Focusing on Spectral Characteristics of Pc5 ULF Waves into 3D Radiation Belt Modeling

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ABSTRACT

The influence of ultra low frequency (ULF) waves in the Pc5 frequency range on radiation belt electrons in a compressed dipole magnetic field is examined. A model is developed describing the magnetic and electric fields associated with poloidal-mode Pc5 ULF waves. Frequency and L dependence of the ULF wave power is included in this model by incorporating published ground-based magnetometer data. This ULF model is used as input to a three dimensional guiding center particle code from which the L, energy and pitch angle dependence of the diffusion rates are analyzed. Results from a dipole magnetic field model are compared to a compressed dipole model in the equatorial plane.

CALCULATING 3D FIELDS

- Magnetic and electric field equations:
  \[ B = B_{\text{dipole}} + B_{c} + B_{\text{pol}} \]
  \[ E = E_{0} + E_{\phi} \]

where: \( B_{\text{dipole}} = B_{r} + B_{0} \)

- 3D dipole equation
  \[ B_{\text{dipole}} = \frac{2B_{0}R_{c}}{r^{3}} \cos(\theta) \hat{r} - \frac{B_{0}R_{c}}{r^{3}} \sin(\theta) \hat{\theta} \]

- Compression added to z component of magnetic fields
  \[ B_{z} = (b_{z} \cos \phi) \hat{z} \]

- ULF fields given by:
  \[ \nabla \times B = 0 \]
  \[ \frac{\partial}{\partial t} B = \nabla \times E_{\phi} \]

- Poloidal fields
  - Includes both radial AND compressional components

- Diffusion Coefficients
  - To calculate, use slope of best fit line of \( \langle AL \rangle_{\phi} \) vs time

RESULTS

- 4 case
  1) \( m_{1}=0, m_{2}=0 \)
  2) \( m_{1}=0, m_{2}=-2 \)
  3) \( m_{1}=1/3, m_{2}=0 \)
  4) \( m_{1}=1/3, m_{2}=-2 \)

- \( H_{(L)} = 100.5m_{1}L \)
- \( F_{(f)} = f^{0.5m_{1}} \)

Symbols:
- \( P_{\phi} \) - power of fields (nT/Hz)
- \( \phi \) - longitude
- \( m_{1} \) - slope of line, figure 2
- \( \omega \) - angular frequency
- \( L \) - background \( L \)
- \( \theta_{0} \) - initial phase
- \( f \) - frequency
- \( \theta \) - colatitude
- \( m \) - azimuthal mode
- \( m_{\phi} \) - slope of line, figure 4

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- \( H_{(L)} = 100.5m_{1}L \)
- \( F_{(f)} = f^{0.5m_{1}} \)

- \( J_{(\alpha,\omega)} = \cos(m_{\phi} - \alpha - \phi_{0}) \)

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value used: \( m = 2 \)

CONCLUSIONS

- Frequency-dependent ULF Wave Power
  - Causes \( D_{L} \) to decrease with increasing energy (fig 4)
  - Less power at higher resonances

- L-dependent ULF Wave Power
  - Changes \( L \) dependence of \( D_{L} \) from \( n=6 \) to \( n=18 \) (fig 5)

- Produces \( D_{L} \) max at equator => 2D good upper limit for space weather forecasting

- Adding compression to dipole
  - Offsets effects of additional resonances (fig 5)

Figure 1: Azimuthal wave power during a simulation of the Sept. 24-26, 1998 geoeagnetic storm using the LFM MHD magnetospheric model from Elkington [2004]. Slope of total power is \( m_{1} \) in \( H_{(L)} \) equation.

Figure 2: Baseband magnetometer observations of power vs frequency reported by Bloom and Singer [1995]. Slope is \( m_{1} \) in \( F_{(f)} \) equation.

Figure 3: Latitude dependence of magnetic and electric fields at \( L=6.6 \). Panel (a) is \( m_{1}=0 \) of \( |B|_{\text{pol}} = |B_{c} + B_{dip} \) off equator. Panel (b) is \( m_{1}=1/3 \) of \( |B| \) at equator. Panel (c) is poloidal electric field: max at equator.

Figure 4: \( W_{d} \) dependence of \( D_{L} \) in a dipole at \( L=5 \) for \( m_{1}=0 \). For (a), \( m_{2}=0 \), \( U_{c} = E^{2} + B^{2} \mu _{0} \) increases as frequency increases. Therefore, as energy gets larger, the drift frequency gets larger and so does the power. This leads to \( D_{L} \) increasing as energy increases. The opposite is true for (b), \( m_{2}=-2 \), \( U_{c} \) decreases as frequency increases \( \Rightarrow D_{L} \) gets smaller as energy gets larger.

Figure 5: \( L \) and pitch angle dependence of \( D_{L} \) in a dipole for \( m_{2}=-2 \). For (a), \( m_{2}=0 \), \( D_{L} \) gives \( n=6 \). For (b), \( m_{2}=1/3 \), \( n=18 \). For (a), \( m_{2}=0 \), \( |B| \) from fig. 3 is relatively constant with a max at equator leading to a weak pitch angle dependence of \( D_{L} \) with a max at \( \alpha_{0}=50-53^{\circ} \). For (b), \( m_{2}=1/3 \), \( |B| \) increases with \( \alpha_{0} \), leading to a larger diffusion coefficient at the equator, \( \alpha_{0}=90^{\circ} \).