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Survey Paper

A Survey of Models, Analysis Tools and Compensation Methods for the Control of Machines with Friction*

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This survey addresses contributions from the tribology, lubrication and physics literatures, as well as the controls literature, which are important for the understanding and compensation of friction in servo machines.

Key Words—Friction; friction compensation; friction modeling; identification; adaptive control; control applications; feedback control; feedforward control; modeling.

Abstract—While considerable progress has been made in friction compensation, this is, apparently, the first survey on the topic. In particular, it is the first to bring to the attention of the controls community the important contributions from the tribology, lubrication and physics literatures. By uniting these results with those of the controls community, a set of models and tools for friction compensation is provided which will be of value to both research and application engineers.

The successful design and analysis of friction compensators depends heavily upon the quality of the friction model used, and the suitability of the analysis technique employed. Consequently, this survey first describes models of machine friction, followed by a discussion of relevant analysis techniques and concludes with a survey of friction compensation methods reported in the literature. An overview of techniques used by practising engineers and a bibliography of 280 papers is included.

1. INTRODUCTION

FRICTION IS PRESENT in all machines incorporating parts with relative motion. Although friction may be a desirable property, as it is for brakes, it is generally an impediment for servo control. The literature relevant to friction and control is very widely scattered; important ideas are to be found in the journals of controls, tribology, lubrication engineering, acoustics, and general engineering and physics. It is the aim of this survey to synthesize the contributions of several hundred articles from the several disciplines, and the input of engineers in industry who have worked with friction and control, to produce a grand picture of models and methods important for friction and control.

Tribology is the science of rubbing contacts. The field is active, with 1000 investigators in North America and a literature that grows by some 700 articles per year; and great progress has been made towards understanding the physical processes of sliding machine contacts: bearings, transmission elements, brushes, seals, etc. For the controls engineer, it is frictional *dynamics* which is of greatest interest. One challenge of this review has been to bring together from the tribology literature an understanding of frictional dynamics. Tribology is concerned with friction; but in recent years the field has been most concerned with issues of wear and machine life on the one hand, and of surface chemistry and physics on the other. Dynamics has not been a focus. Studies in frictional dynamics carried out over the past five decades are brought together in this survey.

Investigations within the field of controls have not capitalized adequately on the friction models available from the experimental and theoretical work of tribology. Many investigations have brought together powerful tools from stability theory, nonlinear control, nonlinear system identification, adaptive control and other areas; but these investigations have been based on the friction models of Leonardo Da Vinci or elementary physics. It is no wonder that consistent results have been elusive and that the analysis tools capable of predicting stick slip and other frictional behavior are not fully reliable. Within tribology there is considerable understanding of the frictional dynamics of lubricated metal-on-metal contacts; and, while perhaps somewhat more complex than Leonardo's static + Coulomb friction model,

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models are available to account for the dynamics observed in a broad range of tribology experiments, some conducted with remarkable resolution of sensing.

In Section 2 of this paper, friction modeling is addressed. Results from a range of experiments reported in the tribology, mechanism, physics and controls literatures are presented and assimilated. At the end of Section 2, an integrated friction model is presented. In Section 3, analysis tools are presented for studying servos with friction. Many of the methods presented have been applied in the controls literature, including analytic methods, the describing function and phase plane analysis; but investigations have also been carried out in the areas of acoustics and mechanics, where frictional instability may be a major contributor to processes of interest. In Section 4, compensation methods for machines with friction are presented. Here the controls literature is the major contributor. The broad classes of compensation strategy are problem avoidance, non-model-based control and model-based control. Problem avoidance deserves special consideration because, as we will see in Section 2, minor modification of the lubrication may have a tremendous impact on the frictional instability, and friction modification is not always a priority of the lubrication engineer. Parameter identification and adaptive control strategies are also addressed in Section 4, as is input from engineers in industry. In Section 5, we conclude with a program for tackling the challenging problems posed by friction in servo-controlled machines.

2. FRICTION IN MACHINES

When methods of feedback control are applied to moving bodies, friction is inevitably among the forces of motion. The field of control has long incorporated sophisticated investigations of other contributions to the forces of motion, such as multibody dynamics, electromagnetics, and aero- or fluid dynamics. But the forces of motion contributed by friction are often studied with simplified models, similar to those employed by Leonardo Da Vinci. The English language literature of tribology grows at a rate of 700 articles per year and represents a vast, modern effort to understand these phenomena. While often academically pursued, tribology is hardly academically motivated: energy loss due to friction and the failure of equipment due to wear represent a considerable percentage of every modern economy.

Feedback control is often applied to mechanical arrangements involving metal-on-metal contact with grease or oil lubrication. Issues of manufacture and performance motivate the choice of metals for working members; and issues of service life motivate the use of fluid lubricants. This study will concentrate on what tribology has to offer towards the modeling of friction in fluid lubricated metal-on-metal junctions. Specialized tribological studies are available which address other combinations of engineering materials, such as plastics on metal and dry lubricated and electrical contacts.

The classic model of friction—friction force is proportional to load, opposes the motion, and is independent of contact area—was known to Leonardo da Vinci, but remained hidden in his notebooks for centuries. Rabinowicz (1965) argues that the scientific study of friction must have been subsequent to the elucidation of Newton's first law (Newton, 1687) and the modern conception of force. This is not quite true. Da Vinci's ideas on the nature of force, of which he knew friction to be an example, provide a fascinating insight into problems of pre-Newtonian natural philosophy (Da Vinci, 1519).

Da Vinci's friction model was rediscovered by Amontons (1699) and developed by Coulomb (1785) among others. Amontons' claim that friction is independent of contact area (the second of Da Vinci's laws) originally attracted skepticism, but was soon verified. Morin (1833) introduced the idea of static friction and Reynolds (1866) the equation of viscous fluid flow, completing the friction model that is most commonly used in engineering: the static + Coulomb + viscous friction model (Morin, 1833; Reynolds, 1886) and shown in Fig. 1(b).

The science of tribology (Greek for the study of rubbing) was born in England in the 1930s. Basic questions of wear mechanisms, true contact area, relationships between friction, material properties and lubricating processes were addressed and answered. It is not possible here to give tribology its due. The interested reader is referred to Bowden and Tabor (1956, 1973), Suh and Sin (1981), Czichos (1978), which provide excellent and readable introductions to the field. Dowson (1979) is an engaging work which illuminates the 3000 year history of man's attempts to understand and modify friction. Hamrock (1986) is a brief handbook survey of the relevant methods of tribology; and Halling (1975) provides a survey that is rigorous but not overly detailed and sufficiently sweeping to address such issues as friction induced instability and solid lubrication. Ludema (1988) is an interesting critique of tribology and cultural barriers to interdisciplinary pursuits; and Rabinowicz (1978), a discussion of priorities for tribology.

2.1. The Tribology of Machine Friction

The majority of servo-controlled machines, of the earth-bound variety at least, are lubricated with oil or grease. Tribologically, greases and oils have more in common than not. Grease is essentially a soap matrix that carries oil, which is released under stress into load bearing junctions. These lubricants are widely used because they provide a fluid barrier between rubbing metal parts that exchanges dry friction for viscous friction and vastly reduces wear. The fluid barrier can be maintained by forcing lubricant under pressure into the load bearing interface, a technique called hydrostatic lubrication. This, however, entails great mechanical complexity and is not applicable to many bearing or transmission designs. The more common technique is that of hydrodynamic lubrication, wherein the lubricant is drawn into the interface by the motion of the parts. Hydrodynamic lubrication is simple to implement, requiring only a bath of oil or

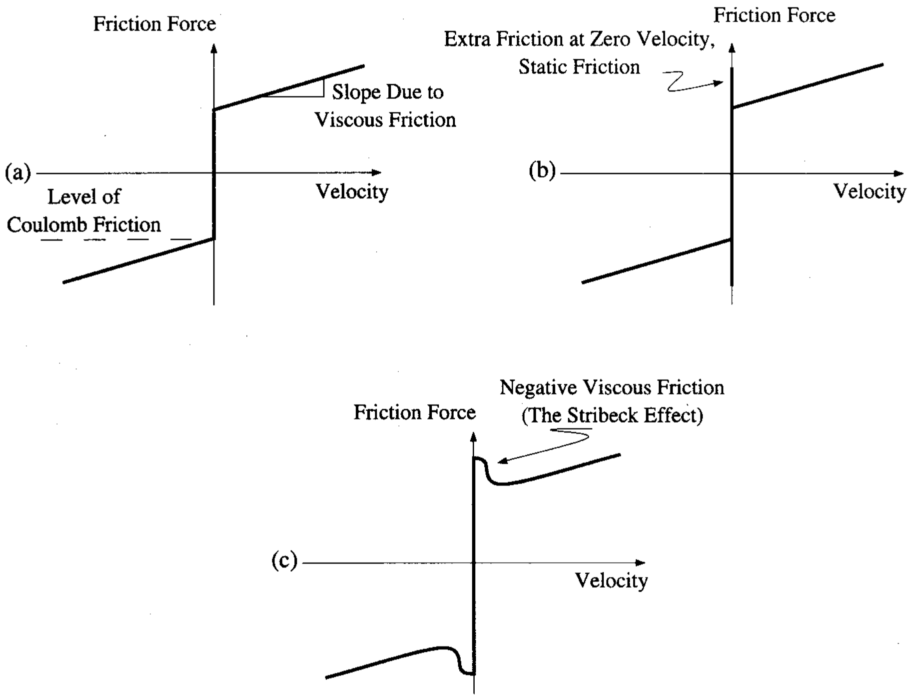

FIG. 1. Friction models: (a): Coulomb + viscous friction model; (b): static + Coulomb + viscous friction model; (c): negative viscous + Coulomb + viscous friction model (Stribeck friction).

grease or perhaps a fluid spray, but suffers the limitation that the fluid film is maintained only above some minimum velocity. Below the minimum velocity solid-to-solid contact occurs.

2.1.1. *The topography of contact*
To understand the tribology of engineering surfaces it is necessary to consider the surface topography. Early models of friction failed because the surface topography was misunderstood. The interactions at contacting surfaces will be examined by considering progressively smaller contacts. In Fig. 2 a conformal contact is shown schematically; part A rests on part B. Kinematically, such contacts are identified as area contacts: the apparent area of the contact is determined by the size of the parts.

Parts that do not enjoy a matching radii of curvature meet at nonconformal contact, as shown in Fig. 3. These contacts are called point or line contacts when considered kinematically; but this is an idealization. In fact the parts deform to create an apparent area of contact, an area that increases with

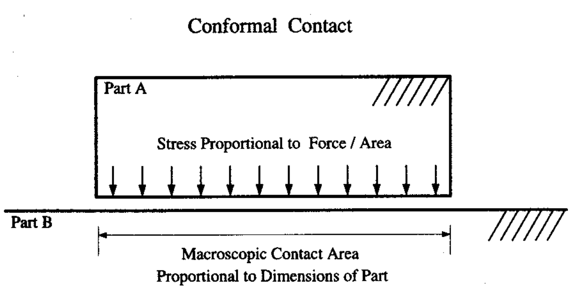

FIG. 2. Conformal contact, such as machine guide ways or journal bearings.

increasing load. The one millimeter contact width suggested in Fig. 3 is typical of small machine parts, such as the transmission gears of an industrial robot.

Tribology as a field is sophisticated in the use of similitude. One widely used transformation maps a nonconformal contact of two radii to one of a flat surface and a single curved part, as suggested in Fig. 3 (Dowson and Higginson, 1966; Hamrock, 1986). This transformation greatly simplifies the study of nonconformal contacts. Nonconformal contacts arise frequently in machinery and may be referred to as Hertzian contacts, after the original analysis (Hertz, 1881). The stresses found in conformal contacts between steel parts are rarely higher than 7 MPa (7 MPa = 1000 psi), whereas in nonconformal contact the peak stress can be 100 times greater (Hamrock, 1986). A stress of 700 MPa corresponds to 100,000 psi, which is greater than the yield strength of many types of steel. This is possible in Hertzian contact because the stress is compressive.

In a BBC radio program, tribology pioneer F. P. Bowden observed that "putting two solids together is rather like turning Switzerland upside down and standing it on Austria—the area of intimate contact will be small" (Bowden, 1950). Crystalline surfaces, even apparently smooth surfaces, are microscopically rough. The protuberant features are called asperities and, as shown schematically in Fig. 4, the true contact occurs at points where asperities come together. In this way, the true area is much smaller than the apparent area of the contact (Bowden and Tabor, 1939). Over a broad range of engineering materials, the asperities will have slopes ranging from 0 to 25 degrees and concentrated in the band from 5 to 10 degrees (Dowson, 1979).

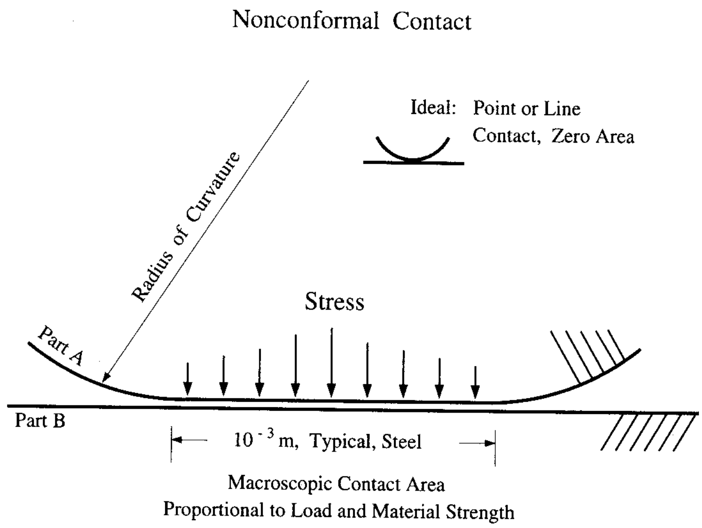

FIG. 3. Nonconformal contact, such as a gear tooth mating or roller bearings.

When asperities come into contact, the local loading will be determined by the strength of the materials. The asperities deform to generate the contact area necessary to take up the total load. As a first approximation, we may consider the local stress at an asperity junction to be in proportion to the yield strength of the material. The contact area, on the other hand, is in direct proportion to the total load. As a rule of thumb, the true contact area, A, is given by $A = W/3Y$, where W is the load and Y is the yield strength of the material. Contact stress at the asperity is taken, by this rule of thumb, to be three times the yield strength. As with the nonconformal contact, stress greater than yield strength is possible because the asperities are under compression.

Friction is proportional to the shear strength of the asperity junctions. As the load grows, the junction area grows; but, to first-order, the shear strength (measured per unit area) remains constant. In this way, friction is proportional to load. If truly clean metal surfaces are brought into contact, the shear strength of the junction (friction) can be as great as the shear strength of the bulk material, and the friction coefficient can be much greater than one (Bowden and Tabor, 1973; Hamrock, 1986). Fortunately for the operation of machines, truly clean surfaces are all but impossible to achieve. Even in the absence of lubricants, oxide films will form on the surface of steel and other engineering materials, producing a boundary layer. In the presence of lubricants, additives to the bulk oil react with the surface to form the boundary layer. The boundary layer additives are formulated to control the friction and wear of the surface. The boundary layer is a solid, but because it has the lower shear strength, most shearing occurs in this film. If the boundary layer has a low shear strength, friction will be low; if it has good adhesion to the surface and can be replenished from the oil, wear will be reduced. Boundary layer thickness varies from a few atomic thicknesses to a fraction of a micron. As suggested in Fig. 4, a tenth of a micron is a typical thickness of the boundary layer formed by the lubricity additives of industrial oil (Wills, 1980; Booser, 1984). Note that this is perhaps two orders of magnitude less than the typical dimension of an asperity in steel junctions. The boundary layer is exactly that, and does not markedly influence the area or local stresses of contact.

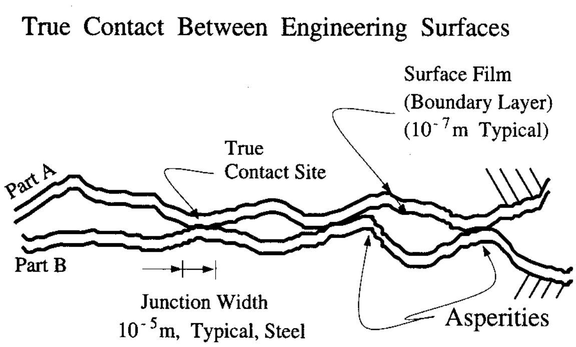

FIG. 4. Part-to-part contact occurs at asperities, the small surface features.

2.1.2. *Friction as a function of velocity: four dynamic regimes*

There are four regimes of lubrication in a system with grease or oil: static friction, boundary lubrication, partial fluid lubrication and full fluid lubrication. These four regimes each contribute to the dynamic that a controller confronts as the machine accelerates away from zero velocity. Figure 5 is known as the Stribeck curve and shows the three moving regimes (Stribeck, 1902; Biel, 1920; Czichos, 1978). The interesting characteristics of regime I, static friction, are not dependent on velocity.

2.1.2.1. *The first regime: static friction and presliding displacement.* In Fig. 4, contact is shown to occur at asperity junctions. From the standpoint of control, these junctions have two important behaviors: they deform elastically, giving rise to presliding displacement; and both the boundary film and the asperities deform plastically, giving rise to rising static friction, discussed in Section 2.1.4 below.

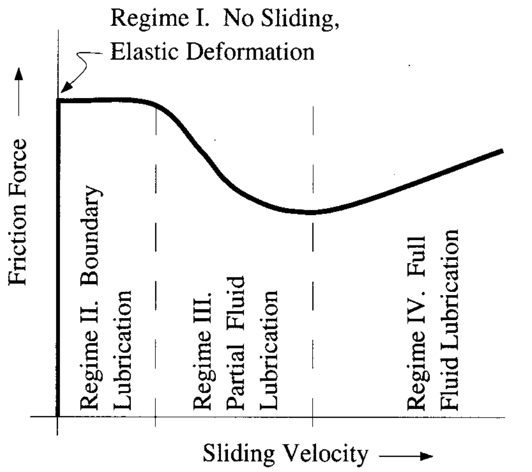

FIG. 5. The generalized Stribeck curve, showing friction as a function of velocity for low velocities.

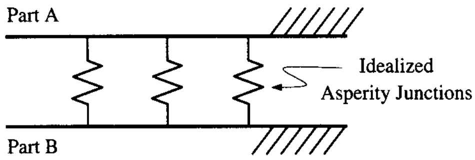

FIG. 6. Idealized contact between engineering surfaces in static friction. Asperity contacts behave like springs.

It is often assumed when studying friction that there is no motion while in static friction, which is to say no motion without sliding; but in mechanics it is well known that contacts are compliant in both the normal and tangential directions, e.g. Johnson (1987). Dahl (1968, 1976, 1977), studying experimental observations of friction in small rotations of ball bearings, concluded that for small motions, a junction in static friction behaves like a spring and considered the implications for control. There is a displacement (presliding displacement) which is an approximately linear function of the applied force, up to a critical force, at which breakaway occurs. The elasticity of asperities is suggested schematically in Fig. 6. When forces are applied, the asperities will deform, as suggested by Fig. 7, but recover when the force is removed, as does a spring. In this regime, the tangential force is governed by:

$$F_t(x) = -k_t x, \tag{1}$$

where F_t is the tangential force, k_t is the tangential stiffness of the contact and x is displacement away from the equilibrium position. F_t and x refer to the force and displacement in the contact before sliding begins, as indicated in Figs 7 and 8. When the applied force exceeds the required breakaway force, the junctions break (in the boundary layer, if present) and true sliding begins, as suggested in Fig. 8. Polycarpou and Soom (1992) have pointed out that static friction is not truly a force of friction, as it is neither dissipative nor a consequence of sliding; but is a force of constraint, and employ the term tangential force. This issue is important for both simulation and analysis.

The tangential stiffness, k_t, is a function of asperity geometry, material elasticity and applied normal force (Johnson, 1987). Note that the tangential stiffness due to presliding displacement is quite different from (and may be substantially less than) the stiffness of the mechanism itself. The asperities, not the mechanism components, are deforming. When normal force is changing, the behavior may be quite complex, because normal force, normal stiffness and tangential stiffness are nonlinear, interacting functions of normal displacement (Martins *et al.*, 1990). To first approximation, it is actually the breakaway displacement that is constant; and the stiffness is then given by:

$$k_t = \frac{F_b}{x_b}, \tag{2}$$

where F_b is the breakaway force and x_b is the maximum deformation of the asperities before breakaway. If normal force is varying and the coefficient of static friction is approximately constant, then k_t becomes proportional to normal force.

The breakaway displacement may be minute in engineering materials, breakaway is observed to occur with deflections on the order of 2–5 microns in steel junctions (Rabinowicz, 1951; Dahl, 1968; Burdekin *et*

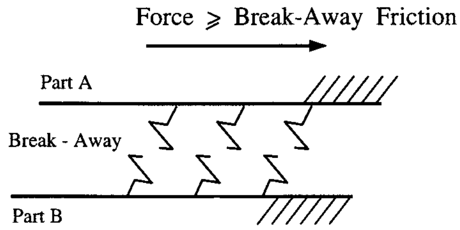

FIG. 8. At breakaway true sliding begins.

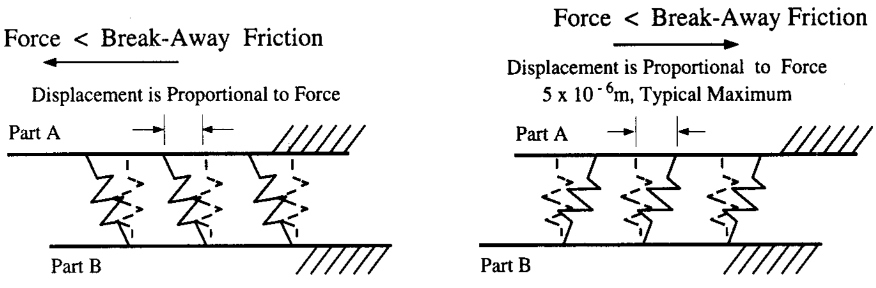

FIG. 7. Asperity deformation under applied force, presliding displacement.

al., 1978; Cheng and Kikuchi, 1985; Villanueva-Leal and Hinduja, 1984; Armstrong-Hélouvry, 1991). But elsewhere in a mechanism a much greater displacement may be observed, displacement significant on the scale of feedback control. This will arise, for example, in robots, where the arm itself acts as a lever to multiply micron motions at the gear teeth to millimeter motions of the output (Armstrong-Hélouvry, 1991).

