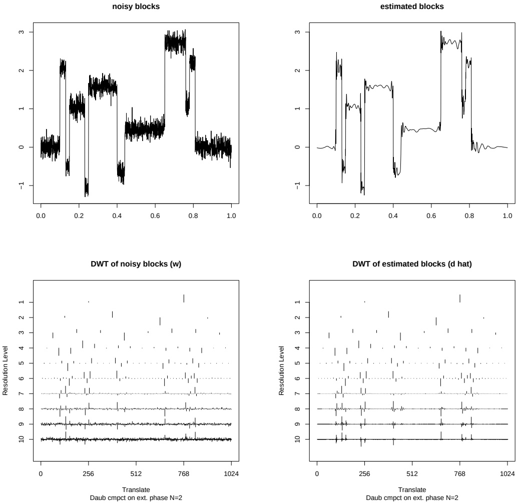

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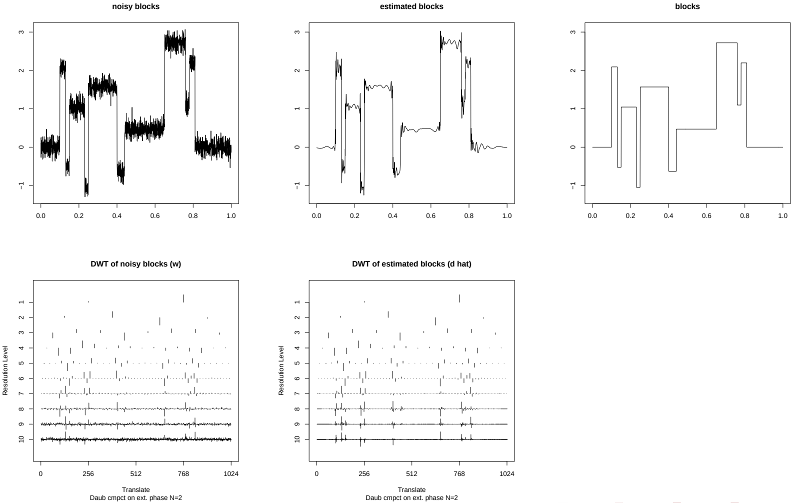

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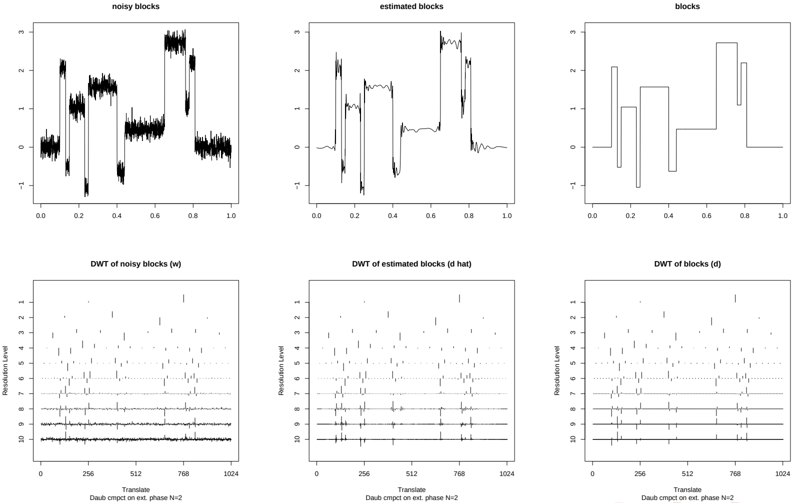

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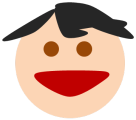

