BE 602: Ordinary differential equations (Fall 2015)

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Class: TTH 2:00 pm – 4:00 pm (Fall 2015 only) Sociology Bldg (SOC), Room B57
Recitation: F 2:00 pm – 3:00 pm (Fall 2015 only) Sociology Bldg (SOC), Room B57

Office Hours: By appointment, preferably after lunchtime!
Office: 44 Cummington St, Room 707

Course documents on Blackboard: https://learn.bu.edu

A linear algebra-centric view of ODEs: Unlike typical undergraduate ODE classes, we will investigate linear ODEs, nonlinear ODEs, and simple extensions into PDEs in the context of:

1. Theoretical linear algebra:
   - Linear operators (Sturm-Liouville problems)
   - Orthogonality, inner products, and finite norms
   - Stability, mapping, and basic topology concepts in nonlinear systems

2. Numerical linear algebra:
   - Decoupling ODE systems using diagonalization (decompositions)
   - Linear system-derived state equation block diagrams for representing the forward Euler / Runge-Kutta, backward Euler, and the Predictor-corrector algorithms
   - ODE Linearization; zero-finding algorithms for stability analysis

Enrollment: All graduate students or senior undergraduates (need department approval) are welcome!

Prerequisites for ODEs (BE 602)

- Linear algebra (finding eigenvalues, eigenvectors, and diagonalizing a matrix)
- Previous exposure to undergraduate ODEs is helpful
- Previous exposure linear or control systems also helps!

(Required)

Fall 2015

BE 601: Linear algebra (2 credits)
BE 602: Ordinary differential equations (2 credits)

Spring 2016

BE 603: Partial differential equations (2 credits)
BE 604: Statistical & numerical methods (2 credits)
Homeworks: Problem sets will be handed out usually on Wednesdays and will be due on the following Friday during recitation.

Recitations: Since we have an unbalanced number of lectures between BE 601 (14 lectures) and BE 602 (13 lectures) during Fall 2015, we may use up to 2 ODE recitations as “mini-lecture extensions” to reinforce topics taught on the Tuesday / Thursday lectures. I anticipate this happening after Lecture #4 (Fri 11/6/15), and depending on the class pace, either after Lecture #6 (Fri 11/13/15) or Lecture 8 (Fri 11/20/15). The rest of the recitations will be dedicated to discussions on problem sets.

Matlab: Since matrix-dependent elements will be in no short supply in this class, we will adopt Matlab as the standard software from which all course materials, homeworks, and take-home tests will be analyzed with.

Exams: Take-home exam at the end of each module
Duration = 1 week

Grading: The breakdown per module is: 80% homeworks (~6 per module) + participation during recitation
20% from take-home exam (1 per module)

BE 602: Ordinary differential equations (reading list)

Reading assignments: I will frequently assign readings from this list (especially the red ones) via Blackboard Learn!!
Selected PDFs will be available for you to download on the class website.

Additional “All-purpose” math textbooks:


ODE books:


Nonlinear ODEs:


Linear algebra (and yes – these books contain goodies on ODEs better than regular ODE books !):


Heat transfer:


Numerical methods:


## ODE syllabus:  
*(2nd half of Fall 2015)*

<table>
<thead>
<tr>
<th>Lectures (theme)</th>
<th>Topics</th>
<th>Key concepts</th>
<th>Applications</th>
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<tbody>
<tr>
<td>1-4 ODEs are related to eigenvalues &amp; eigenvectors</td>
<td>1&lt;sup&gt;st&lt;/sup&gt; and 2&lt;sup&gt;nd&lt;/sup&gt; order homogeneous ODE systems</td>
<td><strong>Intro to linear ODE systems</strong></td>
<td>• Mechanical networks</td>
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<td>Reduction of order</td>
<td>• <strong>Diagonalization</strong> = decoupling of ODE systems</td>
<td>• Chemical kinetics</td>
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<td>Non-homogeneous 1&lt;sup&gt;st&lt;/sup&gt; and 2&lt;sup&gt;nd&lt;/sup&gt; order ODE systems</td>
<td>• State equation block diagram formulations for numerical ODE algorithms</td>
<td>• Allosteric binding / enzyme kinetics</td>
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<td>• Phase-plane analysis for linear systems</td>
<td>• Axon modeling</td>
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<td>• Filters in signals &amp; systems design</td>
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<td>5 Root-finding algorithm</td>
<td>1-D and 2D Newton’s method</td>
<td>• Fixed-point search in nonlinear systems</td>
<td>• Stability analysis</td>
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<td>• The 2D Jacobian matrix contains 2 gradient row vectors that will be used to construct directional derivatives when linearizing ODE systems</td>
<td>• Prelude to understanding the steepest descent / Gauss-Newton algorithms in the numerical module</td>
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<td>6-7 Continuous-time nonlinear ODE systems</td>
<td>ODE Linearization</td>
<td><strong>Continuous-time nonlinear ODE systems</strong></td>
<td>• Biological, chemical, and mechanical oscillators</td>
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<td>Relaxation oscillators</td>
<td>• Fixed point stability, bifurcations, and limit cycles</td>
<td>• Phase-locked loops (PLL) in engineering + biology</td>
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<td>Coupled oscillators</td>
<td>• Using reduction order to analyze high-order systems</td>
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<td>8-9 Discrete-time systems</td>
<td>Difference equations</td>
<td><strong>Discrete-time linear / nonlinear systems</strong></td>
<td>• Biological and digital networks</td>
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<td>Linear Markov chains and Leslie matrices (nonlinear)</td>
<td>• One-to-one analogues between discrete and continuous-time systems (bifurcations, limit cycles, etc.)</td>
<td>• Populations dynamics</td>
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<td>• Period-doubling</td>
<td>• Introduction to chaos</td>
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<td>10-13 Fusion between linear algebra &amp; theoretical ODEs</td>
<td>Sturm-Liouville equations</td>
<td><strong>Survey of boundary value problems (BVP)</strong></td>
<td>• Variations of Fourier’s law in transport problems</td>
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<td>• <strong>Real eigenvalues</strong> -&gt; orthogonal eigenfunctions are guaranteed if the differential equation can be written in ST-form (Hermitian operator)</td>
<td>• Survey of Legendre, associated Legendre, and Bessel functions</td>
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<td>(Crucial for solving linear PDEs)</td>
<td>• <strong>Generalized Fourier series</strong></td>
<td>• Reconstruction of simple Dirichlet BCs for Laplace’s Equation in cylindrical and spherical coordinates</td>
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