Presliding displacement has long been studied in the mechanics community, and is sometimes termed micro-slip (Johnson, 1987). The transition from elastic contact to sliding is not simple. Sliding is observed to originate first at the boundary of a contact and to propagate toward the center (Johnson, 1962). Thus there is no abrupt transition to sliding. Presliding displacement is of interest to the controls community in extremely high precision pointing applications (Dahl 1977; Walrath, 1984) in dynamics (Canudas de Wit *et al.*, 1993) and in simulation (Haessig and Friedland, 1991); and may also be important in establishing that there are no discontinuities in friction as a function of time.

2.1.2.2. *The second regime: boundary lubrication.* In the second regime—that of very low velocity sliding—fluid lubrication is not important, the velocity is not adequate to build a fluid film between the surfaces, e.g. Fuller (1984). As described, the boundary layer serves to provide lubrication. It must be solid so that it will be maintained under the contact stress, but of low shear strength to reduce friction (Bowden and Tabor, 1973). In Fig. 9 sliding in boundary lubrication is shown. Because there is solid-to-solid contact, there is shearing in the boundary lubricant. Because boundary lubrication is a process of shear in a solid, it is often assumed that friction in boundary lubrication is higher than for fluid lubrication, regimes three and four. This, however, is not always the case; it is not necessary that the shear strength of a solid be greater than the viscous forces of a fluid. Consider that glass is a fluid, with a viscosity great enough that centuries are required for it to flow to the bottom of the window frame. Many solids will yield to a lower shear force than the forces of viscous flow in this fluid. Certain boundary lubricants do reduce static friction to a level below Coulomb friction and entirely eliminate stick–slip. Some aspects of these and other boundary lubricants are described in Section 2.1.3 below.

2.1.2.3. *The third regime: partial fluid lubrication.* Shown in Fig. 10 is the process by which lubricant is drawn into the contact zone. Lubricant is brought into the load bearing region through motion, either by sliding or rolling. Some is expelled by pressure arising from the load, but viscosity prevents all of the lubricant from escaping and thus a film is formed. The entrainment process is dominated by the interaction of lubricant viscosity, motion speed and contact geometry. The greater viscosity or motion velocity, the thicker the fluid film will be. When the film is not thicker than the height of the asperities, some solid-to-solid contact will result and there will be partial fluid lubrication. When the film is sufficiently thick, separation is complete and the load is fully supported by fluid.

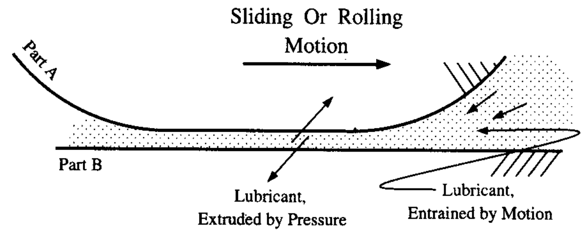

FIG. 10. Motion brings fluid lubricant into the contact zone.

Partial fluid lubrication is shown schematically in Fig. 11. The dynamics of partial fluid lubrication can perhaps be understood by analogy with a water skier. At zero velocity the skier is supported buoyantly in the water. Above some critical velocity the skier will be supported dynamically by his motion. Between floating and skiing there is a range of velocities wherein the skier is partially hydrodynamically supported. These velocities are analogous to the regime of partial fluid lubrication. The analogy is imperfect in that the buoyant support is not like solid-to-solid contact; and the dynamic support of the skier is due to fluid inertia as opposed to viscosity, the dominant force in lubrication. In one aspect, however, the analogy is valid: for both the water skier and the machine, the regime of partial dynamic support is manifestly unstable. As the skier is elevated by his increased velocity, his drag is reduced, allowing him to go even faster. As partial fluid lubrication increases, solid-to-solid contact decreases, reducing friction and increasing the acceleration of the moving part.

Partial fluid lubrication is the most difficult to model of the four regimes. In the case of nonconformal contact, even full fluid lubrication (Elasto-Hydrodynamic Lubrication, or EHL) must be

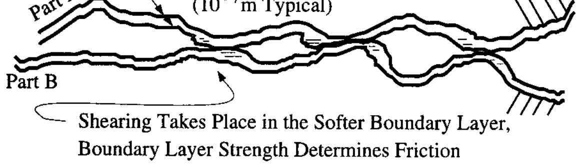

FIG. 9. Boundary lubrication, regime II of the Stribeck curve.

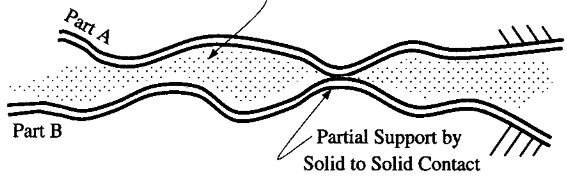

FIG. 11. Partial fluid lubrication, regime III of the Stribeck curve.

investigated numerically. For these contacts, steady state flows over smooth surfaces are well understood (Dowson and Higginson, 1966; Booser, 1984; Pan and Hamrock, 1989); but these are not the true conditions of partial fluid lubrication. Work is proceeding toward an understanding of the interaction of surface roughness and EHL in steady state motion (Zhu and Cheng, 1988; Sadeghi and Sui, 1989). From these papers it appears that the details of surface roughness, asperity size and orientation, have significant impact on the lubricant film characteristics, complicating a general analysis.

Of principal interest to the controls engineer is the dynamics of partial fluid lubrication with changing velocity. Theoretical study of this problem is beginning (Sroda, 1988; Rayiko and Dmytrychenko, 1988). These numerical investigations show a time lag between a change in the velocity or load conditions and the change in friction to its new steady state level. This time or phase lag is called frictional memory and has been observed experimentally in a wide range of circumstances (Rabinowicz, 1958; Bell and Burdekin, 1969; Rice and Ruina, 1983; Walrath, 1984; Hess and Soom, 1990; Armstrong-Hélouvry, 1991; Polycarpou and Soom, 1992; Dupont and Dunlap, 1993). The observed delay may be on the order of milliseconds to seconds, and its impact on stick–slip motion may be substantial (Rice and Ruina, 1983; Dupont, 1994; Dupont and Dunlap, 1993; Armstrong-Hélouvry, 1991, 1992, 1993). Continuing the analogy of the water skier, frictional memory is a consequence of state in the frictional contact, just as the height of the skier is a state variable that does not come to its new equilibrium instantly. Indeed, new work in tribology suggests that frictional memory in fact arises from the normal separation in the frictional interface (see Section 2.3).

2.1.2.4. *The fourth regime: full fluid lubrication.* Hydrodynamic or elasto-hydrodynamic.

Hydrodynamic and elasto-hydrodynamic lubrication (EHL) are two forms of full fluid lubrication. Hydrodynamic lubrication arises in conformal contacts, and EHL in nonconformal contacts. As Fig. 12 shows, solid-to-solid contact is eliminated. In this regime, wear is reduced by orders of magnitude and friction is well behaved. The object of lubrication engineering is often to maintain full fluid lubrication effectively and at low cost. Reynolds (1886) and Sommerfeld (1904) laid the ground work for the investigation of hydrodynamic lubrication, which has been worked out in great detail (see, for example Hersey (1914, 1966), Halling (1975)).

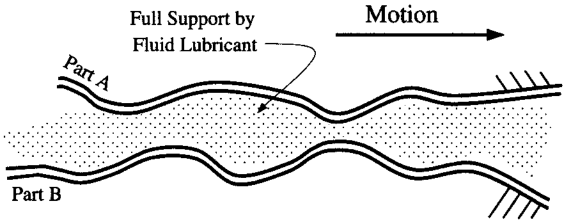

FIG. 12. Full fluid lubrication, regime IV of the Stribeck curve.

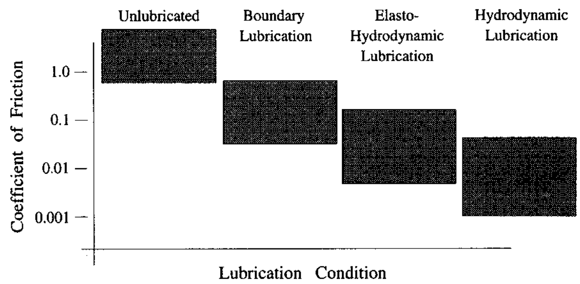

FIG. 13. The range of friction levels [Adapted from Bowden and Tabor (1973)].

EHL is common in servo-controlled machines. As mentioned, it is studied numerically: there is no analytic solution simultaneously satisfying the surface deformation and fluid flow equations. Generally speaking, EHL will give higher friction and wear than hydrodynamic lubrication, as suggested by Fig. 13.

General predictive models of the *steady state* lubricant film thickness are available, e.g. Halling (1975), Hamrock (1986). The film thickness, which determines friction as well as protection from wear, is a function of surface rigidity and geometry, lubricant viscosity and velocity. For control, the value of these results will lie in predicting the velocity of transition to full fluid lubrication. Work is beginning in the exploration of the transient dynamics of elasto-hydrodynamic lubrication (Xiaolan and Haiqing, 1987; Harnoy and Friedland, 1994).

2.1.3. *Boundary lubricants, a domain of many choices*

Boundary lubrication is important to the controls engineer because of the role it plays in stick slip. The key to effective boundary lubrication is the discovery of a molecule that binds with reasonable strength to the metal surface, but is not corrosive; that has sufficient strength to withstand the forces of sliding and yet has a low shear strength to give low friction. Such molecules are added to the bulk lubricant, often comprising only a per cent or two of the total. Lubrication additives may be divided into three broad classes:

- lubricity agents;
- extreme pressure agents; and
- anti-wear agents.

Long chain hydrocarbons with a polar group at one end are commonly used as lubricity agents. The polar group bonds to the metal and the long chain sticks away from the surface, creating, in effect, a mat of bristles (Merchant, 1946; Bowden and Tabor, 1973; Fuller, 1984); the longer the chain (bristle) the lower the friction. These additives are sometimes called oiliness agents, anti-friction agents or friction modifiers. Friction modification refers to reducing the static friction and friction in boundary lubrication. The polar hydrocarbons attach themselves to the metal surface by charge exchange in a process called 'physi-adsorption'. Their application is limited to situations of moderate temperature. At approximately 100°C the polar hydrocarbons desorb and boundary lubrication is lost (Bowden and Tabor, 1973; Fuller, 1984). For this reason the use of long chain hydrocarbons is restricted to applications that generate little frictional heating, which is generally a restriction to conformal contacts.

Use of these polar hydrocarbons as friction modifiers is wide spread in the form of 'way oils', oils specially formulated to eliminate stick slip in machine slideways (Merchant, 1946; Wolf, 1965; Mobil, 1978). Machine slideways are conformal, and thus less affected by frictional heating. A premium is placed on eliminating stick–slip in precision machine tools and great attention has been give to the problem (Merchant, 1946; Wolf, 1965; Bell and Burdekin, 1966, 1969; Kato *et al.*, 1972, 1974). The level of static friction can, in fact, be reduced below the level of Coulomb friction so that there is no destabilizing negative viscous friction and stick–slip is eliminated (Merchant, 1946; Wolf, 1965; Mobil, 1978; Wills, 1980). There are standard procedures for measuring the lubricity of way oils, one is the Cincinnati Milacron stick–slip test (Cincinnati Milacron, 1986). This test procedure measures the friction at breakaway and at a velocity of 0.5 inches per minute. The Cincinnati Milacron test procedure is quite similar to that described in Wolf (1965). The test manual indicates that when ($F_s/F_C < 0.85$), stick–slip will be eliminated. F_s/F_C values as low as 0.55 are observed (Millman, 1990). The possibility of wider use of these lubricants in servo machinery is an intriguing one.

Extreme pressure (EP) agents chemically react with the metal to form a film that will protect the surface from wear. The principal issue in their formulation has been the reduction of wear and seizure (Papay, 1974, 1988; Papay and Dinsmore, 1976), but most EP additives also provide a degree of friction modification and some will pass standard stick–slip tests (Facchiano and Vinci, 1984; Lubrizol, 1988; Cincinnati Milacron, 1986). EP agents are available in a vast variety, and are universally present in gear and other machine lubricants and thus in many servo-controlled machines. EP agents bond with the surface by chemically reacting with the metal, or 'chemi-adsorption'. For this reason they tend to be metal specific. EP additives function at higher temperatures than do lubricity additives and so are serviceable under more severe loading, such as in nonconformal contacts. The chemi-adsorption also offers a generally stronger bond to the surface and thus greater protection against wear. The principal limitations of EP additives are a weaker friction modification than is achieved by the lubricity agents, and chemical reaction with the surface, which is by its nature corrosive. With EP agents one in effect acquires greatly reduced mechanical wear at the price of slow corrosion (Wills, 1980; Fuller, 1984; Papay, 1988).

Anti-wear agents extend the service life of machine parts through a remarkable chemistry that can repair some forms of wear induced surface damage (Estler, 1980; Booser, 1984). The issue of principal concern for the controls engineer is that the anti-wear agent can interfere with the friction modification of lubricity or EP additives. Lubricant additives also perform a host of other functions, including viscosity modification, foam control, corrosion protection, and oxidation stabilization (Papay, 1988). These functions are key to machine and lubricant life, but do not bear directly on mechanism dynamics or control.

Lubricant additives must stay in suspension or solution in the bulk lubricant. In this way they are available to replenish sites on the surface where the lubricant film is damaged by rubbing (Bowden and Tabor, 1973; Hamrock, 1986). Replenishment of the boundary layer from the bulk lubricant may be required after each pass (Vinogradov *et al.*, 1967; Cameron, 1984). Gitis (1986a,b) has studied the relationship between the rates of attrition and replenishment of boundary lubricants and the impact on stick–slip.

Boundary lubricants are standard additives in machine grease or oil; there is a great range of formulations, and they typically constitute less than 2% of the total. Systems with high loading and low relative velocity, such as gear teeth, may operate entirely in boundary lubrication (Mobil, 1971; Wellauer and Holloway, 1976; Wilson, 1979). Much of the attention in boundary lubricant formulation has been focused on reduction of wear. In the design of lubricants, other than way lubricants, friction modification has played a secondary role.

Dry lubricants, such as Teflon®, operate by a variety of mechanisms. Their principal liability is the loss of the protection against wear provided by full fluid lubrication. A good survey of dry lubrication may be found in either Halling (1975) or Fuller (1984). Gassenfeit and Soom (1988) examine and contrast breakaway friction in dry and lubricated contacts. Pope *et al.* (1989) address many issues of concern in space applications. From a control perspective, dry lubricants may offer the substantial advantage of eliminating destabilizing partial fluid lubrication, although negative viscous friction and stick slip may still be present (Martins *et al.*, 1990).

2.1.4. *Relaxation oscillations*

Stick slip was apparent in early studies of low speed motion. The first attempts at explanation were carried out within the static plus Coulomb friction model of Fig. 1(a) (Thomas, 1930). Using a sensitive displacement measuring apparatus, photomicrographs of the rubbing surfaces and hydraulically produced steady motion, Bowden and Leben (1939) demonstrated that sticking occurs and coined the term stick–slip (Rabinowicz, 1956b). They observed welding in the photomicrographs and, using the thermocouple effect between dissimilar metals, they found wide temperature fluctuations that are correlated with the stick cycle. Browden and Leben posited local melting of one rubbing metal as a mechanism for decreased friction during sliding. They found that a similar stick–slip occurs in many lubricated systems, even if there is no welding; and that no stick–slip occurs when long chain fatty acids are used as a lubricant. At the time boundary lubricants were not well understood. The fatty acids used by Bowden and Leben (1939) are now commonly used as lubricity agents.

In 1940 experiments had not yet been conducted which could observe the details of friction during a stick–slip cycle, but it became evident from macroscopic observations, in particular the range of speeds and structural conditions over which stick–slip will occur, that the static plus Coulomb friction model was inadequate to explain the observed phenomena. Dudley and Swift (1949) employed phase plane analysis to study the possible oscillations in slider mechanisms, that is mass-spring-damper systems equivalent to PD control. A negative viscous friction, as shown in Fig. 1(c), was posited and efforts were directed at elucidating its character by fitting predicted oscillations to observed stick–slip (Dudley and Swift, 1949).

Experiments grew progressively more sensitive (Sampson *et al.*, 1943; Dokos, 1946; Rabinowicz, 1951, 1956, 1958; Rabinowicz and Tabor, 1951; Rabinowicz *et al.*, 1955) and evidence mounted both for negative viscous friction, Fig. 1(c), and indicating that changes in friction do not coincide exactly with changes of mechanism state. That is to say that dynamics were found to exist within the surface processes that determine friction. Using experiments designed to directly determine the properties of breakaway (the transition from static to Coulomb friction), Rabinowicz (1951) found that breakaway is not instantaneous, and proposed a model involving translational distance to account for decreasing friction as motion progressed. Rabinowicz (1958) reports an experiment capable of measuring the acceleration of a slider during stick–slip, and observes that the acceleration and deceleration curves are not symmetric. Rabinowicz (1958) is a landmark paper because the two temporal phenomena in the stick–slip process are integrated into a friction model that will at least qualitatively predict the range of speeds and structural conditions over which stick–slip will occur. The temporal phenomena are:

(1) a connection between the time a junction spends in the stuck condition, i.e. dwell time, and the level of static friction (rising static friction); and
(2) a time delay or phase lag between a change in velocity and the corresponding change in friction (frictional memory).

2.1.4.1. *Rising static friction and extinguishing stick slip by increasing velocity.* To understand the role played by rising static friction and frictional memory, it is necessary to consider the stages of a stick-slip cycle; this discussion and Figs 14–16 follow Rabinowicz (1958). In Fig. 14 a pin-on-flat friction machine is sketched. Here the pin is held in place by a spring and the flat moves at a constant velocity. The mechanism is analogous to a servo machine moving with a desired velocity, $\dot{x}_d$, a proportional control gain, k_p, and damping, k_v. The discussion assumes moderate values of damping; extremely large values of damping will influence the qualitative behavior, but moderate values will not (Bell and Burdekin, 1969).

Under some conditions, a system such as that of Fig. 14 will exhibit stick–slip. The spring force (control

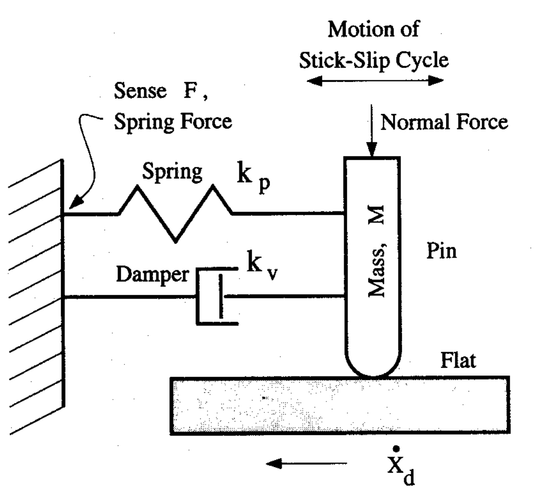

FIG. 14. Pin on flat friction machine, schematic; flat slides under–pin.

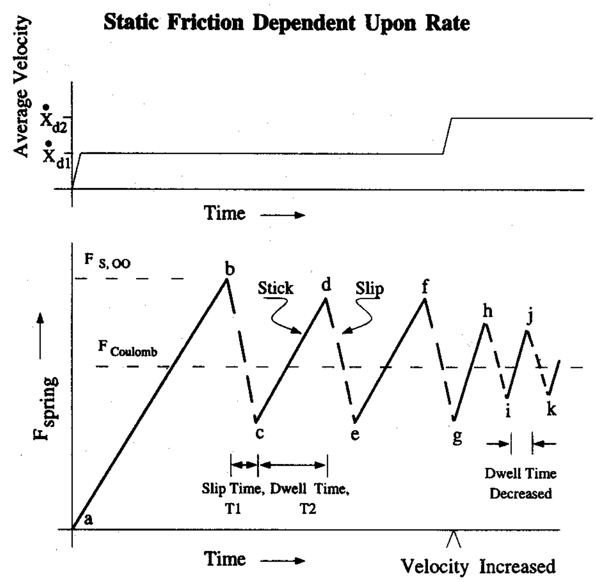

FIG. 15. Spring force profile during stick-slip motion at two velocities; spring force decreases when velocity increases.

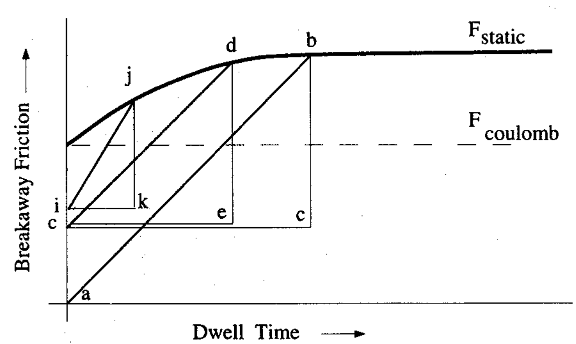

FIG. 16. Static friction (breakaway force) as a function of dwell time, schematic; with stick-slip cycle shown. Dwell time is the time in static friction, shown as T_2 in Fig. 15.

action) observed during motion is sketched in Fig. 15. During the stuck, interval, interval a–b, the force rises at a rate $\dot{F}_{\text{spring}} = k_p\dot{x}_d$. At point b the force reaches $F_{s,\infty}$, the level of static friction when the system has been at rest for considerable time, and slip begins. During interval b–c slip occurs; the exact motion is governed by the mass spring dynamics plus the details of the friction forces. A rapid transit is qualitatively indicated here. At point c, the pin is arrested on the flat and the spring force again begins to rise at rate $\dot{F}_{\text{spring}} = k_p\dot{x}_d$, entering a stable limit cycle of points c–d–e. Point d is somewhat lower than point b because the system has only been at rest for dwell time c–d. At point g the velocity $\dot{x}_d$ is increased. The important empirical fact is that as the velocity is increased, the size of the limit cycle, i–j–k, diminishes (Dokos, 1946; Rabinowicz, 1958; Kato *et al.*, 1972, 1974). If the condition at point j were identical to the condition at point d, a decrease in the slip distance would not be observed, and an analysis based on the static plus Coulomb friction model will not predict that the limit cycle will decrease. In Fig. 16 the limit cycles c–d–e and i–j–k are shown on a plot of static friction as a function of dwell time. The dwell time is the time during which the surfaces are in fixed contact, the time intervals a–b, c–d, e–f, g–h and i–j in Fig. 15. The static friction increases with dwell time and this accounts for the larger limit cycle at lower velocity. Figure 17 is a plot of rising static friction measured directly by Kato *et al.* (1972) who provide a thorough analysis of the processes relating static friction and dwell time. Lubricants A, B, C and D are, respectively, viscous mineral oil, commercial slideway lubricant, castor oil and paraffin oil. Note that in Fig. 16 the time scale is linear, as opposed to logarithmic in Fig. 17. The empirical model of (Kato *et al.*, 1972), relating static friction and dwell time is:

$$F_s(t) = F_{s,\infty} - (F_{s,\infty} - F_C)e^{-\gamma t^m}, \tag{3}$$

where $F_{s,\infty}$ is the ultimate static friction; F_C is the Coulomb friction at the moment of arrival in the stuck condition; γ and m are empirical parameters. Kato *et al.* (1972) examine conformal contacts and find γ to range from 0.04 to 0.64, and m from 0.36 to 0.67.