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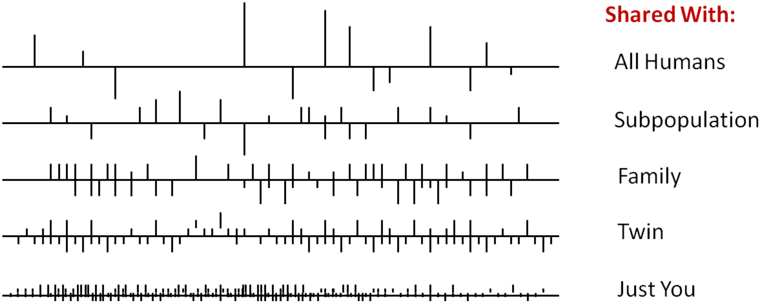

Kidney Stone Treatment

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- Open Surgery (A) vs. Percutaneous Nephrolithotomy (B).
- **Problem:** Stone size and treatment are confounded.

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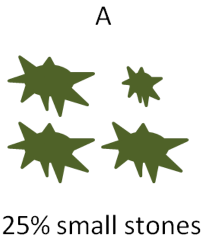

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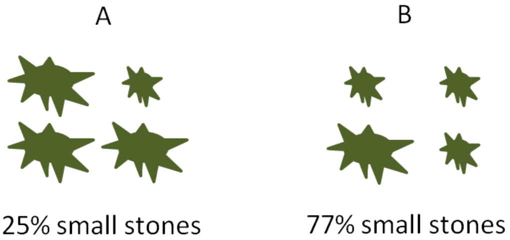

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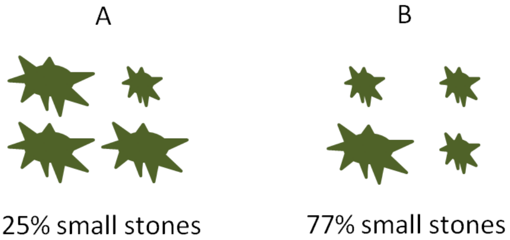

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- **Are more "similar" individuals always more comparable?**

Beauty Is Only Skin Deep

- Are Mark and Ben similar?

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Two Lessons

1. Individuals **apparently** similar may be **intrinsically** dissimilar.

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Two Lessons

1. Individuals **apparently** similar may be **intrinsically** dissimilar.
2. Definition of intrinsic similarity depends on the treatment.

Another Look at Kidney Stones

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- **Which treatment is better assuming randomization?**

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form : ☺ :: particular : $☺_t$

- No direct access to ☺, forms—only see functions $C(☺_t)$.

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- Intrinsic: post-treatment kidney stone size under treatment 1.

Observed Intrinsic Characteristics

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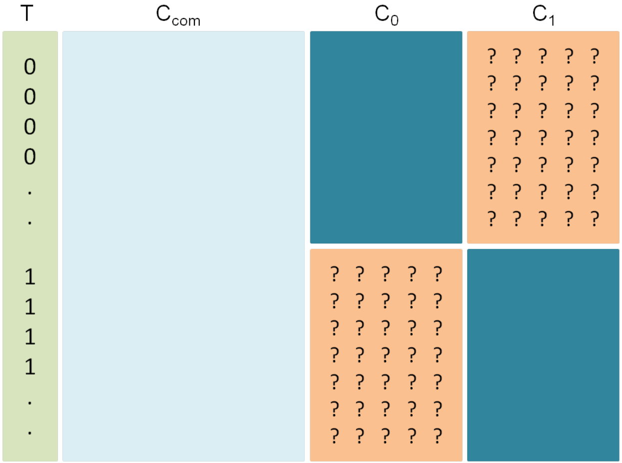

- **Without assumptions**, e.g. $C_0 = C_1$, C_{1-t} is always missing in group $T = t$.

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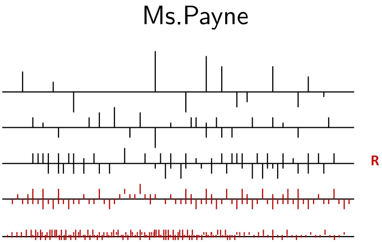

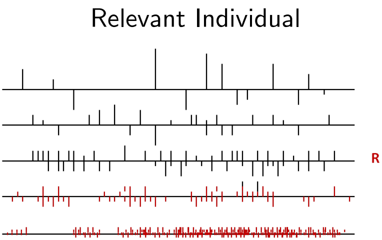

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- The ideal relevant subpopulation contains only Ms.Payne herself (and her exact clones)

$$\Omega_C(☺^*) = \{☺^*\}.$$

C consists of all possible intrinsic characteristics.

