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ACCURATE FORMULA FOR THE SELF-COMPTON X-RAY FLUX DENSITY FROM A UNIFORM, SPHERICAL, COMPACT RADIO SOURCE

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ABSTRACT

A more accurate calculation, plus a correction to the exponent of the $(1 + z)$ factors, in the formula for self-Compton X-ray flux density as given previously by Marscher *et al.*, is presented.

Subject headings: quasars — radio sources: general — X-rays: general

I. INTRODUCTION

The observed correlation of X-ray emission from quasars with radio emission (Zarnorani *et al.* 1981; Ku, Helfand, and Lucy 1980) and millimeter-wave flux (Owen, Helfand, and Spangler 1981) calls out for a theoretical explanation. Synchrotron self-Compton scattering within compact radio components is a likely candidate for the mechanism behind the radio-X-ray connection (Jones, O'Dell, and Stein 1974*a*). Expressions relating the observed radio parameters to the theoretically predicted self-Compton X-ray flux density can be calculated very accurately in the case of a uniform, spherical synchrotron source. Marscher *et al.* (1979) presented such a formula (expression [6] in that paper), which has begun to be used by other investigators (e.g., Urry and Mushotzky 1982). In this paper, the Marscher *et al.* results are improved in terms of accuracy and extended to steeper spectral slopes. In addition, an incorrect exponent to the $(1 + z)$ factor in formula (6) of Marscher *et al.* is amended. Although the extreme sensitivity of the calculated self-Compton flux to errors in the observable parameters allows only rough estimates, it is felt by this author that the errors should not be exacerbated by formulation which contains further inaccuracies.

II. FORMULAE

The "standard" model for a compact synchrotron source assumes a uniform (in strength, but disordered in direction) magnetic field **B**, plus an isotropic, power-law distribution of electrons

$$N(E) = N_0 E^{-(2\alpha+1)} (\gamma_1 mc^2 < E < \gamma_2 mc^2) . \quad (1)$$

A spherical source of radius R and luminosity distance in gigaparsecs $D_{\rm Gpc}$ is adopted here. All of the basic synchrotron and Compton formulas are given by Gould (1979) for this case. From his equations (13), (15), (34), and (40), one can obtain two equations which relate B and N_0 to the observable parameters θ (angular diameter in milliarcseconds), ν_m (frequency of synchrotron self-absorption turnover in gigahertz), S_m (flux density at ν_m in Janskys, extrapolated from the optically thin spectrum, $\nu_{\rm thin} \gg \nu_m$, by the law $S_m = S_\nu[\nu_{\rm thin}][\nu_m/\nu_{\rm thin}]^{-\alpha}$), spectral index α (optically thin flux density $S_\nu \propto \nu^{-\alpha}$), redshift z, and δ $(= \Gamma^{-1}[1 - \beta \cos \phi]^{-1}$ to take into account relativistic bulk motion of the source, ϕ = angle of velocity vector = 0 along line of sight, β = velocity in units of c, $\Gamma = [1 - \beta^2]^{-1/2})$. The result is

$$B = 10^{-5} b(\alpha) \theta^4 \nu_m{}^5 S_m{}^{-2} \left(\frac{\delta}{1+z}\right) \text{ gauss} \quad (2)$$

and

$$N_0 = n(\alpha) D_{\rm Gpc}{}^{-1} \theta^{-(4\alpha+7)} \nu_m{}^{-(4\alpha+5)} S_m{}^{2\alpha+3} \times (1+z)^{2(\alpha+3)} \delta^{-2(\alpha+2)} . \quad (3)$$

The parameters $b(\alpha)$ and $n(\alpha)$ are listed in Table 1 for typical values of the spectral index α.

The first-order Compton flux density can be derived from equation (3) above and Gould's equations (13), (15), (20), and (23) (with his eq. [23] multiplied by $h^2\nu$):

$$S_\nu{}^C(E_{\rm keV}) \approx d(\alpha) \ln (\nu_2/\nu_m) \theta^{-2(2\alpha+3)} \nu_m{}^{-(3\alpha+5)} \times S_m{}^{2(\alpha+2)} E_{\rm keV}{}^{-\alpha} [(1+z)/\delta]^{2(\alpha+2)} \ \mu\text{Jy} , \quad (4)$$

within the limits of applicability (see below).

Here $\nu_2 = 2.8 \times 10^6 B \gamma_2{}^2$ is the upper frequency cutoff to the synchrotron spectrum, and $d(\alpha)$ is given in Table 1. The only inaccuracy of this expression lies in the approximation that the synchrotron flux density is sharply cut off above ν_2 and below ν_m. However, since this error is contained within a logarithmic factor, the resultant uncertainty in $S_\nu{}^C$ owing to this is quite small. It should

TABLE 1

DIMENSIONLESS PARAMETERS OF THE SPECTRAL INDEX α

α	b	n	d
0.25	1.8	7.9	130
0.50	3.2	0.27	43
0.75	3.6	0.012	18
1.00	3.8	0.00059	9.1

be stressed that *expression (4) is valid only within the limits*

$$5.5 \times 10^{-9} \gamma_1{}^2 \nu_m \lesssim E_{\rm keV} \lesssim 0.2 b^{-1}(\alpha) \theta^{-4} \nu_2{}^2 \nu_m{}^{-5} S_m{}^2 [(1+z)/\delta]^2 \ . \quad (5)$$

Another caveat is pertinent: when the angular diameter θ is determined from very long baseline interferometry, it is *not* appropriate to use a Gaussian full-width at half-maximum size, θ_G, in the above formulas, since this is substantially less than the diameter of a spherical source which could be used to reproduce the data. For a partially resolved component (visibility amplitude greater than 0.5), the substitution $\theta = 1.8\theta_G$ is reasonably accurate.

The practical accuracy of the above expressions is limited by the uncertainties in determining θ, ν_m, S_m, α, and ν_2 observationally. Compact radio sources tend to be quite complex, so that one needs to determine the above parameters for each individual component (e.g., Jones, O'Dell, and Stein 1974*b*; Marscher and Broderick 1981). In addition, departures from spherical symmetry and nonuniformities complicate any determination of the Compton X-ray flux (see, e.g., Gould 1979).

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