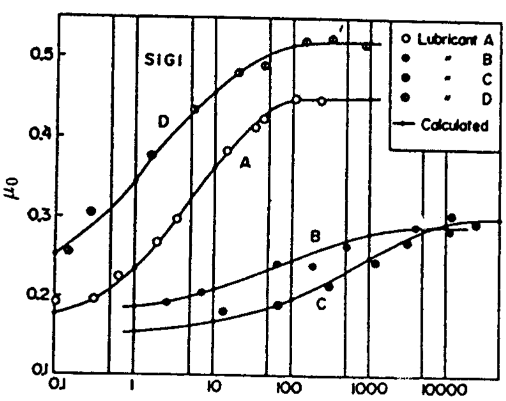

FIG. 17. Measurements of the static friction coefficient (μ_0 in Kato's notation) as a function of T_2, the dwell time or time spent in static friction [from Kato *et al.* (1972), courtesy of the publisher].

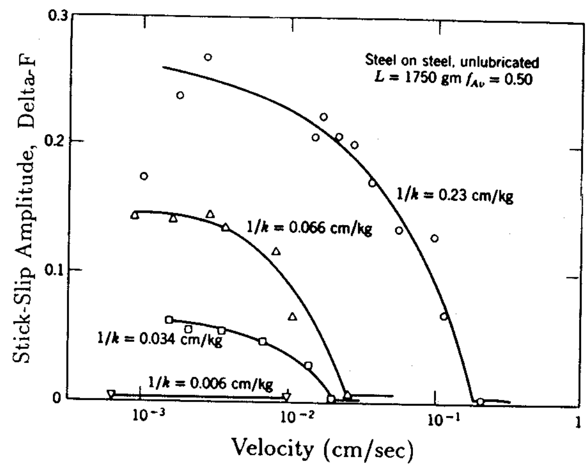

FIG. 18. Stick-slip amplitude as a function of velocity, for several values of spring stiffness [from Rabinowicz (1965), courtesy of the publisher].

Armstrong-Hélouvry (1991) examines a non-conformal contact and finds $\gamma = 1.66$ and $m = 0.65$. A small γ indicates a long rise time and thus resists stick slip.

Armstrong-Hélouvry (1991, 1992) presents a model of rising static friction which is useful for analysis and solves some problems associated with using F_C as the starting point of the static friction rise. The model, which has one fewer parameter than that of equation (3), is:

$$F_{s,b_n}(t_2) = F_{s,a_{n-1}} + (F_{s,\infty} - F_{s,a_{n-1}})\frac{t_2}{t_2 + \gamma}; \tag{4}$$

where F_{s,b_n} is the level of Stribeck friction at the beginning (breakaway) of the nth interval of slip; and $F_{s,a_{n-1}}$ is the Stribeck friction at the end (arrival) of the previous interval of slip. Note that γ, still an empiric factor, will be different in physical dimension from that of equation (3).

Figure 18 presents the amplitude of the spring force cycle during stick–slip, shown as a function of machine velocity, $\dot{x}_d$, for several values of spring stiffness, k_p (Rabinowicz, 1965). Rabinowicz's experiment is shown schematically in Fig. 14. The amplitude of the spring force cycle is a decreasing function of velocity until stick–slip is abruptly extinguished. The amplitude is also a decreasing function of stiffness. These data represent values of several stiffnesses in unlubricated contacts. Brockley *et al.* (1967), Brockley and Davis (1968), Ko and Brockley (1970), present data observed in an experiment with several levels of damping and Kato *et al.* (1972) present data collected with various lubricants. The analysis and data of Kato *et al.* (1972) are the most germane to servo mechanisms as they incorporate engineering materials and lubricants. All of these data present the same pattern: slip amplitude as a decreasing function of velocity up to an abrupt elimination of stick–slip. The process is one of increased velocity leading to reduced dwell time, which lowers the static friction at breakaway, this further reducing the dwell time. At some critical velocity the dwell time is insufficient to build up destabilizing static friction and stick–slip is extinguished. Derjaguin *et al.* (1957), Singh (1990) and Armstrong-Hélouvry (1991) present theoretical treatments that predict the critical velocity for termination of stick–slip as a function of system parameters and rising static friction. For the controls engineer these analyses provide an approach to the question of how slow a machine may be driven before the onset of stick–slip, and on what parameters this limit depends.

Richardson and Nolle (1976) point out that in the experiments of Rabinowicz, Kato and others, force was applied at a steady rate, as shown by the slope of the line from c–d in Fig. 15, creating a connection between force rate and dwell time: the higher the force rate the shorter the dwell time. Johannes *et al.* (1973) and Richardson and Nolle (1976) report experiments designed to allow independent variation of force rate and dwell time. They find that the reduction of static friction is not so much a consequence of short dwell time as of rapid force application rate, posing a challenge for explanations based on creep. Martins *et al.* (1990) propose an explanation based on normal penetration of the friction surfaces. For linear feedback control the distinction is perhaps not great; but for impulsive control designs the implications may be both considerable and favorable.

2.1.4.2. *Frictional memory and extinguishing stick–slip by increasing stiffness.* In Fig. 18 one observes that the trial with the stiffest spring did not exhibit stick–slip at any velocity. It is widely observed that stick–slip can be eliminated by stiffening a mechanism (Bell and Burdekin, 1966, 1969; Rabinowicz, 1965; Armstrong, 1989; Armstrong-Hélouvry, 1991). A stiffness above which there will be no stick–slip is not predicted by a model like that of Fig. 1(a); but increased stiffness is the key to eliminating stick–slip in many mechanical situations (Halling, 1975).

The Stribeck curve, Fig. 19(a), shows a dependence of friction upon velocity. If there is a change in velocity, one might presume the corresponding change in friction to occur simultaneously, as suggested in Fig. 19(b). In fact there is a delay in the change in friction, as suggested by Fig. 19(c), (Sampson *et al.*, 1943; Rabinowicz, 1958, 1965; Bell and Burdekin, 1966, 1969; Rice and Ruina, 1983; Hess and Soom, 1990; Polycarpou and Soom, 1992). Returning to the image of partial hydrodynamic lubrication as a water skier with partial dynamic support, if we imagine the water skier half out of the water, his drag will be a decreasing function of velocity. If the tow boat suddenly increases speed, the skiers drag will decrease, but, as in Fig. 19(c), some time will pass before the new steady state drag is observed. Figure 19 is schematic. Experimental data corresponding to the observation of Fig. 19(c) is presented in Fig. 20.

Rabinowicz (1951) showed that friction level lags a change in system state with an experiment that related delivered impulse to translation distance in a sliding contact. He ascribed the frictional memory to a necessary translation distance for a change in friction, on the scale of surface asperities (Rabinowicz, 1951, 1958, 1965). In fluid lubricated contacts, there is evidence that a simple time lag better describes the effect (Hess and Soom, 1990). At extremely low velocities, evidence supports a state variable model (Rice and Ruina, 1983; Dupont and Dunlap, 1993); see Section 2.1.5. Bell and Burdekin's (1966, 1969) data are particularly applicable to common machine

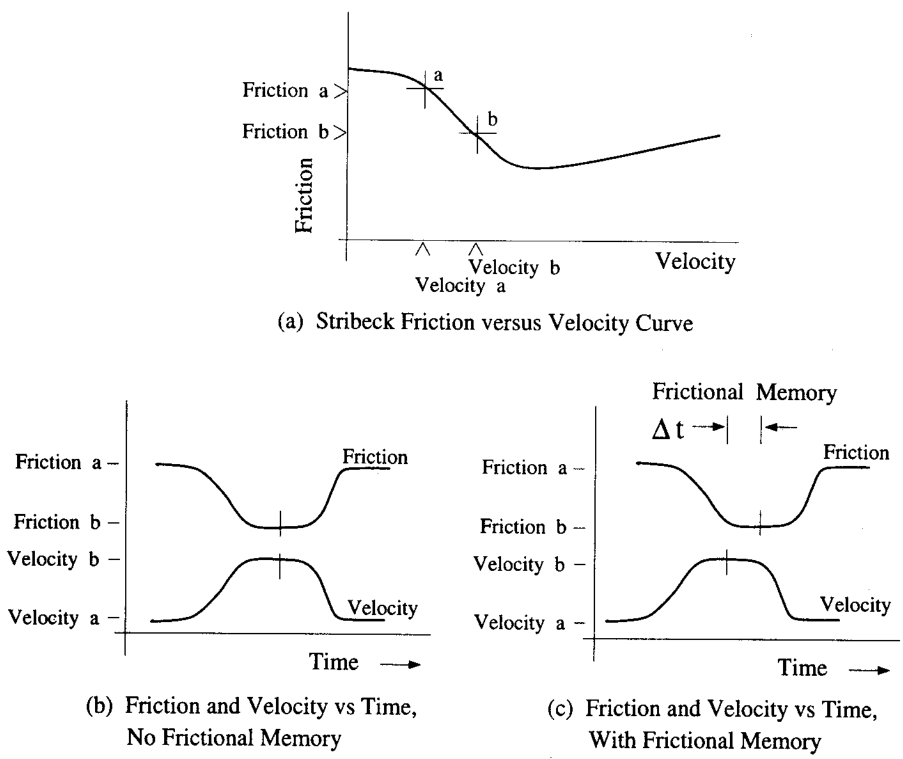

FIG. 19. Time relation between a change in velocity and the corresponding change in friction.

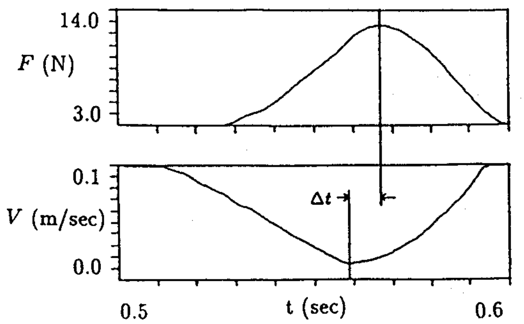

FIG. 20. Typical friction-speed time shift; contact load = 250 N, lubricant viscosity = 0.322 Pa·s, frequency = 1 Hz. $F(N)$: friction, N; $V(\mathrm{m\,s^{-1}})$: velocity [from Hess and Soom (1990), courtesy of the publisher].

configurations. Figure 21 is from (Hess and Soom, 1990) and shows friction data for one oscillation of an oscillatory motion that brings the system into partial fluid lubrication. This experiment was conducted by superimposing a velocity oscillation on steady sliding. After first stabilizing the average motion, the magnitude of the velocity oscillation may be chosen to probe the very low velocity regime without arriving at zero velocity or static friction. "μ" in Figs 21 and 22, as well as Fig. 17, is the friction coefficient, friction force divided by the normal load. Note the vertical separation between the friction curves. The upper friction curve is given during the acceleration away from zero velocity and the lower during deceleration. The solid line of Fig. 21 was generated modeling frictional memory as a pure lag, such that

$$F_f(t) = F_{vel}(\dot{x}(t - \Delta t)), \qquad (5)$$

where $F_f(t)$ is the instantaneous friction force, $F_{vel}(\cdot)$ is friction as a function of steady state velocity, see Fig. 5, and Δt is the lag parameter, the time by which a change in friction lags a change in velocity. Hess and Soom (1990) carefully measure Δt and find it to range from 3 to 9 ms in a range of load and lubricant combinations; the lag increasing with increasing lubricant viscosity and with increasing contact load. The lag appears to be independent of oscillatory frequency (Hess and Soom, 1990). When the period of the oscillation is short relative to Δt, the hysteresis, that is the separation between the friction levels

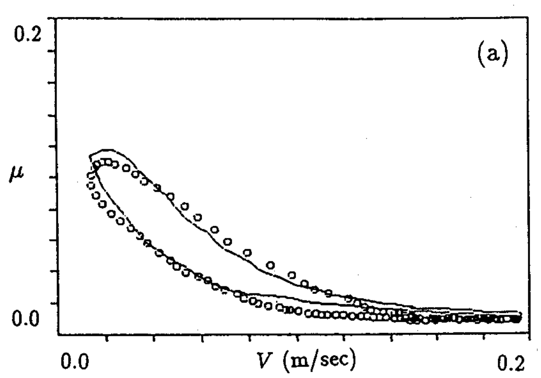

FIG. 21. Friction as a function of velocity; O: experimental; —: theoretical, from equations (7) and (5) [from Hess and Soom (1990), courtesy of the publisher].

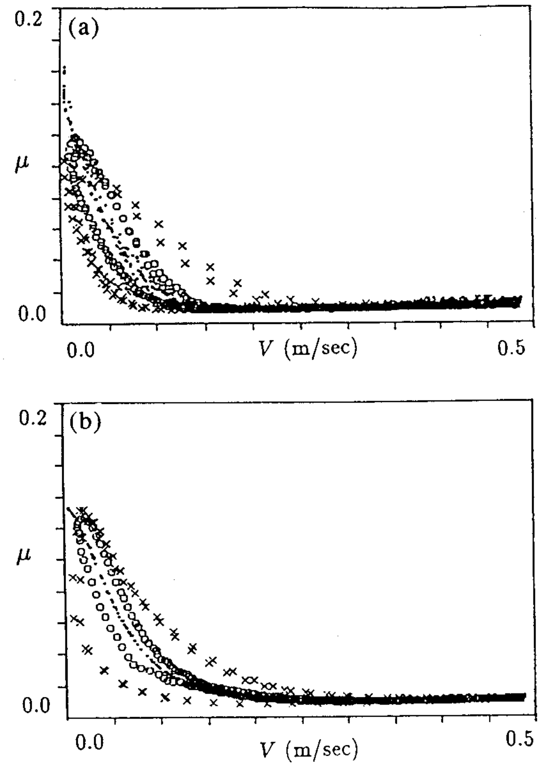

FIG. 22. Friction as a function of velocity; for three different frequencies of oscillation: · · ·: 0.1 Hz; O: 1 Hz; ×: 5 Hz. (a): experimental; (b): theoretical, from equations (5) and (7) [from Hess and Soom (1990), courtesy of the publisher].

during acceleration and deceleration, is greatest. This is illustrated in Fig. 22, also from Hess and Soom (1990). The data presented were acquired driving their pin-on-disk contact at three different frequencies. Figure 22(b) shows the friction curves predicted by their model with frictional memory modeled as a pure lag and should be compared with the experimental data illustrated in Fig. 22(a). Indicative of the progress of tribology, the friction model of Hess and Soom (1990) which accounts for contact geometry and loading, material properties, velocity, lubricant viscosity and Stribeck friction, is to a large degree based on contact and lubricant parameters, only three parameters are fit *a posteriori* to the data.

Evidence for frictional memory is available from a range of experimental sources: Sampson *et al.* (1943), Rabinowicz (1958, 1965), Bell and Burdekin (1966, 1969), Walrath (1984), Rice and Ruina (1983), Hess and Soom (1990). Tribology is not yet able to offer a theoretically motivated model of the frictional memory, though Xiaolan and Haiqing (1987) numerically investigate transient elasto-hydrodynamic lubrication using an analysis that starts with Reynold's equation and Hertzian contact analysis; with this they find a time lag of 3 ms between velocity and friction changes in simulated sliding contact. The physical process giving rise to frictional memory appears to relate to the time required to modify the lubricant film thickness, a process measured by several investigators (Tolstoi, 1967; Bell and Burdekin, 1969; Bo and

Pavelescu, 1982). A period of time required to obtain a new film thickness may be one of several contributing processes, as frictional memory is also observed in dry contacts (Rabinowicz, 1951).

2.1.5. *State variable friction models*

An alternative to the pure time lag model is provided by the state variable models developed by the rock mechanics community (Ruina, 1980; Rice and Ruina, 1983; Gu *et al.*, 1984; Okubo, 1986; Dieterich, 1991; Linker and Dieterich, 1992). Interest in rock friction stems from the hypothesis that earthquakes are fault-line stick–slip events. While these models have been developed from friction experiments on rocks, their properties have recently been observed for a range of materials (Dieterich, 1991; Dupont and Dunlap, 1993). These include lubricated steel, Teflon® on steel, glass, plastic and wood. To date, these experiments have been limited to velocities within the boundary lubrication regime.

The state variable models incorporate a dependence on displacement history. They typically possess the following three properties (assuming constant normal stress):

(1) a steady-state dependence on velocity;
(2) an instantaneous dependence on velocity; and
(3) an evolutionary dependence on characteristic sliding distances.

The steady-state effect, (1), represents the generalized Stribeck curve. The instantaneous effect, (2), means that an instantaneous change in velocity results in an instantaneous change in the friction force in the same direction. The third property indicates that following a sudden change in velocity, the steady-state curve is approached through an exponential decay over characteristic sliding distances. This type of model can reproduce the friction behavior depicted in Fig. 21 (Dupont, 1994).

For constant normal stress, the general model including the n state variables, θ_i, is given by:

$$\begin{aligned} F_f(t) &= f(V, \theta_1, \theta_2, \ldots, \theta_n) \\ \dot{\theta}_i &= g_i(V, \theta_1, \theta_2, \ldots, \theta_n), \quad i = 1, 2, \ldots, n. \end{aligned} \tag{6}$$

This form implies that a sudden change in velocity cannot produce a sudden change in the state, θ, but does affect its time derivative. Hence, the instantaneous velocity effect takes place at constant state. The evolution of the state variables in response to changes in velocity, together with the instantaneous velocity effect, dictate the dynamic behavior.

Physical interpretations of the state variables are possible. Consider a standard dry friction model in which friction stress depends on the yield stress of asperity junctions. For a single state variable and constant normal stress, the state variable can be related to the mean lifetime of an asperity junction. Recently, these models have been enhanced to include dependence on normal stress. In this case, the state variables can be related to the time-dependent growth of the load-bearing junctions (Linker and Dieterich, 1992).

The functional form of the state variable models was deduced from the response to step changes

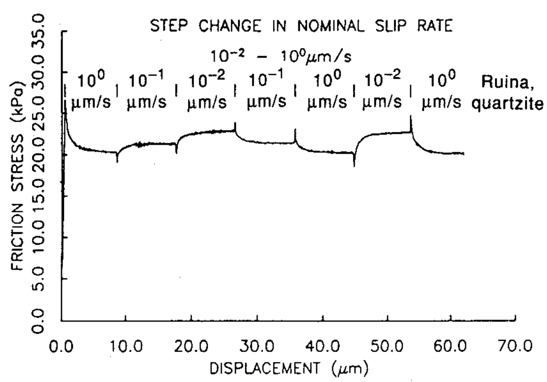

FIG. 23. Friction stress as a function of displacement in trials with unlubricated quartzite. Step changes in velocity produce the instantaneous-effect spikes and subsequent evolution to the new steady-state level [from Ruina *et al.* (1986), courtesy of the authors].

imposed on the velocity at the friction interface. These experiments are a significant improvement over standard tribology experiments because they involve control of the friction interface motion instead of the actuator motion.

Figure 23 depicts friction stress versus displacement data obtained by Ruina *et al.* (1986). The three modeled effects are clearly visible in the data. The fact that very small, steady velocities were achieved through closed-loop control in these experiments is additional evidence that stable, low-velocity control is possible.

State variables models (or additional internal states) have also been proposed whose behavior resembles that of a connection with a stiff (nonlinear) spring (Dahl, 1977). The Dahl model predicts a frictional lag between velocity reversals and leads to hysteresis loops. The mathematical properties of the Dahl model are studied in Bliman (1993). However, this model can only predict Coulomb friction steady-state velocity characteristics; the Stribeck effect is not included. An interesting interpretation of this model by using linear space invariant models (instead of nonlinear differential equations) is presented in Bliman and Sorine (1991). With this new model it becomes clear how frictional forces, predicted by the Dahl model, depend on the curve length associated with the trajectory of relative motions (integral of the velocity absolute value). To introduce the Stribeck effect, it is possible to extend the Dahl model (which is first-order) to a model with a high degree of differentiability (Bliman and Sorine 1991, 1993). The second-order Dahl model can show the Stribeck phenomenon by producing an overshoot in the response of the friction forces. Another possibility is to modify the original Dahl model so as to include the Stribeck effect without increasing the system state dimension (Canudas de Wit *et al.* 1993). In this modified Dahl model, the internal states have a physical interpretation. They describe the Bristles average deformation.

The state variable models of Rice and Ruina (1983), the translation distance of Rabinowicz, and the pure lag of Hess and Soom (1990) are all representations of frictional memory. The effect of frictional memory is a delay in the onset of the destabilizing drop in friction. From a control standpoint, the frictional memory reduces the destabilizing influence of Stribeck friction. If the time constants of a system are short in relation to the frictional memory, which is to say that the mechanism (control) is sufficiently stiff, the stick-slip limit cycle will not be stable (Rabinowicz, 1965). (For the range of frictional memory time constants, see Table 1). *This is the process whereby increasing stiffness eliminates stick–slip.*

2.1.6. *Friction as a function of steady state velocity: variants of the Stribeck curve*

Friction is a function of velocity because the physical process of shear in the junction changes with velocity. Figure 24 presents several friction–velocity curves. Details of the (f–v) curve depend upon the degree of boundary lubrication and the details of partial fluid lubrication. Curves such as (a) arise when lubricants that provide little or no boundary lubrication are employed. The data of Bell and Burdekin (1966, 1969) and Hess and Soom (1990) indicate such a curve. When boundary lubrication is more effective, the friction is relatively constant up to the velocity at which partial fluid lubrication begins to play a role. Vinogradov *et al.* (1967) and Khitrik and Shmakov (1987) present data supporting a flat (f–v) curve through the region of boundary lubrication, as suggested by curve (b) of Fig. 24. Fuller (1984) cites data contrasting a specific lubricating oil with and without a lubricity additive. The plain oil gives a curve of type (a); with the lubricity additive a curve of type (b) is observed [see Fuller (1984), Figs 11–14; the reference offers considerable discussion of boundary lubrication]. One must be careful in discussing friction as a function of steady state velocity. Data collected during velocity transients will exhibit the effects of frictional memory, equation (5), and a curve of type (b) may be observed even if the underlying steady state (f–v) curve is of type (a). Bell and Burdekin (1969) present a thorough analysis of this phenomenon. A curve of type (c) is given by way lubricants (Merchant, 1946; Wolf, 1965). The boundary lubrication provided by the additives to these oils

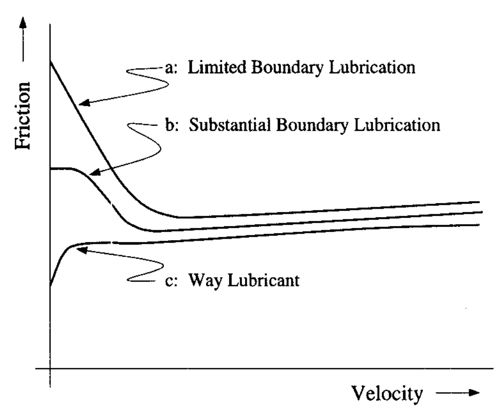

FIG. 24. Friction as a function of steady state velocity for various lubricants; the (f–v) curve [after Fuller (1984)].

reduces static friction to a level below Coulomb friction.