Study Data

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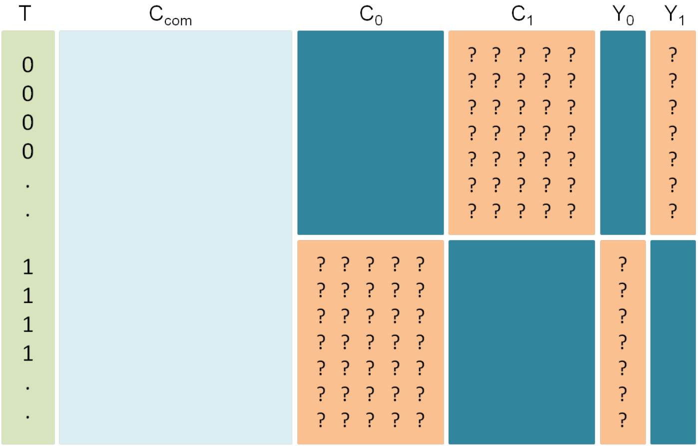

- We will always condition on C_{com}, denoted by $C = \emptyset$.

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$$Y_0 | ☺ \in \Omega_C(☺^*) \qquad Y_1 | ☺ \in \Omega_C(☺^*)$$

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	$Y_0 \mid ☺ \in \Omega_C(☺^*)$	$Y_1 \mid ☺ \in \Omega_C(☺^*)$
$C = \emptyset$	**Yes**	**Yes**

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	$Y_0 \mid ☺ \in \Omega_C(☺^*)$	$Y_1 \mid ☺ \in \Omega_C(☺^*)$
$C = \emptyset$	**Yes**	**Yes**
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The Estimation Question and Data Resolution

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$C = (C_0, C_1)$	**No**	**No**

The Gap Between Our Ideal and the Data

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- **BUT**, if C is too rich, we cannot estimate $\text{Ave}_C(Y_t)$ from data.

Which Philosophy for Big Data?

Frequentist OR Bayes

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More fundamentally...

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Coherent Joint Model OR Incoherent Piecemeal Models

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- **How to do principled corner cutting?**

Component-Wise Maximal Resolution

Estimate $\text{Ave}_C(Y_0)$

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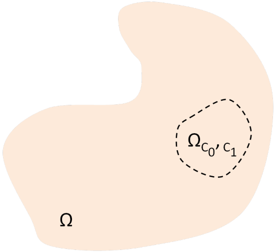

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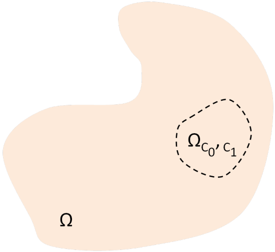

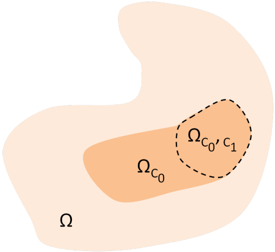

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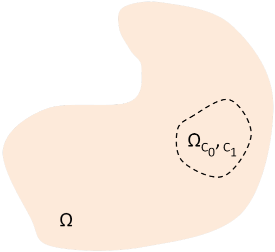

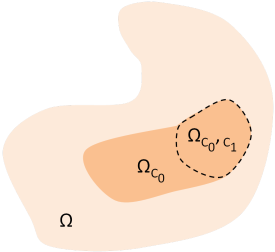

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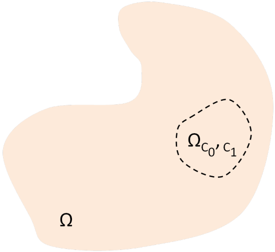