For analysis or simulation it is important to have a mathematical model of the steady-state friction–velocity dependence. Hess and Soom (1990) employ a model of the form

$$F(\dot{x}) = F_C + \frac{(F_s - F_C)}{1 + (\dot{x}/\dot{x}_s)^2} + F_v\dot{x} \qquad (7)$$

and show a systematic dependence of $\dot{x}_s$ and F_v on lubricant and loading parameters. Bo and Pavelescu (1982) review several models proposed in the literature and adopt and then linearize an exponential model of the form:

$$F(\dot{x}) = F_C + (F_s - F_C)e^{-(\dot{x}/\dot{x}_s)^\delta} + F_v\dot{x}, \qquad (8)$$

where F_s is the level of static friction, F_C is the minimum level of Coulomb friction, and $\dot{x}_s$ and δ are empirical parameters. The viscous friction parameter, F_v, is added here; a viscous term was not incorporated by Bo and Pavelescu (1982). In the literature surveyed by Bo and Pavelescu (1982), they find δ to range from 1/2 to 1. Armstrong-Hélouvry (1990, 1991) employs $\delta = 2$; and the data cited by Fuller (1984), observed in a system with an effective boundary lubricant, would suggest δ very large. The exponential model (8), with $\delta = 2$, is a Gaussian model. The Gaussian model is nearly equivalent to the Lorentzian model of Hess and Soom (1990), equation (7).

The exponential model (8), is not a strong constraint. By appropriate choice of parameters, curves of types (a), (b) and (c) can be realized. What is needed are data such as that of Hess and Soom (1990) over a broad range of engineering materials, conditions and lubricants. For specific lubricant formulations, lubrication engineering firms can provide measures of lubricity and other qualities based on standard industrial tests. The standard tests of lubricant qualities are not the equivalent of the data of Hess and Soom (1990), but are none-the-less useful. Industrial testing for lubricity is still evolving (Ludema, 1988).

2.2. An Integrated Friction Model

This discussion of friction has focused on sliding between hard metal parts lubricated by oil or grease. For reasons of machine life and performance, these engineering materials make up many of the machines encountered by controls engineers. When these materials are used, the state of understanding supports a friction model that is comprised of four velocity regimes, two time dependent properties and several mechanism dependent properties.

(1) The four velocity regimes.
   (I) Static Friction: displacement (not velocity) is proportional to force [see Fig. 7 and equation (1)].
   (II) Boundary Lubrication: friction is dependent on surface properties and lubricant chemistry.
   (III) Partial Fluid Lubrication: if static friction is greater than Coulomb friction, friction decreases with increasing velocity.

(IV) Full Fluid Lubrication: friction is a function of velocity, a viscous plus Coulomb friction model may model the friction quite accurately. [Regimes II–IV in Fig. 5, see also equation (8).]

(2) The two time-dependent properties.

(I) Rising Static Friction with Increasing Dwell Time [see Fig. 16 and equation (3)].

(II) Frictional Memory: in partial fluid lubrication, friction is dependent upon velocity and load; a change in friction will lag changes in velocity or load [see Fig. 20 and equation (5)].

2.2.1. *The seven parameter friction model.* Theoretically motivated models for the components of friction are not yet available, and a variety of empirically motivated forms have been presented. One choice of model is the seven parameter model, where the friction is given by:

Not sliding (pre-sliding displacement).

$$F_f(x) = -k_t x \tag{9}$$

Sliding (Coulomb + viscous + Stribeck curve friction with frictional memory).

$$F_f(\dot{x}, t) = -\left(F_C + F_v\,|\dot{x}| + F_s(\gamma, t_2)\frac{1}{1+\left(\dfrac{\dot{x}(t-\tau_L)}{\dot{x}_s}\right)^2}\right)\operatorname{sgn}(\dot{x}). \tag{10}$$

Rising static friction (friction level at breakaway).

$$F_s(\gamma, t_2) = F_{s,a} + (F_{s,\infty} - F_{s,a})\frac{t_2}{t_2+\gamma}, \tag{11}$$

where:

- $F_f(\cdot)$ is the instaneous friction force;
- F_C (*) is the Coulomb friction force;
- F_v (*) is the viscous friction force;
- F_s is the magnitude of the Stribeck friction (frictional force at breakaway is $F_C + F_s$);
- $F_{s,a}$ is the magnitude of the Stribeck friction at the end of the previous sliding period;
- $F_{s,\infty}$ (*) is the magnitude of the Stribeck friction after a long time at rest (with a slow application of force);
- k_t (*) is the tangential stiffness of the static contact;
- $\dot{x}_s$ (*) is the characteristic velocity of the Stribeck friction;
- τ_L (*) is the time constant of frictional memory;
- γ (*) is the temporal parameter of the rising static friction;
- t_2 is the dwell time, time at zero velocity;
- (*) marks friction model parameters, other variables are state variables.

The magnitudes of the seven friction parameters will naturally depend upon the mechanism and lubrication, but typical values may be offered. Ranges suggested elsewhere in this section, originating principally with Bowden and Tabor (1973), Kato *et al.* (1974), Fuller (1984), Armstrong-Hélouvry (1991), Hess and Soom (1991a, b), Polycarpou and Soom (1992), are summarized in Table 1. The friction force magnitudes, F_C, F_v and $F_{s,\infty}$ are expressed as a function of normal force, i.e. as coefficients of friction. Δ_x is the deflection before breakaway resulting from contact compliance.

TABLE 1. APPROXIMATE RANGES FOR THE PARAMETERS OF THE SEVEN PARAMETER FRICTION MODEL

	Parameter range	Parameter depends principally upon
F_C	$0.001 - 0.1 * F_n$	Lubricant viscosity, contact geometry and loading
F_v	0-very large	Lubricant viscosity, contact geometry and loading
$F_{s,\infty}$	$0 - 0.1 * F_n$	Boundary lubrication, F_C
k_t	$\frac{1}{\Delta_x} * (F_s + F_c)$; $\Delta_x \simeq 1 - 50[\mu\text{m}]$	Material properties and surface finish
$\dot{x}_s$	$0.00001 - 0.1\left[\frac{\text{meter}}{\text{secod}}\right]$	Boundary lubrication, lubricant viscosity, Material properties and surface finish, Contact geometry and loading
τ_L	$1 - 50\,[\text{ms}]$	Lubricant viscosity, contact geometry and loading
γ	$0 - 206\,[\text{s}]$	Boundary lubrication

Each of the seven parameters of the model represents a different friction phenomenon. The seven rows of Table 2 indicate the effect of these

TABLE 2. FRICTION MODEL CAPABILITIES

Friction model	Predicted/observed behavior
Viscous	Stability at all velocities and at velocity reversals.
Coulomb	No stick–slip for PD control; No hunting for PID control
Static + Coulomb + Viscous	Predicts stick slip for certain initial conditions under PD control; predicts hunting under PID control.
Stribeck	Needed to correctly predict initial conditions leading to stick–slip.
Rising static friction	Needed to correctly predict interaction of velocity and stick-slip amplitude.
Frictional memory	Needed to correctly predict interaction of stiffness and stick-slip amplitude.
Presliding displacement	Needed to correctly predict small displacements while sticking (including velocity reversals).

phenomena on sliding behavior. Alternatively, and more appropriately, the table can be used to select a friction model based on experimental observations.

Polycarpou and Soom (1992) have recently reported dynamic measurements of friction in lubricated metal contacts made with a remarkably sensitive apparatus. Except for viscous and rising static friction, each of the components of the seven parameter model is evident in the data of Polycarpou and Soom (1992); and the authors observe that rising static friction may have been present on a time scale other than that observed. Furthermore, although a detailed parameter identification is not presented, the authors are able to account for all of the qualitative phenomena with reference to presliding displacement, Coulomb and Stribeck friction, and frictional memory.

In practical machines there tend to be many rubbing surfaces—drive elements, seals, rotating electrical contacts, bearings etc—which contribute to the total friction. In some mechanisms, a single interface may be the dominant contributor, as transmission elements often are. In other cases where there are several elements contributing at a comparable level, it may be impossible to identify their individual contributions without machine disassembly. In these cases, a model, such as the one above, can be used to represent the aggregate friction.

2.2.2. *Special mechanical considerations*
Much of this survey has dealt with sliding lubricated metal contacts; but other contacts may be important. This section provides a brief overview of rolling friction as well as other friction phenomena which may arise in complex machines.

2.2.2.1. *Rolling friction.* Rolling elements typically generate much less friction than sliding elements at comparable loads and speeds. For this reason, the friction contribution of roller bearings is usually insignificant in comparison with that of the sliding contacts in a machine and, thus, often plays a minor role in machine design. Some important exceptions include disk drives; ball screws (Ro and Hubbel, 1993) and ball-bearing slideways (Futami *et al.*, 1990) used in precision engineering; and the gimbal bearings of pointing and tracking devices (Gilbart and Winston, 1974; Walrath, 1984; Himmell, 1985; Maqueira and Masten, 1993).

To gain an appreciation of the level of friction involved, consider that for ball and roller bearings operating at typical loads and speeds, the friction coefficients range between $\mu = 0.001$ and 0.005 (Eschmann, 1985). For roller bearings, the friction coefficient is related to friction torque by:

$$\mu = \frac{\tau_f}{Fd/2}. \tag{12}$$

Here, τ_f is the friction torque, F is the resultant bearing load, including both radial and axial components, and d is the bearing bore diameter. Starting from rest, a slightly higher stiction level of 'rolling' friction may exist, but in ball bearings this effect is usually quite small (Palmgren, 1945).

Several friction models have been proposed over the years. Roller bearing texts typically provide semi-empirical equations of the basic form:

$$\tau_f = \tau_0 + \tau_1, \tag{13}$$

where τ_0 is the no-load component of friction torque and τ_1 usually depends strongly on bearing load, but only lightly on velocity (Eschmann, 1985). While the model described above is meant to apply to a broad range of operating conditions, the Dahl model was developed to explain the hysteretic behavior of precision ball bearings undergoing very small amplitude oscillations (Dahl, 1968, 1977). The Dahl model has been widely used to study the simulation and control of machines.

Mechanisms of rolling friction.
There are two effects associated with the elasticity of the contact zone which contribute to rolling friction (Harris, 1984). These effects, however, make up a small portion of the total rolling friction. It is a surprising fact that most of the friction in roller bearings is due to sliding motion. This sliding is one of the major reasons that roller bearings must be lubricated with oil, grease or sometimes, and with less effect, a dry lubricant. To understand how sliding can occur, first consider that pure rolling would require point contacts or line contacts parallel to the bearing axis of rotation. Owing to elastic deformation, ball bearings on flat or curved raceways have curved contact regions. In addition, rollers and raceways are usually crowned in order to prevent edge loading (Harris, 1984). Thus the contact region is curved.

Consider Fig. 25. With the ball rolling at a particular velocity, there will be only two curvilinear segments within the elliptical contact zone at the proper radius to undergo pure rolling. The velocity profile for the major axis of the contact ellipse is shown. The points D and D' lie on the rolling

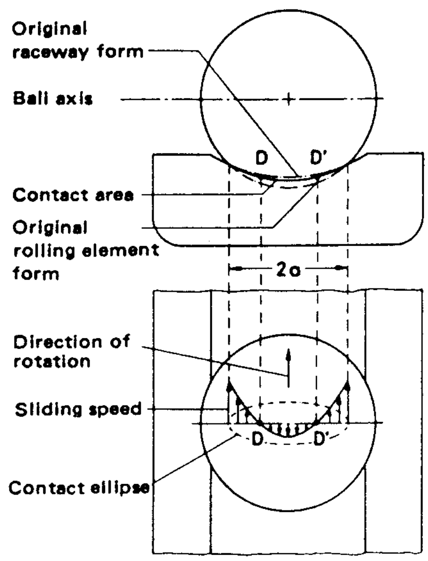

FIG. 25. Sliding in the contact ellipse of a ball rolling on a curved raceway [from Eschmann (1985), courtesy of the publisher].

segments. Between these points, slip will occur opposite the direction of rolling. Outside the points, slip occurs in the direction of rolling.

According to the type of roller bearing, sliding friction will also arise from contact between the rolling elements and the cage, between the rolling elements themselves and between the roller faces and the raceway lips. There is also viscous drag on the rollers caused by the lubricant and friction due to the bearing seals. Seal friction can be considerable and can far exceed the total of all other sources of bearing friction (Harris, 1984).

2.2.2.2. *Other machine elements.* The preceding discussions apply to simple sliding or rolling friction; in complex machines there may be additional considerations. One such consideration is different friction magnitudes in different directions of motion. Different Coulomb and viscous friction levels in the left and right rotation directions have been observed experimentally on many occasions, e.g. Mukerjee and Ballard (1985), Canudas de Wit *et al.* (1987), Armstrong-Hélouvry (1991). Theoretically, this may be due to anisotropies in material or geometry (Zmitrowicz, 1981; Ibrahim, 1992a). And the phenomenon is a sufficient consideration that a standard stick-slip test calls for separate measurements in the left and right directions (Cincinnati Milacron, 1986).

Some mechanisms will exhibit position-dependent friction (Mukerjee and Ballard, 1985; Candas de Wit *et al.*, 1987; Armstrong, 1988; Armstrong-Hélouvry, 1991). This is particularly true of transmissions with spatial inhomogeneities, i.e. contact geometry or loading which varies as a function of position. Gear drives are a common example and give rise to position-dependent friction. With accurate friction measurements, Armstrong-Hélouvry (1991) was able to count the transmission gear teeth, and incorporating this factor into the friction model substantially increased the accuracy of predicted friction. In part to eliminate position-dependent friction, Salisbury *et al.* (1988) and Townsend (1988) study designs with homogeneous transmissions.

2.2.2.3. *External friction.* Sources of internal friction, such as bearings, are often designed so as to minimize friction. Mechanisms which must make contact with their environment, however, can have quite different design goals. In robotic dextrous manipulation, for example, high friction coefficients are desirable. Very soft fingers, made of rubber or elastomeric material, can provide friction coefficients greater than one. As a result, objects can be grasped gently while inhibiting both tangential sliding and rotation about the contact normal (Cutkosky and Wright, 1986).

Due to the complexity of the dextrous manipulation problem, many simplifying assumptions are made in the system modeling. For instance, most studies involving sliding assume quasistatic conditions (Kao and Cutkosky, 1992; Peshkin and Sanderson, 1988; Trinkle, 1989). This is done under the assumption that fine assembly operations are typically performed slowly (Trinkle, 1989). Recently, attention has been given to issues of control arising from the details of friction in grasp (Howard and Kumar 1993; Schimmels and Peshkin 1993).

The modeling of friction in contacts involving rubber or elastomers has received attention, but its description is beyond the scope of this paper. The following references on this topic are provided by Cutkosky and Wright (1986), Cutkosy *et al.* (1987), Howe *et al.* (1988), Moore (1972, 1975) and Schallamach (1971). The issue of stick slip as it affects motion planning and control in dextrous manipulation has apparently not been studied. Other examples of external friction, such as deburring or drilling operations, pose quite different modeling and control challenges as these tasks involve deliberate operation within the severe wear regime for one surface (Smith, 1989).

2.2.2.4. *Run-in and friction noise.* In developing our friction model, we have, for the most part, dwelt on factors such as velocity and load which can be considered as exogenous variables. There are also internal factors at work which depend on time, sliding cycles or total sliding distance. These effects are due to such things as loss of lubricant, deformation of surface material, change in temperature due to generated heat or accumulation of wear debris.

These factors all contribute to produce changes in the mean friction force even while the exogenous variables of velocity and load are held constant. These effects are perhaps most evident at the beginning and end of the life of a tribo-system. During the run-in period, the friction level of a new machine may increase or decrease until a long-term steady-state condition of mild wear is reached. The end of a tribo-system's useful life is marked by a transition to severe wear.

In addition to variations in the mean friction level, the 'noise' level can also vary over time depending on such properties as surface roughness and accumulation of wear debris. Often, variation in the friction force is highest during the run-in period and after the transition to severe wear (Blau, 1987).

These factors are important in terms of friction identification and control for the following reasons:

- A new machine may exhibit a higher or lower (and noisier) level of friction than the 'steady-state' level achieved after run-in.
- After a period of machine inactivity (at the start of the day, for example), it may be worthwhile to perform machine calisthenics. This will allow for circulation of the lubricant, temperature stabilization and thus stabilization of friction level.
- The average friction level obtained from very noisy data may not be correct. While the maximum friction magnitudes may well be due to the microscopic geometric and structural properties of the interface, the minima may depend more on the machine stiffness and sensor response (Blau, 1987).

In distributed parameter systems, such as a violin string or railway wheel, friction can induce chaotic motions. Popp and Stelter (1990) have investigated frictionally induced chaos in lumped and distributed parameter systems, and find that PID control of a single mass with static + Coulomb friction is not expected to exhibit chaotic motion; but that a two-mass, spring system under the same conditions will, as will a distributed mass system (such as a railway wheel or break drum) under a broad range of conditions. they present both theoretical and experimental results, including a proposed method for distinguishing chaos from noise in empirical data.

2.2.3. *Normal force and the coefficient of friction*
For much of this discussion, friction has been addressed as a force, rather than as a coefficient of friction, and normal force has not been addressed in depth. The frictional force, normal force and coefficient of friction are, of course, related through:

$$F_f(t) = \mu_f F_n(t), \tag{14}$$

where $F_f(t)$ is the instantaneous force of friction, $F_n(t)$ is the instantaneous normal force and μ_f is the coefficient of friction. The coefficient, μ_f, is not constant, but may depend upon velocity, velocity history, normal force and normal force history (Pavelescu and Tudor, 1987; Martins *et al.*, 1990). In control applications, situations exist in which it is possible to know the normal force, such as in a machine way carrying a known load; there are situations in which it may or may not be possible to know the normal force, such as in a bearing where the external load is known but internal force may not be; and there are situations in which it is not at all straight forward to know the normal force, such as in a preloaded gear train or motor brushes. In some cases the normal force may be constant and in others it may vary. In systems which exhibit stable friction, such as joint 1 of the PUMA robot (Armstrong, 1988; Armstrong-Hélouvry, 1991), normal force, along with temperature and other factors, must be well behaved.

In mechanisms where the normal force is varying, the prediction of friction becomes more complicated. This is particularly true where normal force is determined by control effort, as will be the case in transmissions that are not preloaded. The characteristic velocity of the Stribeck curve, state associated with frictional memory and the stiffness of presliding displacement are all influenced by instantaneous normal force, and by the history of applied normal force (Martins *et al.*, 1990; Soom, 1992). Normal force history has been shown to influence friction in geophysical systems (Linker and Dieterich, 1992). It is beyond the current state of the art to completely model the influence of changing normal force, though attention within tribology is turning to what appears to be the central issue: the normal displacement, e.g. Tolstoi (1967), Oden and Martins (1985), Martins *et al.* (1990), Hess and Soom (1991a, b). For the moment, the most viable approach to problems of dynamic normal force employs the integrated model, with Coulomb, viscous and Stribeck friction components represented as coefficients of friction and the stiffness of presliding proportional to normal force.

2.3. Future Trends in Tribology and Implications for Control

The overwhelming majority of treatments of friction have viewed the part-to-part interaction as a one degree of freedom motion: tangential, sliding motions are considered. Normal force has always been considered, but normal motions have been neglected. A school of thought is developing that normal motions play a central role in determining friction; including the realization of frictional memory and the Stribeck curve (Tolstoi, 1967; Tudor and Bo, 1982; Oden and Martins, 1985; Martins *et al.*, 1990; Goyal *et al.*, 1991). Tolstoi and others have made careful observations of friction and sub-micron normal displacements and find a strong correlation between instantaneous friction and instantaneous normal displacement, as shown in Fig. 26 (Tolstoi, 1967; Budanov *et al.*, 1980).

Described heuristically, as the contact begins to slide, impacts between the contacting asperities increase the separation between surfaces. Because the friction is a strong and nonlinear function of asperity penetration (normal separation), friction is modified by the changing normal separation (Martins *et al.*, 1990). The friction–velocity curve and frictional memory are thus in part manifestations of the normal dynamics. Different mechanisms, such as preloaded gears or a slider on a machine way, may have very different normal stiffness and damping, giving different frictional dynamics, even though material,

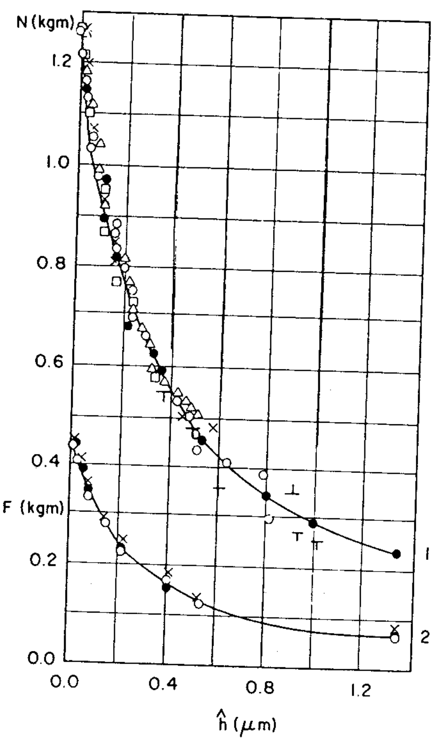

FIG. 26. Normal load (N, curve 1) and static friction force (F, curve 2) versus the normal separation, $\hat{h}$ (arbitrarily, $\hat{h}$ is taken to be zero for the maximum normal load used in the experiments). Dry steel surfaces [from Oden and Martins (1985), courtesy of the publisher, adapted from data reported in Tolstoi (1967)].

lubrication, geometry and normal loading may be the same.

As a demonstration of the potential of this framework, Martins *et al.* (1990) have been able to account qualitatively for a broad range of previously irreconcilable experimental observations, using computer simulations based on a simple model of friction physics and a more detailed model of the normal direction contact dynamics. To date, this work has concentrated on dry friction contacts; the impact of fluid lubricants must be considered for the results to be directly applicable to common control situations. The payoff for controls is the possibility of predictive, physically motivated models for machine friction. At issue are the physics underlying the Stribeck curve and frictional memory, both of which play leading roles in determining stick slip.