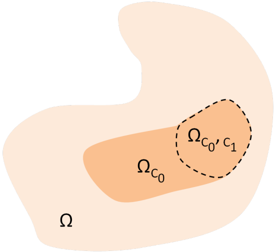

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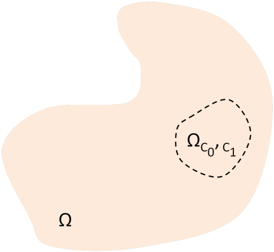

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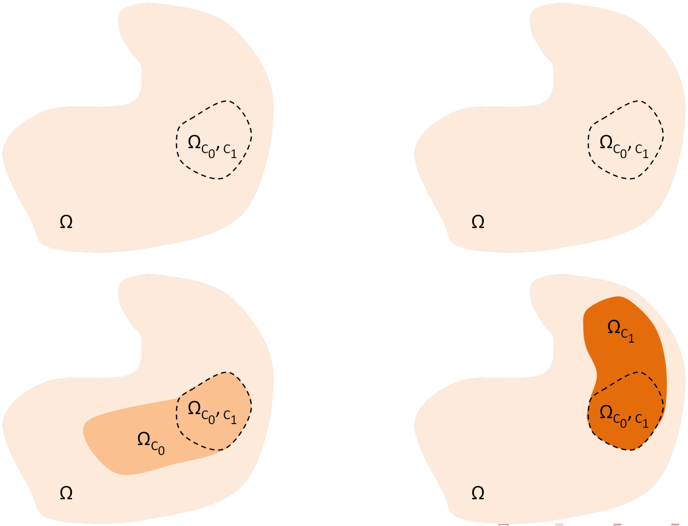

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The Inferential Quesiton and Resolution

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How best to choose an operational C**?**

Revisiting The Signal/Noise Dichotomy

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The Inferential Question

How should we choose $\Omega_C(☺^*)$?

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Revisiting The Signal/Noise Dichotomy

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$$Y_t(☺^*) = \text{Ave}_C(Y_t) + \epsilon = \text{signal} \quad + \quad \text{noise.}$$

In reality, signal and noise are two sides of the same coin.

$$\text{noise} = \text{unmodelled signal} \quad + \quad \text{intrinsic noise}$$

Noise Is Just Resolution Bias

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- Let $C^{(1)}, C^{(2)}, \ldots$ be a sequence of vectors: (i) $C^{(r)}$ contains intrinsic characteristics (ii) $C^{(r)}$ is nested in $C^{(r+1)}$.

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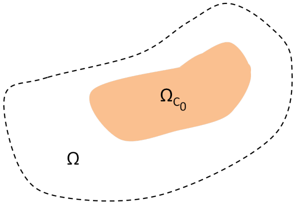
Signal at Resolution s for $s > r$.

$$\text{signal at } s = \text{Ave}_{C^{(s+1)}}(Y) - \text{Ave}_{C^{(s)}}(Y)$$

2 × Rule



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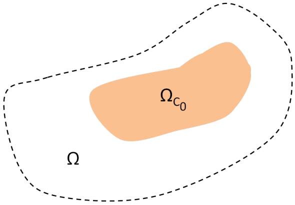

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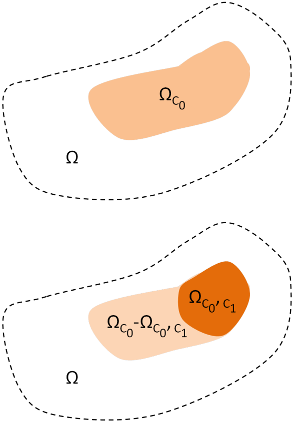

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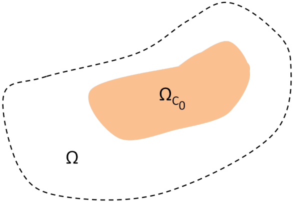

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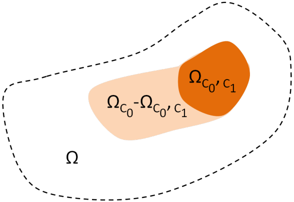

- We do not observe C_1.