2.4. A Final Word on Models

As evidenced by the recent works of Oden, Martins, Soom and others, tribology has found a renewed interest in frictional dynamics, and new paradigms that may overcome conundrums left by the investigations of the 1950s. The direct motivation often stems from vibrationally induced noise, fatigue and wear—active feedback is never addressed in the tribology literature—but the possibility of spin-off technology for the controls community seems great, especially in as much as both camps are concerned with interfaces of engineering materials and mass-spring-damper systems. Even if predictive models of friction are never genuinely achieved, benefit for mechanism design and controls will come in the forms of more certain model structure, better identification strategies, bounds on parameter ranges, a broader range of frictional interfaces which are understood, and a richer pallet of design strategies for friction modification. All of which will contribute to better price/performance in machines.

3. ANALYSIS TOOLS THAT HAVE BEEN APPLIED TO SYSTEMS WITH FRICTION

Analysis of the motions of machines with friction have been made employing four types of tools: describing functions, algebraic analysis, phase plane analysis and simulation. Simulation is not normally considered an analysis tool; but when sufficient trials, perhaps thousands of trials, are conducted, the structure of the system behavior may be illuminated or empiric relations identified. We include simulation as an analysis tool here because its use is common in applications.

In almost all cases where these tools have been applied, the goal has been to predict the conditions for stick slip. The character of the result depends heavily upon the friction model, task and control structure considered. For a slip cycle during which velocity does not reverse—the common case with tracking tasks (Derjaguin *et al.*, 1957)—a Coulomb friction model permits an exact integration of the acceleration through a slip cycle and thus exact algebraic results. In all other cases approximations are involved. The character of the approximations and their impact on the validity of the conclusions drawn are important issues in all analyses. A relatively small number of investigators have verified their analysis with either experiment or extensive simulation.

The works applying nonlinear analysis techniques to systems with friction are all relatively specific in their focus. As a general introduction to analysis techniques for these systems several books have been written in the last decade, such as Slotine and Li (1991), Vidyasagar (1991) and Khalil (1992). Among older texts Atherton (1975) is often cited. Mees (1984) provides an interesting discussion of recent results regarding the describing function.

3.1. Describing Functions

The application of describing function analysis to study the motions of machines with friction has a long history (Tou and Schultheiss, 1953; Satyendra, 1956; Silverberg, 1957; Shen, 1962; Woodward, 1963; Brandenburg, 1986; Brandenburg and Schäfer, 1987, 1988a, b, 1989, 1991: Schäfer and Brandenburg, 1990, 1993; Townsend and Salisbury, 1987; Wallenborg and Åström, 1988; Canudas de Wit 1988; Canudas de Wit and Seront, 1990; Canudas de Wit 1987, 1991; Ehrich, 1991). The technique is an approximate one consisting of representing the input–output map of a single-input/single-output nonlinear element by the magnitude and phase relationship between a sinusoidal input and the fundamental harmonic of the corresponding output. This relationship constitutes a sort of transfer function; it can be represented by a complex number, and will in general be frequency and magnitude dependent. Underlying describing function analysis is the requirement that the element be single-input/single-output, and the assumption that investigation of the first harmonic provides a reasonable approximation to the behavior to the true system (Brogan, 1991). The advantage of the describing function is that it permits the use of frequency domain tools for the analysis of control.

There is an important special case in the study of describing functions: the memoryless, odd function. A memoryless element is one without state; and for such an element, the describing function will depend solely upon magnitude of the input. When the function is odd (friction as a function of velocity, with symmetric friction in the left and right directions, is a memoryless, odd function), the describing function will be strictly real (Brogan, 1991). Recent authors Brandenburg (1986), Brandenburg and Schäfer (1987), Townsend and Salisbury (1987), Wallenborg and Åström (1988), Canudas de Wit (1988), Canudas de Wit and Seront (1990), Canudas de Wit *et al.* (1987, 1991) and Ehrich (1991) have taken advantage of this special case, which we will call the 'memoryless element' construction. Earlier authors Tou and Schultheiss (1953), Satyendra (1956), Silverberg (1957), Shen (1962) and Woodward (1963) formed describing functions of the combined plant with friction. This construction is not memoryless because

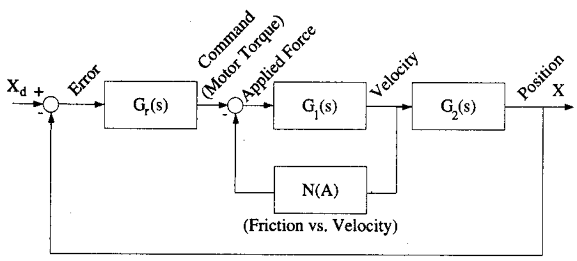

FIG. 27. A single mass system with friction, friction modeled as a function of velocity.

it places plant state within the modeled nonlinear element; this we term the 'integrated friction/plant' construction.

The describing function analysis of a position controlled one degree of freedom rigid mass system with sliding friction may be constructed as shown in Fig. 27. Simple or complicated systems may be considered with appropriate choices of $G_r(s)$, $G_1(s)$ and $G_2(s)$, [see, for example, Townsend and Salisbury (1987), Brandenburg (1986), Brandenburg and Schäfer (1987)]. The block diagram must be manipulated to arrange all of the linear elements as one block, leaving the nonlinear element as a second block giving the block diagram of Fig. 28.

The transfer function of the system of Fig. 27 may be written:

$$\frac{X(s)}{X_d(s)} = \frac{G_r \dfrac{G_1}{1 + G_r G_1 G_2} G_2}{1 + N(A)\left(\dfrac{G_1}{1 + G_r G_1 G_2}\right)}. \tag{15}$$

Stick–slip is a limit cycle, that is a motion giving a closed path in the phase plane. A limit cycle may be either stable, which means that nearby paths converge onto it; or unstable, which means that nearby paths do not converge to the limit cycle, but does not imply that the system is exponentially unstable. Using the describing function, a limit cycle is detected when there exists an amplitude, A, and frequency, $s = j\omega$, such that the denominator of (15) goes to zero; giving:

$$-\frac{1}{N(A)} = \frac{G_1}{1 + G_r G_1 G_2} \tag{16}$$

or, referring to Fig. 28:

$$-\frac{1}{N(A)} = G_L(s); \quad G_L(s) = \frac{G_1}{1 + G_r G_1 G_2}. \tag{17}$$

This condition is easily tested by drawing a Nyquist plot, with $-1/N(A)$ as one branch, and $G_L(s)$ as a second branch; as seen in Fig. 29. An intersection predicts a limit cycle. The limit cycle must additionally

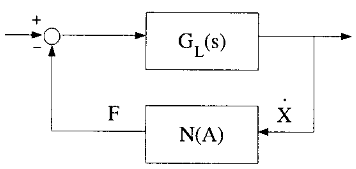

FIG. 28. A single mass system with friction, block diagram manipulated to lump linear elements.

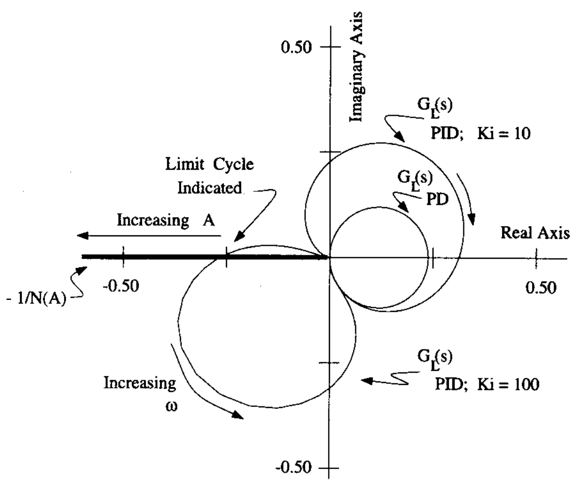

FIG. 29. Nyquist plot for limit cycle detection in a system with integral control [following Townsend and Salisbury (1987)].

be tested for stability, [see e.g. Brogan (1991)]. Three contours of $G_L(s)$ are plotted in Fig. 29, corresponding to a damped mass with a PD or PID controller. For the system under consideration, equation (16) gives:

$$\begin{aligned} \text{PID:} \quad & -\frac{1}{N(A)} = \frac{s}{Ms^2 + (b + k_v)s + k_p} \\ \text{PID:} \quad & -\frac{1}{N(A)} = \frac{s^2}{Ms^3 + (b + k_v)s^2 + k_p s + k_i}. \end{aligned} \tag{18}$$

Definitions and values for the parameters of equation (18) and Fig. 29 are given in Table 3.

Of the three $G_L(s)$ contours of the Nyquist diagram, Fig. 29, two predict stable motion. Those are the PD contour and the PID contour with $K_i = 10$. Following Townsend and Salisbury (1987) a contour is shown with a large integral control gain; this contour indicates stick slip. Note that the contours of Fig. 29 are plots of $G_L(s)$ as given by equation (17), rather than the transfer function customarily presented in Nyquist plots.

The memoryless element describing function is straightforward to apply. Several authors have made extensions to this analysis. Wallenborg and Åström (1988) present an interesting proof that a system with Coulomb friction (no static friction) and state feedback can be unstable only for the special case of an unstable controller. Amin (1993) has studied Coulomb and Coulomb + static friction with the memoryless element and integrated plant/friction describing function, coupled to a single mass and PID

TABLE 3. DEFINITIONS AND VALUES FOR THE PARAMETERS OF EQUATION (18) AND FIGURE 29

Quantity	Symbol	Value in Figure 29
Mass	M	1.0
Damping	b	0.1
Position gain	k_p	10
Velocity gain	k_v	5
Integral gain	k_i	10 or 100

control. He derives the predictions of the describing function analyses for all possible parameter combinations, and compares these with analytic results. Canudas de Wit *et al.* (1991) employ describing functions to study the behavior of adaptive systems in the presence of negative viscous friction and find that overcompensation can lead to instability. Brandenburg and Schäfer (1987, 1988a, 1991) have applied describing functions to the case of unidirectional sliding by local linearization of the Stribeck curve and application of the Popov stability criterion.

The ease of use seems to come at considerable cost however. Brandenburg (1986), Brandenburg and Schäfer (1987) and Townsend and Salisbury (1987) both report extensive simulations done to verify the predictions of describing function analysis, implicitly recognizing that the describing function analysis adds a layer of approximation above and beyond the approximations within the friction model itself. Townsend and Salisbury (1987) report:

> "Dynamic simulations show that the SIDF (single input describing function) predictions for Coulomb friction are qualitatively useful though quantitatively inaccurate. The (single input describing function) predictions for stiction become even qualitatively incorrect."

Brandenburg and Schäfer have carried out an extensive series of studies of a two mass, flexible system with multi-loop feedback; and report that the describing function analyis (extended to the harmonic balance technique) qualitatively agrees with the results of simulation for the case of Coulomb friction and unidirectional sliding, but fails substantially in the cases of Coulomb + static friction or integral control (Brandenburg, 1986; Brandenburg and Schäfer, 1987, 1988a, b, 1989, 1991; Schäfer and Brandenburg, 1990, 1993). The authors attribute this to the infinite-valued branch at $\dot{x} = 0$ and, as Townsend and Salisbury also point out, the oscillation is not well approximated by its fundamental harmonic. Amin (1993) shows that, for the case of a single mass and PID control, the memoryless element describing function will predict stick slip if and only if the system without nonlinear friction is unstable. Figure 29 is an example of this, the linear portion of the system is unstable when $k_i = 100$.

A further challenge arises in representing static friction by a memoryless element describing function. The describing function is the complex gain between a sinusoidal input and the fundamental harmonic of the output of a nonlinear element. The sinusoidal input spends zero time at zero, thus any phenomena occuring precisely when the input is equal to zero will be modeled as having zero extent. When friction is modeled as a function of velocity, static friction will make no contribution to the fundamental harmonic of the output, and thus cannot be represented by the describing function.

These challenges to application of the memoryless element describing function are consequences of a deeper issue, illuminated by Ehrich's effort to construct two describing functions: one a function of applied force and the other a function of velocity

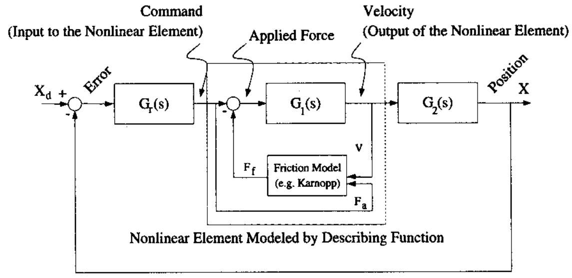

FIG. 30. A single mass system with friction, friction modeled as a function of velocity and applied force.

(Ehrich, 1991). The issue is this:

> Friction is neither a function of velocity nor a function of applied force, but of both.

When the body is sliding, friction—in a simplified model—is a single valued function of velocity; but when the body is at standstill, friction is not friction at all, properly speaking, but is a constraining force: a function of applied force (Polycarpou and Soom, 1992). Virtually all authors who have undertaken simulation have reflected this fact with some form of switching function that handles the case of $\dot{x} = 0$.

The early investigators (Tou and Schultheiss, 1953; Satyendra, 1956; Silverberg, 1957; Shen, 1962; Woodward, 1963) did not employ the memoryless element construction, but developed describing functions for the composite of the elements from the force input to the velocity output, as shown in Fig. 30. With this approach a describing function can be worked out by piecewise integration of the response. This procedure, illustrated in Fig. 31, gives rise to a describing function that is a curve on the complex plane, or a family of curves for the case of Coulomb + static friction, as illustrated in Fig. 32. Here the family of complex valued describing function curves is parameterized by the ratio of the static + Coulomb friction and plotted as $-1/N(A, \omega)$, as described in equation (16). This analysis predicts that the series compensator will exhibit stick–slip while the parallel

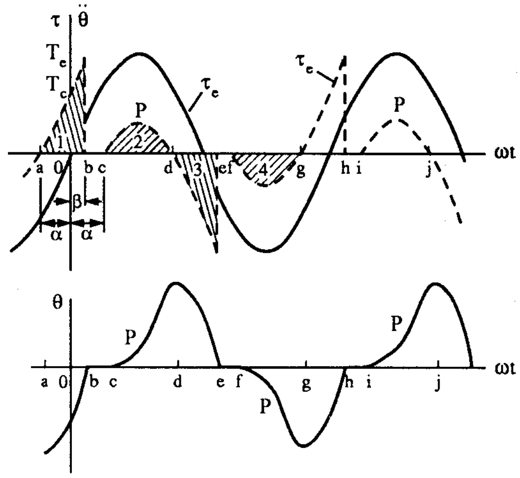

FIG. 31. Piecewise evaluation of the output of a system plus friction element with sinusoidal applied force and periods of standstill [from Tou and Schultheiss (1953), courtesy of the publisher].

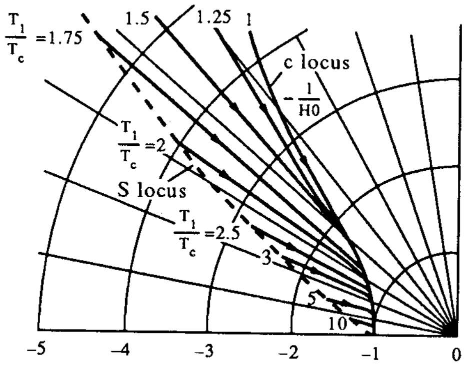

FIG. 32. Nyquist diagrams showing complex valued describing function of an integrated plant/friction model. The family of curves corresponds to different ratios of static: Coulomb friction [from Tou and Schultheiss (1953), courtesy of the publisher].

compensator will not; a prediction verified in the laboratory (Tou, 1953).

Amin (1993) has recently revisited the integrated plant/friction describing function of Tou and Schultheiss and compared its predictions with exact analytic results. He finds that in spite of the representation of standstill and static friction, Tou's construction is not, in general, able to correctly predict the presence of a limit cycle in the idealized single mass, Coulomb + static friction, PID controlled system. The breakdown appears to relate to the poor representation of the forces and motions during stick slip by a sinusoidal approximation.

Using the integrated friction/plant construction, Tou and Schultheiss (1953) study a system which includes a type I controller constructed of analog components, and Coulomb or Coulomb + static friction. Limit cycling is predicted; the authors conclude that it can be extinguished by the addition of sufficient velocity feedback, which they demonstrate in the laboratory. Satyendra (1956) studied mechanisms with backlash as well as Coulomb + static + viscous friction. As did Tou, Satyendra studied stability at zero velocity. Silverberg (1957) strives to separate the describing function of a Coulomb + static + viscous friction element into frequency dependent and amplitude dependent parts, which greatly aids interpretation. This effort is necessitated by the fact that he is working with an integrated plant/friction element, such as in Fig. 30.

Shen (1962) and Brandenburg and Schäfer (1987, 1988a, 1991) have studied systems with static + Coulomb friction tracking a ramp position input. The shift in emphasis from the stationary to the slowly moving problem is important. The describing function—whether memoryless or integrated with the plant—changes when a steady velocity is introduced. This occurs because the sinusoidal input (velocity or force) must be superimposed on a DC level, shifting its relation to the (friction–velocity) curve. For the memoryless element construction of Fig. 27, the shift of level will upset the odd function property of the nonlinear element, and the describing function, $N(A)$, will be complex rather than strictly real. Brandenburg gets around this problem by forming the describing function of a local linearization of the Stribeck curve. He finds the results to give predictions in qualitative agreement with observed behavior.

Shen (1962) presents a describing function for nonlinear, low-velocity friction. He verifies the reliability of his (approximate) describing function analysis by comparison with the results of algebraic analysis—integration of equations of motion through a slip cycle—for the case Coulomb friction. He finds roughly 10% difference in the range of the parameters for which stick slip is predicted by the integrated plant/friction describing function and algebraic analyses. None of the authors of this period carried out extensive simulation.

3.2. Algebraic Analysis

An alternative approach to predicting stick slip lies in integrating the equations of motion through a slip cycle and then determining whether the system arrives again in the stuck condition. Arrival in the stuck condition may be determined by a test on system state (Derjaguin et al., 1957; Shen, 1962; Shen and Wang, 1964; Cockerham and Cole, 1976; Amin, 1993), or by a test on system energy (Armstrong-Hélouvry, 1992, 1993). The possibility of stable motion can also be determined by investigating the stability of equilibrium points of the system dynamics (Dupont, 1994).

When a Coulomb, or Coulomb + viscous friction model is applied during sliding the equations of motion may be integrated exactly (Derjaguin et al., 1957; Shen, 1962; Shen and Wang, 1964; Cockerham and Cole, 1976). Static friction may then be modeled as a modification of the initial condition of the trajectory. Considering static + Coulomb + viscous friction, Derjaguin et al. provide perhaps the clearest exposition of this technique. The authors focus on a second-order system (analogous to a single mass with PD control) and study the influence of rising static friction. Applying dimensional analysis, integrating the equations of motion during sliding and detecting stick–slip by examining whether the trajectory arrives again at zero velocity; they determine that there is a critical level of static friction below which stick–slip will be extinguished, and that this critical level can be expressed as an implicit function of desired velocity and system parameters:

$$\frac{\theta}{\sqrt{(1-\theta^2)}}\tan^{-1}\left(\frac{\phi_c\sqrt{(1-\theta^2)}}{\phi_c\theta-1}\right) = \ln\left(\sqrt{\phi_c^2-2\phi_c\theta+1}\right), \tag{19}$$

where, following the notation of Derjaguin,

$$\omega=\sqrt{k_p/m};\quad \theta=\frac{k_v/m}{2\omega}$$

$$\phi_c=\frac{\Delta F_c}{mv\omega}=\frac{\Delta F_c}{v\sqrt{mk_p}}$$

and k_p is system stiffness; m is mass; k_v is velocity feedback or damping; ΔF is the excess of breakaway friction over Coulomb friction and ΔF_c is the critical value required for stick slip; θ is dimensionless damping; and ϕ_c is dimensionless excess of breakaway

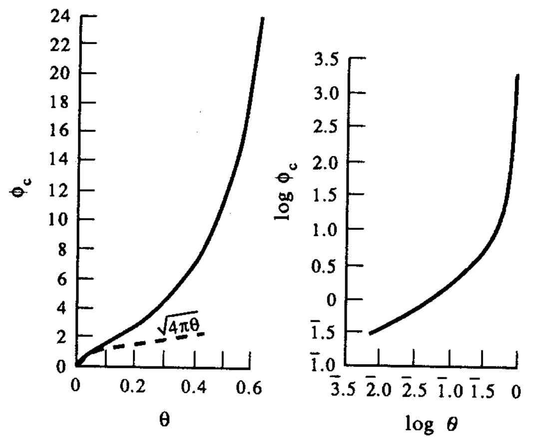

FIG. 33. Variation of critical parameter, ϕ_c, with damping parameter θ; ——: according to exact equation (19); - - - -: according to the first-order approximation $\phi_c \approx \sqrt{4\pi\theta}$; [from Derjaguin *et al.* (1957), courtesy of the publisher].

friction over Coulomb friction. The relationship between ϕ_c and θ given implicitly by equation (19) is plotted in Fig. 33. Rising static friction is modeled as a modification of ΔF_c. Derjaguin *et al.* (1957) find, as the experiments of Rabinowicz (1951, 1958) and others had shown, that the abrupt extinction of stick slip with increasing average velocity is strongly influenced by the time constant of the rising static friction. This work is also notable for its use of dimensional analysis.

Shen (1962) carries out an exact analysis as a method of verifying the results of a modified describing function analysis. He finds that for low velocity ramp inputs to a PD controller, i.e. slow tracking, stick slip will be observed under a wide range of conditions and may be extinguished by sufficient derivative feedback. Shen and Wang (1964) extend this result to systems with an integral control term. They observe that the damping required for stabilization is a decreasing function of desired velocity and propose a variable structure system comprising a very high-gain velocity feedback with saturation. The saturation serves to reduce the impact of high-gain velocity feedback on the system above those velocities where stick slip is observed. Integral feedback with a deadband is also considered. Because it is useful to achieve high tracking accuracy at velocities above the stick-slip range, a deadband is proposed that is a function of system velocity. Constructions such as these are also found in industrial applications.

Cockerham and Cole (1976) consider a model that is based on the data of Bell and Burdekin (1969); which shows a friction curve as shown in Fig. 34 [linearized by Cockerham and Cole (1976)]. This friction curve is an approximation to the Stribeck curve with frictional memory. As a linear approximation, it has the important property of making analytic results possible.

Amin (1993) has considered Coulomb + static + viscous friction, PID control and the pointing task (see Section 4). By considering motions between the instant after breakaway and the instant before stick, and establishing the solution to the equations of motion for all possible combinations of parameters, Amin finds that a PID, position-control system will hunt when there is static friction. And that it will do this for all combinations of parameters for which the linear portion of the system is stable. He also finds that the system will not hunt for any stable combination of parameters and Coulomb friction alone. The result establishes an important baseline for the study of describing function and other approximate analysis methods.

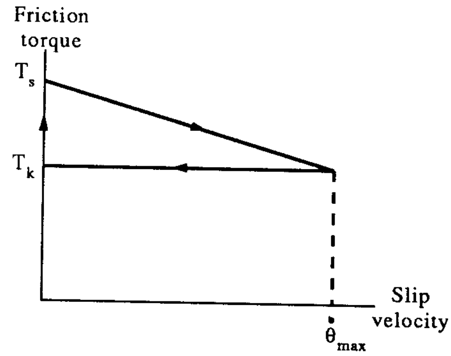

FIG. 34. Friction curve showing linearized Stribeck friction and frictional memory [from Cockerham and Cole (1976), courtesy of the publisher].

Armstrong-Hélouvry (1991, 1992, 1993) has applied first-order perturbation theory to a two degree of freedom system with PD control and friction described by a Coulomb + Stribeck + frictional memory + rising static friction model. He studies a tracking problem comparable to that of Derjaguin *et al.* (1957) or Shen (1962). In this analysis the trajectory of the system unperturbed by friction is used to determine the energy transactions due to the full friction model. The influence of frictional memory and rising static friction are mapped out and the required damping for stable motion is found as a function of system stiffness (position feedback gain) and desired velocity. The predictions of the analysis are verified by simulation and experiment. The frictional memory is found to dominate the extinction of stick slip in very stiff systems, and rising static friction to dominate as the average velocity increases. A test on the total change in system energy during a slip is used to detect the presence of stick slip; and gives the prediction that steady motion will obtain when:

$$-2\pi\rho\dot{\xi}_d^2 + F^*_{s,b_n}\dot{\xi}_d\Delta'_{Ef_s} < -\frac{1}{2}\left(F^*_{s,b_n}\frac{\dot{\xi}_d^2}{1+\dot{\xi}_d^2}\right)^2 \quad (21)$$

where, following the notation of Armstrong-Hélouvry (1993), ρ is dimensionless damping; $\dot{\xi}_d$ is dimensionless desired velocity; F^*_{s,b_n} is dimensionless excess breakaway friction and is influenced by rising static friction; and Δ'_{Ef_s} reflects the energy contribution due to Stribeck friction with frictional memory, and is given by an integral that is a function of two parameters and must be evaluated numerically.

ρ, $\dot{\xi}_d$ and F^*_{s,b_n} are respectively analogous to θ, v and ΔF in equation (19). The resultant stick-slip extinction boundary for the case of a specific robot

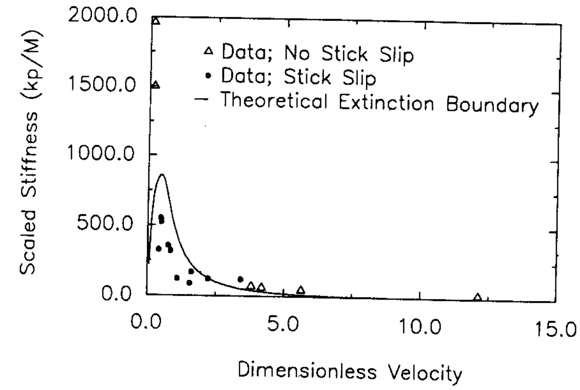

FIG. 35. Experimental stick-slip data; the contour is the stick-slip extinction boundary calculated with equation (21). Dimensionless velocity is given by $\xi_d = \dot{x}_d/\dot{x}_s$, cf. equation (10) [from Armstrong-Hélouvry (1993)].

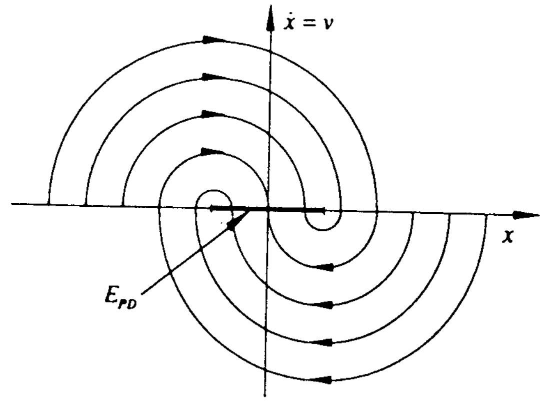

FIG. 36. Phase trajectories of a PD regulator and mass with Coulomb friction [from Southward et al. (1991), courtesy of the publisher].

arm, along with experimental data, are presented in Fig. 35.

Southward *et al.* (1991) propose a variable structure controller comparable to sliding mode control (Utkin, 1977). A hard switch in control action about $\dot{x} = 0$ is used to overcome the frictional nonlinearity. A Lyapunov proof of stability is achieved with the use of Dini-derivatives to handle the discontinuity in friction at $\dot{x} = 0$. The analysis requires only that an upper bound on friction be known. Implementation problems, such as chatter or the impact of controller and actuator bandwidth limitations, are not addressed. But sliding mode control—suitably modified to be implementable—has been remarkably successful elsewhere (Slotine, 1984).

Heck and Ferri (1991) apply singular perturbation theory to a fourth-order model of a turbine system with a Coulomb friction model, and find that a first-order correction term is sufficient to produce a system simulation that agrees well with a full model simulation. Their effort is focused on model order reduction.

The investigations noted above have sought to establish the presence of stick slip by testing for the existence of a stable limit cycle. For constant-velocity sliding motion, an alternative approach is to consider the stability of the steady-sliding fixed point. If the fixed point is stable and its domain of attraction is large enough, the possibility of smooth sliding is established. Rice and Ruina (1983) and Dupont (1994) have pursued this approach to illuminate the interaction of stiffness and frictional memory. Their contribution is the demonstration of the role of frictional memory in the stability of steady sliding.

3.3. Phase Plane Analysis

The phase plane may be used to graphically represent the trajectories of systems, linear and nonlinear. An example is shown in Fig. 36. In cases where theoretical restrictions on the character of trajectories are not possible, the phase plane serves as a graphical presentation of the results of simulation; and can be said to demonstrate general results about the system when trajectories spanning a sufficient range of initial conditions are explored. In other cases, where theoretical restrictions on the behavior of trajectories are possible, the phase plane serves to organize and clarify the interactions and consequences of these restrictions. In Fig. 36, for example, all trajectories lying sufficiently near the origin can be shown to enter the set of multiple equilibria, E_{PD}.

The dimensionality of the phase plane is the order of the system state. A mass with PD control thus has a two-dimensional phase plane, one that can be easily realized and interpreted. The same system with integral control, however, has three dimensions, and might already be difficult to visualize. A two mass system with flexibility and integral control, such as that considered by Brandenburg *et al.*, e.g. Brandenburg (1986) has five states and is probably not usefully studied with the phase plane.

In two dimensions there are a number of strong theorems belonging to phase plane analysis; for example, the Poincaré–Bendixson theorem (Slotine and Li, 1991):

> If a trajectory of a second-order system remains in a finite region Ω, then one of the following is true:
>
> (a) the trajectory goes to an equilibrium point,
> (b) the trajectory tends to an asymptotically stable limit cycle,
> (c) the trajectory is itself a limit cycle.

This result and several others, however, do not extend to higher dimensions.

Many authors have used phase planes to illustrate various points about system trajectories (Shen, 1962; Shen and Wang, 1964; Kubo *et al.*, 1986; Townsend *et al.*, 1987; Radcliffe and Southward, 1990; Southward *et al.*, 1991; Armstrong-Hélouvry, 1992). Radcliffe and Southward (1990) go beyond illustration and use the phase plane to make several important points. They are concerned with the ability of various friction models to predict stick slip, and show that all trajectories of an otherwise stable linear system with Coulomb friction and PD control will converge to a set of multiple equilibria, and will thus fail to show stick slip. Extending the phase plane to three dimensions, the authors proceed to show that a system with Coulomb friction, and typical controller gains and friction values, cannot exhibit hunting (back and forth oscillation about the set point) with PID control.

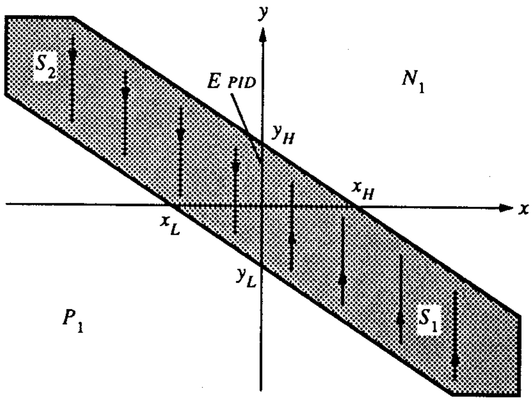

FIG. 37. Parallel trajectories inside the sticking band S have zero acceleration until they reach the border [from Radcliffe and Southward (1990), courtesy of the publisher].

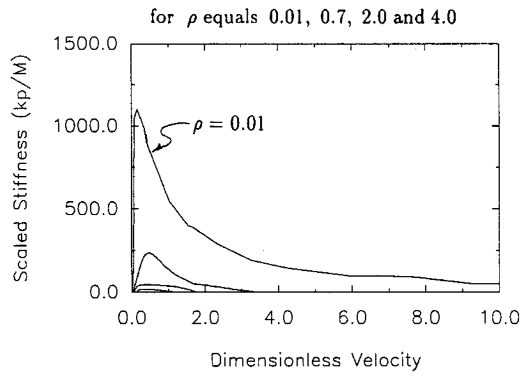

FIG. 38. Plot of boundary above which there is no stick-slip, determined by 8000 simulation trials. ρ is the dimensionless damping coefficient, equation (21) [from Armstrong-Hélouvry (1991)].

This is done by investigating the properties of a two-dimensional slice through the three-dimensional phase space. The slice yields the x–y plane, as shown in Fig. 37; y is the integral error state. The phase space is partitioned into six regions, trajectories in each region are shown to enter another region; and the map from one region onto itself, similar to a Poincaré map, is established. Specifically, the position-error coordinate at breakaway is mapped to the position-error coordinate at the next breakaway event. Convergence, limit cycling or divergence are determined by establishing contraction, stability or expansion in this one-dimensional map. Integrations in the method are performed numerically, and so only for specific combinations of parameters. In the cases studied, it is shown that excess breakaway friction over Coulomb friction—both static and Stribeck friction models are considered—is required to excite stick slip. The authors propose that friction models more sophisticated than the Coulomb friction model need to be studied to predict even the qualitative behavior of frictional limit cycles.

3.4. Analysis by Simulation

For controller analysis and synthesis, the tools described in the previous sections are applied to system models. When the models permit exact analysis, the validity of the results depends only on that of the model. When more complicated systems are analyzed with approximate techniques, such as describing functions, the validity of the results rests not only on the validity of model, but also on the assumptions of the analysis technique.

Simulation provides a means of verifying both models and analysis techniques. Comparison of experiment with simulation can be used to validate models. Comparison of analytic results with simulation can be used to validate approximate analysis techniques. While simulation can certainly be abused, when used in conjunction with both experiment and analysis, it is a powerful tool for closing the loop between them.

A great deal can often be learned by simulating a handful of carefully chosen trajectories. Alternatively, by performing hundreds or thousands of simulation trials, one can numerically map stability boundaries in parameter space or domains of attraction in state space. Figure 38 provides an example of the former. Here, the "stick-slip extinction boundary" is plotted as a function of system parameters. A valid analysis technique should be used to verify these numerically computed contours. By extensive simulation, one gains a measure of parameter sensitivity and experiment/analysis verification which is lacking when only a few trajectories are simulated. The increasing availability of massively parallel computers makes this approach practical and efficient. In the following paragraphs, the special problems associated with simulating friction are presented along with a discussion of solution techniques.

For systems with multiple degrees of freedom, such as robots, the rigid-body dynamic equation including friction can be written in the form:

$$\tau = D(x)\ddot{x} + h(x, \dot{x}) + f(x, \dot{x}, \ddot{x}). \qquad (22)$$

The vectors of joint displacements and actuator torques are x and τ, respectively. Their dimension equals the number of degrees of freedom of the mechanism. The configuration-dependent inertia matrix is denoted by D. It is both symmetric and positive definite. The vector h consists of centrifugal, Coriolis and gravity terms. The vector f includes all friction terms and is a function of joint positions, velocities and accelerations.

The forward dynamics problem is to solve for the joint positions, velocities and accelerations given the input torques or forces and the initial conditions. This is the problem of simulation. At each time step, the known joint torques, positions and velocities are used to compute the joint accelerations. In the absence of friction, this typically involves solving a set of linear algebraic equations for the accelerations. Using the values of acceleration and velocity, numerical integration yields the velocity and position at the next time step.

By considering the standard Coulomb friction equation, we can gain insight into the computational issues involved in simulating a broad class of friction models. Independent of the area of contact, the

Coulomb friction force always opposes relative motion and is proportional to the normal force of contact. This force can be expressed as

$$F_C = \mu\,|F_N|\,\mathrm{sgn}\,(\dot{x}_r), \tag{23}$$

where μ is the coefficient of friction, F_N is the normal force and $\dot{x}_r$ is the relative sliding velocity. The signum function is defined:

$$\mathrm{sgn}\,(\dot{x}) = \begin{cases} +1, & \dot{x} > 0 \\ 0, & \dot{x} = 0 \\ -1, & \dot{x} < 0 \end{cases}. \tag{24}$$

Due to its dependence on the sign of velocity, the friction force is discontinuous at zero velocity. This indicates that the governing differential equations are discontinuous in the highest order derivative terms. In addition, as indicated by $\mathbf{f}(x, \dot{x}, \ddot{x})$, the normal forces in machine components can depend not only on link positions and velocities, but also on accelerations. Because this dependence is often nonlinear, equation (22) can only be solved explicitly for $\ddot{x}$ in special cases. Both difficulties are discussed in detail in the following paragraphs.

3.4.1. *Discontinuous friction models*
When integrating discontinuous ordinary differential equations, the appropriate value of the derivative must be used on each side of a discontinuity. Unfortunately, discontinuities generally occur inside integration subintervals. A standard technique is to employ switching functions which flag the presence of a discontinuity in the last subinterval. For the initial value problem,

$$\dot{x} = f(x, t)$$
$$x(0) = x_0$$

a switching function, $\phi(x, t)$, is defined such that $\phi(x, t) = 0$ when $f(x, t)$ is discontinuous and $\phi(x_n, t_n) \cdot \phi(x_{n+1}, t_{n+1}) < 0$ implies a discontinuity in the subinterval $x_n \le x \le x_{n+1}$ (Fatunla, 1988; Dupont, 1993). For a velocity zero crossing, $\mathrm{sgn}\,(\dot{x}_r)$ is such a function.

In addition to detecting a discontinuity, the integrator must also provide a mechanism for locating the point of discontinuity within the subinterval. Next, integration up to the point of discontinuity is repeated and then the integration routine is restarted from the discontinuity using the appropriate derivative value. Initially, small steps should be taken to accurately capture any transients which follow the discontinuity. If the friction model includes stiction, the integrator must also include tests to detect when sticking occurs.

Variable-step-size, variable-order methods are appropriate for integrating discontinuous equations. For synchronization purposes in real-time simulation and control, however, Morgowicz (1988) suggests the use of fixed-step-size methods. By choosing the controller period as a multiple of the fixed-step-size, the simulated machine state is available at controller sampling times.

To locate discontinuities occurring during the previous subinterval, Morgowicz (1988) uses linear interpolation. Approximate values of the state derivatives on both sides of the discontinuity are computed. They are used to reintegrate the subinterval in one step. Unless very fine motions are under consideration, simplifications of this type can give quite adequate simulation results. The time saved in simulating a given trajectory will depend on the number of velocity zero crossings involved.

3.4.2. *Alternate friction models*
A number of researchers have proposed alternate friction models with the goal of producing accurate results while minimizing algorithm complexity and simulation time. A general approach is to replace the discontinuity of the static + Coulomb model by a curve of finite slope (Threlfall, 1978; Bernard, 1980; Rooney and Deravi, 1982; Haessig and Friedland, 1991). This type of model eliminates the need to search for the switching point within an integration subinterval. If the slope is large, however, small step sizes are needed and the numerical integration remains slow. More importantly, these models do not provide a true stiction mode. The system creeps through zero velocity instead of sticking. This effect may be important when the period of the stick-slip limit cycle is long.

Several techniques have been proposed to include stiction while still avoiding the search for the switching point. In one method, best described in Karnopp (1985), friction is given by:

$$F_f(\dot{x}, F) = \begin{cases} -\mathrm{sgn}\,(\dot{x}) F_C & |\dot{x}| > D_V \\ -\mathrm{sgn}\,(F)\max\,(F, F_H) & |\dot{x}| \le D_V \end{cases}. \tag{25}$$

A small neighborhood of zero velocity is defined by D_V, as shown in Fig. 39 (Bernard, 1980; Karnopp, 1985; Johnson and Lorenz, 1991; Younkin, 1991). Outside this neighborhood, friction is a function of velocity. Inside the neighborhood, velocity is considered to be zero and friction is force dependent. As long as the resultant force is less than the maximum stiction force, the small velocity remains constant within the $\pm D_V$ neighborhood or is set to zero. Karnopp's (1985) implementation of this method for a block of mass m sliding on a flat frictional surface with an applied force, F, is depicted in the block diagram of Fig. 40. The deadband of the gain block between the two integrators forces the velocity to exactly zero during sticking. These models allow discontinuity of static to Coulomb friction force at the neighborhood boundary.

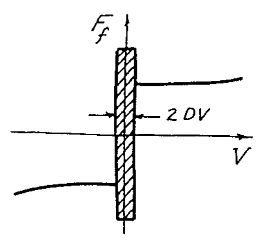

FIG. 39. Karnopp's friction–velocity model. By allowing stiction within the interval $\pm D_V$, the integrator does not have to search for velocity zero crossings [from Karnopp (1985), courtesy of the publisher].

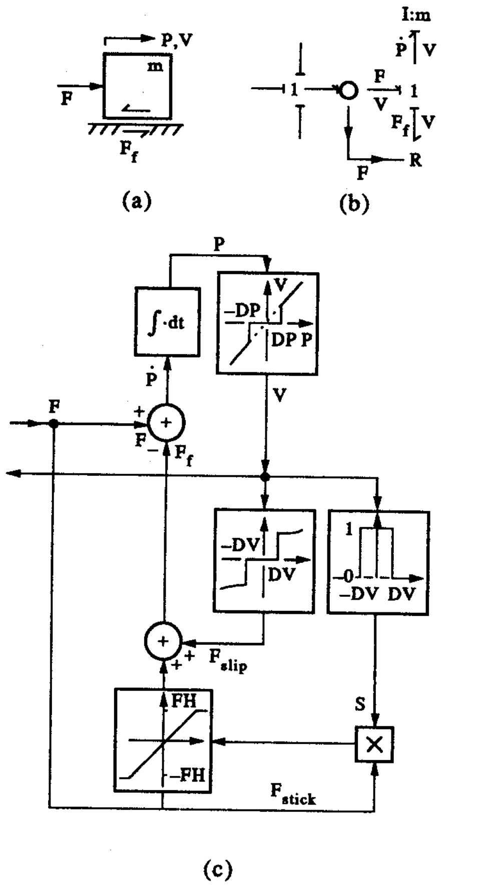

FIG. 40. Block diagram of Karnopp's friction simulator for a force F, applied to a block of mass m moving on a flat frictional surface [from Karnopp (1985), courtesy of the publisher].

A second approach is based on experimental observation of presliding displacement, that is, at velocity reversals, friction may be more appropriately modeled as a continuous function of displacement. This can be represented graphically with a hysteresis loop as shown in Fig. 41. For small displacements, this can be interpreted as the straining and eventual rupture of many small bonded contacts between the two sliding or rolling surfaces (Dahl, 1968, 1977; Threlfall, 1978; Haessig and Friedland, 1991). If very small displacements are to be accurately simulated, these position-dependent models could be more accurate than a velocity-dependent model. Threlfall (1978) and Haessig and Friedland (1991) propose switching between position- and velocity-dependent models. For large displacements, Coulomb friction together with spring stiffness can sometimes be modeled in a similar fashion (Bernard, 1980).

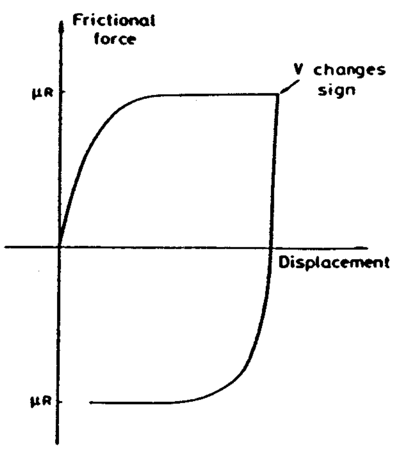

FIG. 41. Friction–displacement hysteresis loop [from Threlfall (1978), courtesy of the publisher].

3.4.3. *Load-dependent friction*

The significance of varying applied loads in machine friction has not been thoroughly studied. Some experiments have suggested that, in certain mechanisms, friction does not exhibit a measurable dependence on the transmission and reaction loads which determine the frictional normal forces (Armstrong-Hélouvry, 1991; Canudas de Wit *et al.*, 1991). Other experiments indicate that it can be an important factor in transmission elements (Newman *et al.* 1992; Dohring *et al.* 1993; Dupont, 1993). The situations are distinguished by the degree of preloading of the elements.

In general, the inclusion of a load-dependent Coulomb or static friction term, $f(x, \dot{x}, \ddot{x})$, in the dynamic equation (22), renders it implicit in the joint accelerations, $\ddot{x}$. The forward solution requires an iterative root-finding procedure (such as a modified Newton method) at each step of the integration to compute the accelerations. The cause of the implicitness is the dependence of friction on the magnitude of the normal force. The normal force itself is a function of the resultant force and moment at a joint. Expressed in a local coordinate frame, the components of the resultant force and moment can be formulated in terms of the joint positions, velocities and accelerations. These components will be affine transformations of the accelerations (Dupont, 1993).

If the direction of the normal force is constant in the local coordinate frame, the normal force can be expressed as a function in which the net force and moment components appear linearly. This is true for friction in translational joints and certain transmissions. Since the sign of the normal force can change, its absolute value must be used to obtain its magnitude.

When the direction of the normal force is not constant in a local joint coordinate frame, the magnitude of the normal force will involve the square root of sums of squares of net force and moment components. As an example, consider a radially loaded revolute joint. More generally, there are multiple bearings at a joint and one must consider joint geometry and reaction torques as well as reaction forces (Gogoussis and Donath, 1988).