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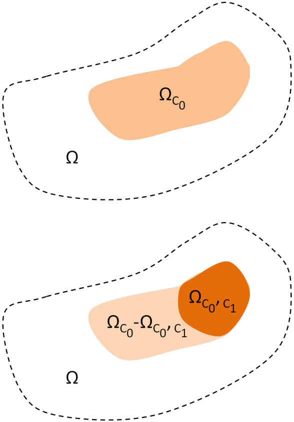

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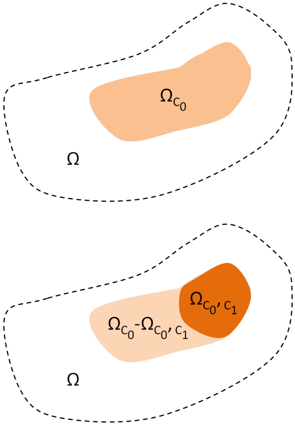

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- Discriminatory power in the observed data must be twice as substantial as discriminatory power in the missing data.

There Is Actually Math Involved...

- $\hat{Y}_t \equiv \mathrm{E}(Y_t|C_t)$, $R_t \equiv Y_t - \mathrm{E}(Y_t|C_t)$, $\sigma_t^2 \equiv V[\hat{Y}_t|C_{\text{com}}]$,

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The $2\times$ Rule

- $\mathrm{Ave}_{\tilde{C}_1}(Y_1) - \mathrm{Ave}_{\tilde{C}_0}(Y_0)$ beats $\mathrm{Ave}_{C_{\text{com}}}(Y_1) - \mathrm{Ave}_{C_{\text{com}}}(Y_0)$ in MSE if and only if

$$2[w\beta^{\text{mis}}_{1|0} + (1-w)\,\beta^{\text{mis}}_{0|1}] \le \left[w\left(1-\beta^{\text{obs}}_{1|0}\right) + (1-w)\left(1-\beta^{\text{obs}}_{0|1}\right)\right]$$

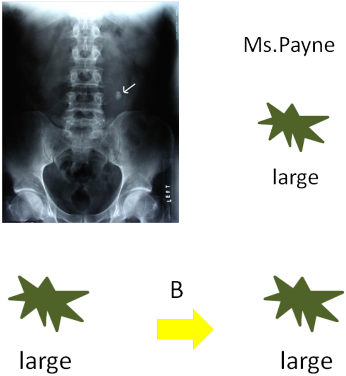
where $w = \sigma_0^2 / \left(\sigma_0^2 + \sigma_1^2\right)$.

Some Personalized Information

Ms.Payne

large

Some Personalized Information



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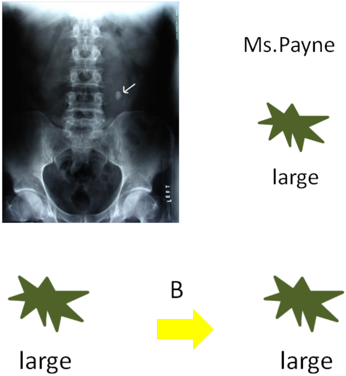

Problem

- The clinical trial data do not allow us to calculate the average of $Y_B - Y_A$ conditional on size $B =$ large.

What Comparison Should We Make?

Treatment A	Treatment B
78% (273/350)	**83%** (289/350)

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Small Stone	**93%** (81/87)	87% (234/270)
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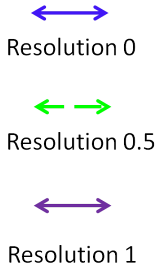

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- **Plausible Range**: $[-0.31, 0.43]$

A Multi-Resolution View of Big Data

Population Resolution

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Population Resolution

Individual Resolution