Thus, load-dependent friction involves the absolute value or square root of sums of squares of acceleration-dependent terms. Substituting either type of expression into the original dynamic equations renders them implicit in the accelerations. Consequently, it is necessary to solve iteratively for joint accelerations at each time step of a simulation.

To minimize the number of iterations, Threlfall (1978) proposes a predictor–corrector method which uses the reaction forces from the previous seven time steps to predict the new values. Morgowicz (1988) uses only the reaction forces from the preceding time step. Both authors indicate that convergence is usually obtained in two iterations.

The cost of implicitness is significant, however. Using two iterations effectively requires two forward solutions in addition to solving the inverse problem once for those force and torque components needed to compute the normal forces. To avoid an iterative solution, Gogoussis and Donath (1990) propose a hybrid analog/digital computer implementation solving the implicit portions in the analog computer. Analog simulations are discussed by Cockerham and Cole (1976) and by Dahl (1968) although neither paper considers varying normal force.

For those cases in which the magnitude of frictional normal force involves only an absolute value, there is a finite number of possible solutions based on the combinations of signs of normal forces. By adopting an efficiency formulation and tracking the signs of the normal forces using switching functions, the number of iterations can be minimized (Dupont, 1993). This is an important case because it applies to transmission elements which, when present, often dominate machine friction.

3.4.4. *Existence and uniqueness*

In the absence of friction, the rigid-body forward dynamic equations can be shown to possess a unique solution for the accelerations at each time step. For most cases of interest involving low-friction mechanisms, rigid-body models yield a unique solution as well. For sufficiently high coefficients of friction, however, systems with load-dependent friction can be shown to possess either multiple consistent solutions or none at all. The problem arises because the normal-force reactions cannot be written as functions of the system state, but instead must be expressed in terms of the unknown accelerations. Since the resulting set of equations is nonlinear in the unknowns, it is not surprising that solution existence and uniqueness is problematic.

The existence and uniqueness problem associated with Coulomb friction between rigid bodies has been studied in Painlevé (1895), Löstedt (1981), Rooney and Deravi (1982), Rajan *et al.* (1987), Mason and Wang (1988), Wang *et al.* (1992) and Dupont (1992a, b). Dupont (1992a) shows that a single degree of freedom is sufficient to exhibit these problems and that, according to the value of input force or torque and velocity, there can be either zero, one, two or three feasible solutions. Rooney and Deravi (1982) demonstrate similar behavior with a quasistatic analysis of a slider-connecting rod mechanism.

Mason and Wang (1988) address the case of no consistent solution and model it as an impact with zero approach velocity. The hypotheses used to govern frictional impact can sometimes violate the principles of dynamics such as energy conservation. Wang *et al.* (1992) provide a good discussion of this topic and propose discretizing the normal-force contact zones into compliant patches with lumped linear stiffness to ensure a unique solution.

As did Wang *et al.* (1992), Dupont (1992b) points out that the existence and uniqueness problems associated with load-dependent friction can be resolved by relaxing the rigid-body assumption. He proposes to make the otherwise ambiguous normal reactions functions of system state by introducing a lumped compliance with a component normal to the friction contact surface. Using this approach, he addresses the case of multiple dynamic solutions and shows that for a system of finite stiffness, the "extra" solutions are dynamically unstable.

In many situations, existence and uniqueness is not a concern. In these cases, it is a straightforward task to develop computer code for simulating the appropriate friction model. Alternatively, several commercial software packages allow for the numerical integration of nonlinear differential equations including those with discontinuities. Friction simulation can be a valuable analysis tool when used judiciously and in combination with experiments and other analysis techniques.

3.5. Summary of Analysis Tools

A general analysis tool—one which will illuminate both the positioning and tracking tasks; PD, PID and other control structures; and incorporates an adequate friction model—has not yet been presented. But as shown in Table 4, many partial steps have been taken.

Radcliffe and Southward (1990) have shown that systems with a broad range of friction models and PD control will not exhibit hunting. This result has been arrived at by a number of routes, including those of Kubo *et al.* (1986) and Wallenborg and Åström (1988). Furthermore, as Kubo *et al.* (1986) have established, a system with Coulomb friction alone will not stick slip while tracking with PD control. This point was made long ago in the mechanics community, and was the motivation for early proposals in the tribology literature that, based on the observation of self-exciting stick slip during sliding, the Stribeck friction model might apply to a wide range of situations (Thomas, 1930). The algebraic analyses of Derjaguin *et al.* (1956, 1957), and others, consider Coulomb + static friction and the tracking problem in a general way, but only a PD control; and the possibility of stick–slip is found. Armstrong–Hélouvry (1993), likewise, presents a general analysis, incorporating a rich friction model, but considers only PD control. The result of Wallenborg and Åström (1988) is quite rigorous and extends to full state feedback, but rests on a describing function description that is limited to Coulomb friction and the positioning task. The results of Tou (1953) and Shen (1962), among others are specific to particular systems and are not thoroughly verified, but overcome the difficulties of reflecting standstill or steady motion with the describing function.

Too much attention has been focused on the simplest problem: positioning of a machine with Coulomb friction and PD control. The repeated demonstration that this case will neither hunt nor stick slip, coupled with the evident presence of limit-cycles in the laboratory and field, should provide adequate

TABLE 4. PAPERS PROVIDING ANALYTIC PREDICTION OF STICK SLIP IN MACHINES; THE INDICATIONS NO OR YES REFER TO THE POSSIBILITY OF STICK SLIP DETERMINED BY THE CITED ANALYSES

Friction model	Task/controller: Positioning, PD	Positioning, PID or Lag*	Tracking, PD	Tracking, PID or Lag*
Coulomb	NO Kubo, Radcliffe, Wallenborg	NO Radcliffe, Amin	NO Kubo	?
Coulomb + Static	NO Tou† Radcliffe†	YES Tou†, Amin Radcliffe†	YES Derjaguin‡	YES Shen†
Coulomb + Stribeck	NO Radcliffe†	YES Radcliffe†	YES Dupont§ Armstrong‡§	YES

* Parameters giving stable linear portion assumed.
† Specific system parameters considered.
‡ Incorporates rising static friction.
§ Incorporates frictional memory.
Tou: (Tou, 1953; Tou *et al.*, 1953) Derjaguin: Derjaguin *et al.* (1956, 1957) Shen: (Shen and Wang, 1964) Radcliffe: (Radcliffe *et al.*, 1990) Kubo: (Kubo *et al.*, 1986) Wallenborg: (Wallenborg *et al.*, 1988) Dupont: (Dupont, 1994) Armstrong: (Armstrong-Hélouvry, 1991, 1993) Amin: (Amin, 1993)

motivation to move on to the more challenging cases of PID control, the tracking task and a Stribeck friction model. Furthermore, to be useful in practice, analysis tools must comprehend not only the nonlinearities of friction, but the nonlinearities of control as well. Even for nominally linear PID control, nonlinearities such as deadband and saturation operations are common.

The possibilities have not been exhausted. The projective phase plane techniques of Radcliffe and Southward (1990), for example, coupled with the analytic techniques of Derjaguin *et al.* (1957) or Armstrong-Hélouvry (1993), might yield more general analysis tools. Perhaps these routes, or others, will bring tools for the most general case: the tracking task with a general control structure and adequate friction model. It seems likely that an analysis capable of handling this case would comprehend the other possibilities as special cases, and greatly extend our ability to analytically predict stick slip in machines.

4. COMPENSATION TECHNIQUES FOR MACHINES WITH FRICTION

An extensive body of literature exists relating to friction compensation; this survey, for example, cites over one hundred papers in this area. This body of literature can be viewed as corresponding to the multidimensional character of the problem. As suggested by Fig. 42, the coordinate axes might be the mechanism, task, friction model, analysis technique, and compensation technique under consideration. Many papers in this area are very specific, such that they can be viewed as a single pixel in this multi-dimensional space. Allowing for discussion that there might be:

- four classes of mechanisms—roller bearings, sliding bearings with dry or fluid lubricated contacts, and hydrostatic or magnetic bearings;
- four different tasks—precision positioning, smooth velocity reversal, low velocity tracking and high velocity tracking;
- five different friction models—Coulomb friction, Coulomb + static friction, Coulomb + Stribeck friction, sliding friction with frictional memory, sliding friction with rising static friction;
- four different analysis techniques—describing function, exact integration, phase plane and extensive simulation; and
- seven different compensation techniques—low friction machine design or lubricant choice, stiff position control, integral control with deadband, direct force feedback, impulsive control, Coulomb friction feedforward, position-dependent friction feedforward;

one finds that 2240 specific papers are possible. Despite the extensive literature in this area, we are not aware of any comprehensive attempt to piece together the many threads running through this space.

In compiling this section, we had several goals. The

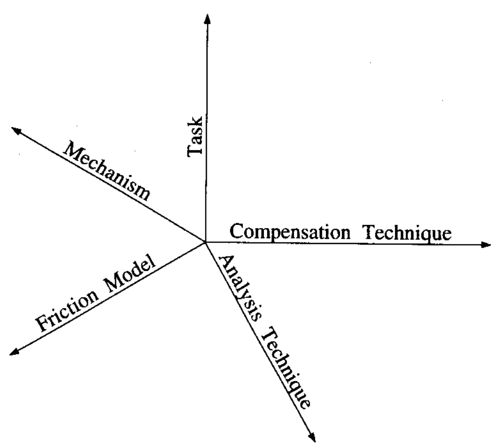

FIG. 42. Axes of friction compensation space.

first was to provide an overview of the major friction compensation techniques. More importantly, we have tried to assemble the many pieces of this problem into their whole and to point out where work remains to be done. In the next section, we propose a classification of friction control tasks. For each task, the problematic behavior is described and the contributing friction components identified. Following that, we describe how the control problem can sometimes be avoided through the selection of alternative mechanisms, materials or lubricants. Next, both model-based and non-model-based compensation methods are presented along with techniques for friction identification and adaptive control. The section concludes with a survey of the compensation techniques in current use by industry.

4.1. Compensation Tasks

A classification of compensation tasks appears in Table 5. Of the four tasks, one is the regulator and the remaining three are versions of the tracking problem. They are listed along with the associated controller error and the dominant friction effect. A specific machine application could involve several of these tasks.

Task I, the Regulator, is encountered with positioning and pointing systems. Application examples include telescopes, antennas, machine tools, disk drives and robots. In this case, a system spends most of its time either near or within the stiction regime. When the friction–velocity curve of a system is negatively sloped at the origin, the equilibria of a PD position regulator consist of an interval on the position axis in phase space. Only one point in this set corresponds to the desired goal position as was shown in Fig. 36. By adding integral control, the equilibria set consists only of points with the desired position (and velocity); however, this set can be unstable with nearby trajectories diverging away to a limit cycle (Radcliffe and Southward, 1990). This integral-induced stick-slip oscillation about the goal position is referred to as "hunting". This task is discussed in Blackwell *et al.* (1988), Stockum *et al.* (1988), Yang and Tomizuka (1988), Ostertag *et al.* (1989), Radcliffe and Southward (1990), Auslander and Dass (1990), Brandenburg and Schäfer (1991) and Southward *et al.* (1991).

As to its frictional cause, **Task II, Tracking with Velocity Reversals**, is closely allied with Task I. Due to a higher static level of friction, motion through zero velocity is not smooth. A system may pause at zero velocity until sufficient force is applied to exceed the maximum stiction level. This task is encountered with machine tools, tracking mechanisms and robots under position or force control. An important example of the effect of friction on this task occurs for machine tool slideways, where it is known as stand still or quadrant glitch. In multiple degree of freedom motion, the joint undergoing velocity reversal pauses while the others continue unimpeded. The resulting motion manifests itself as an aberration in the workpiece contour. Suzuki and Tomizuka (1991) address this problem in machining circular paths. In many papers, tracking with velocity reversal is studied for a sinusoidal reference signal, including Gilbart and Winston (1974), Canudas de Wit *et al.* (1987), Canudas de Wit and Seront (1990), Walrath (1984), Schäfer and Brandenburg (1990) and Brandenburg and Schäfer (1991). Maqueira and Masten (1993) have investigated tracking with spectrically broad band inputs.

Task III, Tracking at Low Velocities, differs from Task II in that the desired motion is of constant direction and perhaps constant velocity. This task arises for machine tools, tracking mechanisms and robots under position or force control. It is the task most often associated with stick slip. The common pictorial representations include the pin on flat apparatus of Fig. 14 and its kinematic inversion depicted in Fig. 43.

The potential for stick-slip limit cycling exists when the operating point, V_0, lies on a negatively sloped portion of the steady-state friction–velocity curve such as Regime III of Fig. 5. This task is often studied by addressing one of two criteria for smooth motion:

(1) Does a stick-slip limit cycle exist and for what system parameter values will it be stable?

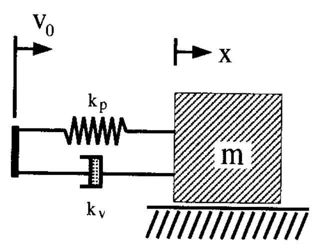

FIG. 43. Tracking at low velocities. The free end of the spring and damper are moved at constant velocity, V_0.

TABLE 5. COMPENSATION TASKS

	Compensation task	Control error	Dominant frictional contributor
I.	Regulator (pointing or position control)	Steady-state error, hunting (limit cycle around fixed point)	Stiction.
II.	Tracking with velocity reversal	Stand still, lost motion	Stiction.
III.	Tracking at low velocities	Stick–slip	Negatively-sloped Stribeck curve; stiction.
IV.	Tracking at high velocities	Large tracking errors	Viscous behavior of lubricant.

(2) For what system parameter values is the equilibrium point $\dot{x} = V_0$ stable?

The former has been addressed most often in the literature. Examples include Harmer (1952), Singh (1960), Shen and Wang (1964), Kato and Matsubayashi (1970), Kato *et al.* (1972, 1974), Armstrong-Hélouvry (1991, 1993). Investigations of equilibrium-point stability have been carried out by Rice and Ruina (1983) and Dupont (1994). Analyses involving the determination of stable stiffness and damping values can be applied in controller design by relating these quantities to position and velocity gains. Controller design for tracking at low velocities is also studied in Gilbart and Winston (1974), Walrath (1984), Kubo *et al.* (1986).

Task IV, **Tracking at High Velocities**, arises for machine tools, position-controlled robots and tracking mechanisms. High-speed operation not only increases productivity, it may actually be necessary to meet process constraints. For example, in high-speed machining, a critical cutting velocity must be exceeded in order to avoid the excessive tool temperatures which lead to premature failure (Suzuki and Tomizuka, 1991).

This task is significantly different from the previous three tasks because high-velocity friction is dominated by viscous effects. The friction–velocity curve is positively sloped and stability is usually not a problem. Instead, tracking error is observed to increase as a function of velocity. For example, the radial error in machining circular contours is approximately proportional to the square of the angular-velocity feed rate (Suzuki and Tomizuka, 1991).

Often, machines performing high-velocity tracking must also cope with velocity reversals. Due to the nonlinearity of friction, a linear fixed-gain controller that is tuned for low velocities may perform poorly at high velocities and vice versa. This suggests the need for nonlinear compensation, as described in Section 4.4, and for variable structure controllers, whose industrial use is addressed in Section 4.5.

The effectiveness of a particular compensation technique depends strongly on the task. This is due in a large part to the task defining the dominant frictional effect. Figure 44 connects task and compensation technique pairs which have been investigated in the literature. Each of these compensation methods is described in the following sections. Figure 44 also lists the common applications associated with each task.

The first item listed under compensation techniques, Friction Problem Avoidance, is not truly a compensation technique. In this method, one attempts to replace the given system with one which is easier to control. Model-based methods are distinguished from non-model-based methods in that they employ a

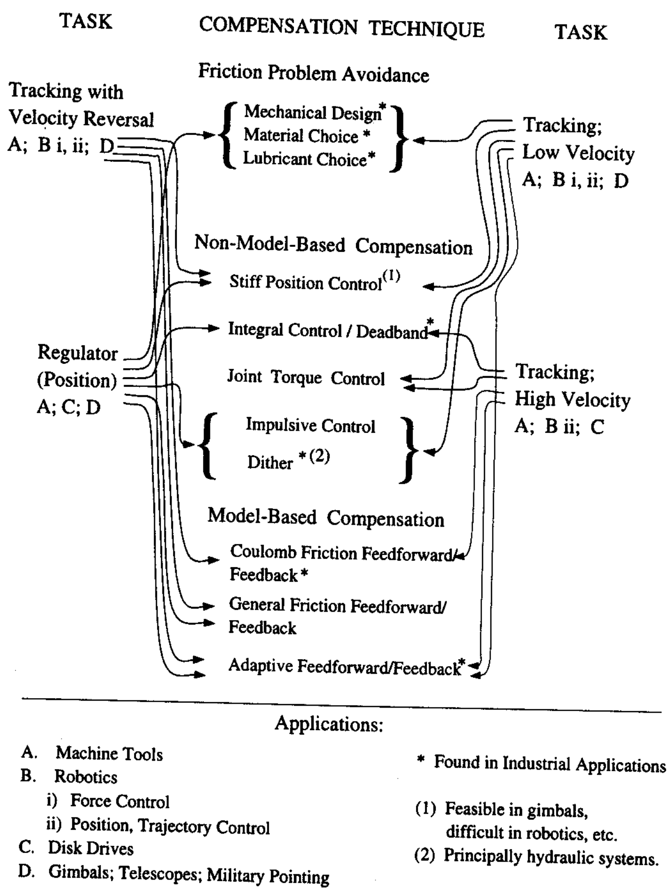

FIG. 44. Tasks and their associated compensation techniques as reported in the literature. Typical task applications appear in the legend.

friction model in a feedback or feedforward loop. The model parameters may be identified once through a set of identification experiments. Otherwise, adaptive control techniques may be applied.

4.2. Problem Avoidance: Design for Control

While the latest control techniques are finding acceptance in industry, problem avoidance, or design for control, is usually the first strategy employed to defeat friction problems. Many studies have shown that the amplitude of stick slip can be reduced—even to zero under some circumstances—by decreasing the mass, increasing the damping or increasing the stiffness of a mechanical system (Rabinowicz, 1959; Singh, 1960; Kato *et al.*, 1974). Damping is usually controlled through selection of the lubricant and the sliding surfaces. The latter may be of different composition than the underlying bulk material either applied as a coating or liner or embodied as a bearing. Inertia and stiffness are determined to a great extent by the geometry and composition of the mechanism's bulk material. The selection of actuators, bearings and sensors can affect system damping, stiffness and inertia. For example, rolling element bearings may possess damping and stiffness characteristics which are considerably different than sliding bearings. These topics are discussed briefly below.

4.2.1. *Lubricant selection*

In a control context, the goal of lubricant selection is usually to reduce the negative slope of the friction–velocity curve near zero velocity. Slope reduction is equivalent to increased damping. If the slope remains negative, the system is still unstable, but is, however, easier to stabilize by active control.

Dry lubricants, such as polytetrafluoroethylene (PTFE or Teflon®) and molybdenum disulfide, can be used to produce a stabilizing positive slope at very low velocities. For example, the friction–velocity curve for PTFE has a positive slope up to $10\,\mathrm{cm\,s^{-1}}$. One limitation of these soft lubricating films, however, is their high wear rate. In addition, while these solids may stabilize and decrease friction at low velocities, they may generate more friction than would otherwise be encountered at higher velocities. In certain applications, such as machine tools, this extra damping is desirable for stability. Solids, such as Rulon®, are often used in conjunction with liquid lubricants as slideway liner materials.

4.2.2. *Bearings*

Bearing friction can be a problem in high-precision positioning, pointing and tracking systems. Several schemes have been proposed to overcome motion errors from ball bearings. One involves active control of the bearing outer race (Bifano and Dow, 1985). In another method, the outer bearing race is not rigidly mounted to the machine frame. Instead, it is connected to the frame through torsional springs as shown in Fig. 45 (Clingman, 1991). At motion initiation and direction reversal, the friction torque and spring torque act in series reducing the effective slope of the friction–displacement curve. A controller

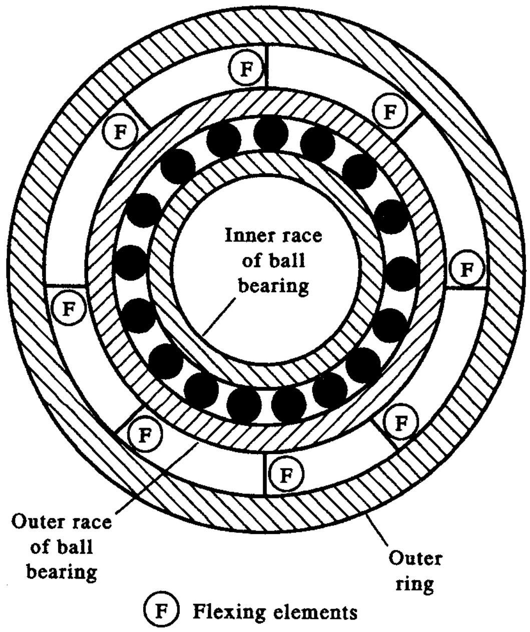

FIG. 45. Outer race of ball bearing is attached to static outer ring through torsional flexing elements [from Clingman (1991), courtesy of publisher].

can be added to the bearing which drives the torsional springs to null displacement (Clingman, 1991).

In order to avoid the nonlinearity of low-velocity friction, oil or air hydrostatic bearings can be used. While producing extremely low friction, air bearings exhibit low stiffness and damping in the normal direction which can make them sensitive to profile errors of guideways and to external disturbance forces. To actively control stiffness and damping, lubricant supply pressure can be controlled or piezoelectric actuators can be placed in series with the air pads (Horikawa *et al.*, 1991). Alternatively, active magnetic bearings represent a promising non-contact technology which is currently used only in high-velocity applications. In addition to producing minimal friction, their ability to provide active damping to high speed rotors eliminates critical speed vibrations (O'Connor, 1992).

4.2.3. *Stiffness and actuation*

The most common source of excessive machine compliance is transmission elements. The obvious solution, which has been considered by a number of researchers, is to eliminate the transmission (Asada and Youcef-Toumi, 1984), or at least to locate it at the joint (Townsend and Salisbury, 1987). Even in the absence of transmission elements, machine compliance due to shafts and couplings can be significant (Brandenburg and Schäfer, 1991). Very high machine stiffnesses are sought when designing experimental apparatus for studying friction (Dieterich, 1979; Ruina, 1980; Polycarpu and Soom, 1992).

Elimination of the transmission can necessitate the use of oversized or special high-force/torque motors. For fine displacement applications, piezoelectric actuators with or without sliding contacts can be used.

To overcome their stroke limitation, they may be used in conjunction with conventional long-stroke actuators.

4.2.4. *Inertia*
While inertia reduction has a stabilizing effect on stick-slipping systems, it is not always possible or practical to reduce inertia and, in some cases, reduced inertia can be detrimental to system performance. An important example is the stabilization of gimballed pointing systems. Consider, for example, a camera mounted through a gimbal to a base where the base is subject to disturbances. At high disturbance frequencies, the bandwidth of the controller may preclude active compensation. In these cases, the mechanism must be designed for passive inertial stabilization. In the single degree of freedom case, the governing equation is

$$\tau_f = I\ddot{\theta}, \tag{26}$$

where τ_f is the friction torque generated by the base disturbance, I is the moment of inertia of the camera about the gimbal axis and $\ddot{\theta}$ is the resulting camera acceleration. To minimize $\ddot{\theta}$, it is the ratio of friction torque to inertia, τ_f/I, which is important (Ellison and Richi, 1983; Stockum *et al.*, 1988). In this case, reducing inertia without reducing friction as well can worsen high-frequency performance. In addition to this direct connection with friction, inertial components are sometimes added to pointing systems to passively cancel either inertial disturbance terms (Ellison and Richi, 1983) or inertial reactions during active motion control (Germann and Braccio, 1990).

While design for control does not guarantee the passive elimination of stick–slip, it usually produces a system which is easier to control and which possesses better performance characteristics. Further improvement can be achieved by friction compensation as described in the following sections.

4.3. Non-model-based Compensation for Friction

4.3.1. *Stiff PD control*
While the regulator problem is stable under PD control, the tracking problem does exhibit stick slip at low velocities. For many years, it has been known that by increasing the damping or the stiffness of a system, stick–slip can be eliminated. In a control context, this is accomplished by increasing the PD gains.

PD control, along with integral action, is widely used in industry. Perhaps because of its widespread use, few papers appear reporting the experimental performance of PD and PID controllers in machines with friction. Instead, most related papers use experiment to obtain a system model, including friction, which is then analyzed to produce stabilizing PD gains for a given input trajectory. Since these results depend on the friction model used in the analysis, frequent reference to friction modeling in this section is unavoidable.

The success of stiff PD control can only be fully understood by considering frictional memory. While the tribology community has been aware of this frictional effect for almost 50 years, it has only recently been incorporated into friction models used for control. Consequently, most of the literature on stiff PD control, while correct for the friction models considered, is of limited practical value. While these simplified models can mimic stick–slip limit cycling, they predict steady sliding only through derivative action, even though experiments have shown that proportional control is also effective.

Analyses based on friction models which are single-valued functions of velocity for nonzero velocity include Harmer (1952), Derjaguin *et al.* (1957), Cockerham and Symmons (1976), Brockley *et al.* (1967), Bannerjee (1968), Armstrong-Hélouvry (1990) and Gao and Kuhlmann-Wilsdorf (1990). These analyses may have direct implications for systems in which frictional memory is of such a magnitude that its neglect is justified.

Beginning with Sampson *et al.* (1943), researchers observing stick-slip limit cycles noted that friction was higher during the acceleration phase of the slip cycle than during the deceleration phase. (Recall Fig. 21). This led to two-valued models which were employed in the stability analyses of Bell and Burdekin (1969), Cockerham and Symmons (1976), Cockerham and Cole (1976) and Bo and Pavelescu (1982). This work represents a significant step toward the recognition of frictional memory. Alternatively, Kato and Matsubayashi (1970) observe the multi-valued behavior, but propose the use of a mean friction coefficient during the slip phase.

Recently, it has become possible to explain the stability of high-stiffness systems which exhibit negative steady-state damping through the inclusion of frictional memory effects. For example, Armstrong-Hélouvry (1992) considers a friction model composed of Stribeck, viscous and rising-static-friction components. The Stribeck component is subject to the pure time lag proposed by Hess and Soom (1990). Armstrong-Hélouvry employs a perturbation method in combination with numerical integration to determine the stability of a stick-slip limit cycle. His analysis first considers the frictional memory and rising static friction models separately to obtain stability criteria. These are then combined to determine the effect of each. Considering only time lag with a friction model exhibiting negative damping at low velocities, a single degree of freedom system consisting of a sliding mass, M, will not experience stick slip for moderate friction if the system stiffness (including proportional gain) meets or exceeds a critical value, k_{cr} where

$$k_{cr} = M\frac{\pi^2}{\tau_L^2}. \tag{27}$$

Clearly, as the time lag, τ_L, approaches zero, the critical stiffness approaches infinity. The addition of the rising-static-friction model is of greatest import for very lightly damped systems, increasing the critical stiffness at low velocities and decreasing it at higher velocities. The combined model provides a good match with experimental data from the base joint of a PUMA robot.

In contrast to studying the stability of stick-slip limit cycling, Rice and Ruina (1983) employ the concept of a nonlinear friction model which depends on slip history and consider the stability of the equilibrium point associated with steady sliding. This class of friction models includes the state variable models described in Section 2.1.5. They consider small perturbations by linearizing the system about steady sliding. The steady-state friction-velocity slope is assumed to be negative while the instantaneous slope is positive either due to the friction itself or external damping. Using a root-locus argument, they obtain a critical stiffness above which the equilibrium point is stable under small perturbations. For a friction model involving a single exponential decay over the characteristic length, L, the critical stiffness, k_{cr}, can be expressed as

$$k_{cr} = -\frac{Vd\,\tau_{ss}(V)/dV}{L}\left[1+\frac{mV}{L\,\partial\tau/\partial V}\right], \quad (28)$$

where V is velocity, τ is frictional shear stress, τ_{ss} is steady-state frictional shear stress and m is slider mass. Dupont and Bapna (1992) have developed expressions for critical stiffness in systems where frictional memory is also associated with changes in normal stress.

In summary, stick slip can be eliminated through either high derivative (velocity) feedback or high proportional (position) feedback. They are best used together as they are complementary. While derivative feedback is additive with inherent system damping, this is not the case with proportional (position) feedback. System stiffness acts in series with controller stiffness. Thus, high gain proportional control is most successful in systems which can be designed for high rigidity. This topic is discussed further in Section 4.5.

4.3.2. *Integral control*
While stiff PD control can be used to achieve stable tracking, integral control of position or velocity is almost always introduced to minimize steady-state errors. Using integral action, systems are found to limit cycle when tracking at low or zero velocities. Integral action, and the limit cycling it induces, are rarely discussed in the controls literature except as motivation for more complicated control methods. This is in direct contrast with its widespread use and with the variety of techniques developed to circumvent its shortcomings.

To overcome limit cycling, one standard technique is to employ a deadband as the input to the integrator block. This, of course, imposes its own steady-state error—less, it is hoped, than that before the integral action was added. Shen and Wang (1964) employ a static-Coulomb model to study position-ramp inputs. They find that the size of the stabilizing deadband decreases almost linearly with ramp rate. To avoid sluggish response and large steady-state errors at high ramp rates, they propose setting the deadband limit as a function of input ramp rate.

In addition to inducing limit cycling, integral control can be ineffective and even deleterious at velocity reversals (Tung et al., 1993). Integral windup from prior motion can actually inhibit breakaway. To prevent this, the integral term is typically reset at velocity reversals. While this eliminates the windup problem, the ensuing integral action produces minimal effect when needed the most to overcome stiction. In a system with multiple degrees of freedom, other joints may be moving at high velocity during the reversal. Consequently, the reduced effect of the integral action in concert with the higher level of static friction at reversal can lead to significant tracking errors. Suzuki and Tomizuka (1991) consider this issue in the context of high speed machining of circular contours. As an alternative to integral control, they propose a model-based controller which applies a pulse to overcome stiction at breakaway. Hansson et al. (1993) apply a fuzzy rule system to controlling windup in PID controllers. Brandenburg and Schäfer (1988a, 1989, 1991) consider integral control in the context of a variety of tasks and compensation techniques. Industrial use of integral control is discussed further in Section 4.5.

4.3.3. *Dither*
Dither is a high frequency signal introduced into a system to modify its behavior. Dither can stabilize unstable systems (Bogoliubov and Mitropolsky, 1961), and is used to improve performance by modifying nonlinearities in adaptive control (Anderson et al., 1986), communication systems (Chou, 1990), optics (Hirel, 1990) and image processing (Chau, 1990). For machines with friction, the controls community has focused on the capability of dither to smooth the discontinuity of friction at low velocity. Early discussions of dither employ the describing function, (MacColl, 1945; Atherton, 1975). Later treatments bring to bear averaging theory (Mossaheb, 1983), functional analysis and other methods of nonlinear system analysis (Zames and Shneydor, 1976, 1977; Cebuhar, 1988; Bentsman, 1990; Lee and Meerkov, 1991).

How smoothing arises with dither can be seen in an example [following Cebuhar (1988)]: the relationship

$$y(t) = \mathrm{sgn}\,(u(t)) \quad (29)$$

is discontinuous. However, when a dither of amplitude α and frequency ω is added to the input, the averaged output becomes:

$$\bar{y}(t) = \int_{t-2\pi/\omega}^{t} \mathrm{sgn}\,(u(\tau) + \alpha\sin(\omega\tau))\,\mathrm{d}\tau. \quad (30)$$

Through averaging, $\bar{y}(t)$ can be a continuous function of $u(t)$.

4.3.3.1. *Tangential and normal dither.* The analyses presented in the control literature focus on a dither signal added to the command input, which, for the configuration shown in Fig. 46, will give rise to vibrations that are tangential to the sliding contact. In the tribology literature, on the other hand, the impact of vibrations normal to the contact have been considered (Friedman and Levesque, 1959; Godfrey, 1967; Oden and Martins, 1985; Hess and Soom, 1991a, b). The distinction between normal and

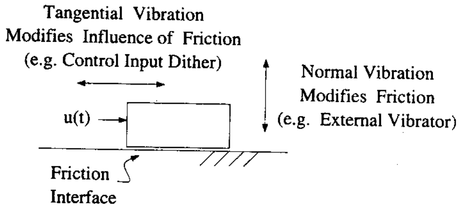

FIG. 46. Direction and effect of dither.

tangential dither in a friction contact is a considerable one: the effect of tangential dither is to modify the influence of friction (by averaging the nonlinearity); the effect of vibrations normal to the contact is to modify the friction [by reducing the friction coefficient, (Godfrey, 1967; Martins *et al.*, 1990)]. This survey is apparently the first time that these two possibilities have been considered together.

Working with a simple Coulomb + viscous friction model one would not expect friction to be reduced by normal vibrations, so long as contact is not broken; but when contact compliance (the origin of presliding displacement) and asperity contacts are considered, more sliding is seen to occur during periods of reduced loading and less during periods of increased loading. This arises because the mechanical bandwidth of individual asperities may be orders of magnitude higher than the bandwidth of the macroscopic mechanical elements. Godfrey (1967) reports a reduction of the coefficient of friction from 0.15 to 0.06 in a lubricated steel contact with the addition of 1000 Hz normal vibrations.

For the controls engineer, the importance of this distinction lies in how dither is to be applied. The original applications of dither involved external mechanical vibrators (Bennet, 1979). And on gun mounts and other large pointing systems, such vibrators, sometimes called "dipplers," are still used. While any form of dither will result in both tangential and normal forces through the coupling arising out of asperity contacts (Martins *et al.*, 1990), more freedom exists for orienting the dither when an external vibrator is used. It is likely that more than one servo functions because of a badly balanced fan in the vicinity is providing dither.

4.3.3.2. *Depth of discontinuity and dither in hydraulic servos.* When dither is applied, filtering exists between the source of the vibration and the point where it is to have its effect, as shown by the transfer function $G_1(s)$ in Fig. 47. In all cases, but particularly when dither is applied at the control input, this filtering is important. Cebuhar (1988) and evaluates the filtering in terms of the 'depth' of the discontinuity, and defines a formal measure of this depth relating to the number of integrators in $G_1(s)$. He finds that when the depth is great, the designer is

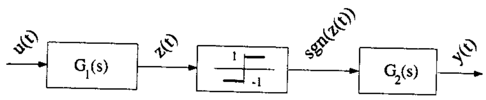

FIG. 47. A transfer function between the input and the nonlinearity where dither is to have its influence.

more restricted in the application of a dither. Both Cebuhar (1988) and Lee and Meerkov (1991) investigate the optimal dither parameters with a construction like that of Fig. 47. The ratio

$$\frac{G_1(i\omega)}{G_2(i\omega)} \tag{31}$$

plays an important role in the effectiveness of the dither and Cebuhar proposes that ω_d, the dither frequency, should be chosen to maximize a slightly modified form of equation (31). Horowitz *et al.* (1991) consider limitations on range of applicable ω_d and propose an adaptive construction which allows lower frequency dither. Based on input from engineers in industry (see Section 4.5), it seems that dither is only occasionally applied to motor servos, but is often applied and with great effect to spool valves in hydraulic servos. The larger $G_1(i\omega)/G_2(i\omega)$ achievable in the hydraulic servo may account for the greater success of dither in these systems.

4.3.4. *Impulsive control*

A number of investigators have devised controllers which achieve precise motions in the presence of friction by applying a series of small impacts (Yang and Tomizuka, 1988; Suzuki and Tomizuka, 1991; Armstrong, 1988; Armstrong-Hélouvry, 1991; Deweerth *et al.*, 1991; Hojjat and Higuchi, 1991). These are 'impulsive controllers'. Impulsive control is distinguished from dither in that it is the impulses themselves which are to carry out the desired motion. The impulses used are not zero mean and must be calibrated to produce the desired result. Impulsive control is also distinct from standard pulse width modulation (PWM) controllers, where voltage pulses are applied to a motor. In PWM controllers, the motor inductance averages the relatively high frequency (perhaps 20 kHz) voltage pulses to produce a nearly constant motor current, and therefore nearly constant torque.

In Yang and Tomizuka (1988), Suzuki and Tomizuka (1991) Armstrong (1988), Armstrong-Hélouvry (1991), Hojjat and Higuchi (1991), the impulses are applied when the system is at rest, i.e. in the stuck condition. The effect of the impulse is a small displacement (Armstrong-Hélouvry, 1991; Hojjat and Higuchi, 1991) or a controlled breakaway, leading to transition to another controller which regulates macroscopic movements (Yang and Tomizuka, 1988; Suzuki and Tomizuka, 1991). By making the impulses of great magnitude but short duration; the static friction is overcome and sensitivity to the details of friction is reduced. A typical behavior is shown in Fig. 48. Hojjat and Higuchi (1991) present an apparatus designed especially to demonstrate impulsive control. They reliably achieved a remarkable 10 nm per impulse motion and speculate that repeatable 1 nm per impulse motions may be possible. Their mechanism was not unlike a machine slideway; the slider measured about 3 cm on a side and weighed 155 g. Hojjat and Higuchi (1991) control the amplitude of their impulses, typically applying a force about 10 times the static friction for about 1 ms, and

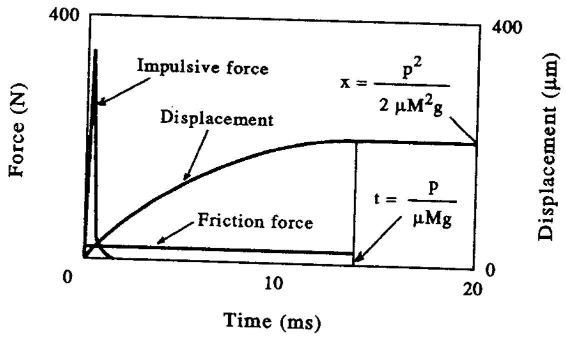

FIG. 48. Behavior of motion under impulsive control [from Hojjat and Higuchi (1991), courtesy of the publisher].

show that displacement is given by the square of the amplitude times an empirical constant.

Yang and Tomizuka (1988) present a variable structure controller. Away from zero velocity, a standard linear controller operates. At or near zero velocity, the adaptive pulse width control takes over. The impulse is tuned by varying the pulse width. A current pulse is applied, as opposed to the voltage pulses used in standard PWM control. The impulses are of a predetermined force which must be greater than the static friction; Yang and Tomizuka (1988) chose a force about four times the level of Coulomb friction. The duration is then selected to achieve the desired displacement. The authors present a rigorous demonstration of the stability of both their controller and the adaptive element. Suzuki and Tomizuka (1991) focus on the challenges of high speed machining, and demonstrate a controller which includes an impulsive element which operates near zero velocity.

Deweerth *et al.* (1991) describe a neural network based controller, with functionality comparable to a PD controller, but which generates a pulse train. The impulses are a natural consequence of the neural structure. The authors demonstrate that the controller is effective in controlling low-speed, friction-limited motions.

Armstrong (1988) and Armstrong-Hélouvry (1991) demonstrate impulsive control of a PUMA robot. Their objective was very high precision force control, needed to manipulate an object with a crush strength of only 1/60th the level of static friction in the mechanism. Using a calibrated table of impulse magnitude and duration, Armstrong-Hélouvry (1991) applies impulses that are only 10–20% greater in magnitude than the static friction and achieves 10 micro-radian per impulse motions of the industrial manipulator.

The impulsive controllers of Hojjat and Higuchi (1991), Yang and Tomizuka (1988), Armstrong (1988) and Armstrong-Hélouvry (1991) have in common the use of a long sampling interval. Underlying these impulsive controllers is the requirement that the system be in the stuck condition when each impulse is applied. The method is in essence a small bang followed by an open-loop slide. Returning to the stuck condition imposes a number of limitations, but improves the predictability of the response to the impulse. The variable structure controller demonstrated by Yang and Tomizuka (1988), which employs the impulsive control only at zero velocity, perhaps best exploits the capability of impulsive control.

The interaction of force application rate and breakaway friction has not been considered by the authors presenting impulsive controllers. But if force application rate is at the heart of the observed rising static friction (Richardson and Nolle, 1976; Johannes *et al.*, 1973; see Section 2.1.4.2), the implications may be substantial for impulsive control.

4.3.4.1. *Dither, impulsive control and contact compliance.* None of the authors describing either dither or impulsive control address contact compliance, discussed in Section 2.1.2. In the above cited works, it is presupposed that high frequency forces applied to the sliding elements act fully on the friction contact. But the elastic response of the junction serves as a low pass filter, increasing the depth of the frictional discontinuity by adding one or perhaps two poles to $G_1(s)$ of equation (31). As the works of Cebuhar (1988) and Lee and Meerkov (1991) make clear, these methods will be influenced by $G_1(s)$ at the applied frequencies. The importance of contact compliance is illuminated by the fact that the micro-radian movements of Armstrong (1988) and Armstrong-Hélouvry (1991) and the nanometer movements of Hojjat and Higuchi (1991) are both smaller than—and in the later case three orders of magnitude smaller than—the typical presliding displacement. This suggests that actual motion is occurring only in some percentage of asperity contacts, with movement of the body a consequence of the shifted equilibrium of the stresses of all of the contacts. Contact compliance depends upon contact loading, surface finish and the material properties of the parts, indicating that the success of either impulsive control or dither will depend upon these aspects of machine design.

4.3.5. *Joint torque control*

Joint torque control is a sensor-based technique which encloses the actuator–transmission subsystem in feeback loop to make it behave more nearly as an ideal torque source (Wu and Paul, 1980). Disturbances due to undesirable actuator characteristics (friction, ripple, etc) or transmission behaviors (friction, flexibility, inhomogeneities, etc) can be significantly reduced by sensing and high gain feedback. The basic structure is shown in Fig. 49; an inner torque loop functions to make the applied torque, T_a follow the command torque, T_c.

Joint torque control has been implemented as a means of compensating for actuator and transmission friction (Wu and Paul, 1980; Luh *et al.*, 1983; Pfeffer *et al.*, 1986, 1989; Vischer and Khatib, 1990a, b; Karlen *et al.*, 1990; Hashimoto *et al.*, 1992), as a means of compensating or more precisely controlling transmission flexibilities (Karlen *et al.*, 1990; Furusho *et al.*, 1990; Hashimoto *et al.*, 1992), and as a means of sensing and compensating for the nonlinear rigid body dynamics and gravitational loads experienced in

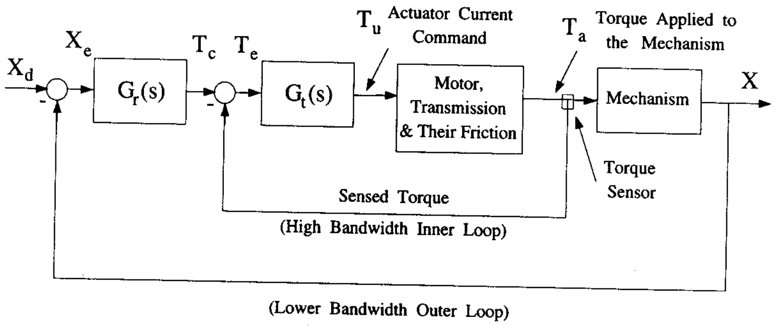

FIG. 49. Block diagram of a joint torque control (JTC) system.

robotics (Kosuge *et al.*, 1988; Hashimoto, 1989). The above implementations all incorporate one axis of sensing per joint. Six axes of force/torque sensing per joint has been proposed in robotics (Mukerjee and Ballard, 1985) for inertial parameter identification.

Implementation of joint torque control requires torque or force sensing as near as practical to the output element of the system so that all or nearly all of the actuator and transmission friction will be enclosed in the joint torque feedback loop. A number of authors have employed strain gages, typically mounted on supports for transmission elements (Wu and Paul, 1980; Luh *et al.*, 1983; Pfeffer *et al.*, 1989; Hashimoto, 1989; Hashimoto *et al.*, 1992). Hashimoto (1989) and Hashimoto *et al.* (1992) focus specifically on harmonic drives. Vischer and Khatib (1990a) present a sensing package that may offer advantages of sensitivity, robustness and stiffness. It is based on differential inductive sensing.

In joint torque control, the sensor and actuator are often non-collocated, separated by the compliance of the transmission and perhaps that of the sensor itself. This gives rise to the standard challenges of non-collocated sensing. All of the authors cited above who demonstrate joint torque control discuss the issues of system identification and Laplace or Z domain compensator design.

As shown in Fig. 49, the controller design is a nested one, with an inner loop which maintains applied torque and compensates for friction, and an outer loop which governs the execution of the mechanism task. The multi-loop structure would in general require a full multi-loop analysis; the authors cited, however, have focused their attention on the inner torque loop, and have required that the frequency domain separation between the inner and outer loops be sufficient to protect against dynamic interactions. Eismann, who has substantial experience with the commercial Robotics Research arm, suggests that a 4:1 ratio is required between the cross over frequency of the inner joint torque controlled loop and that of the outer task control loop (Eismann, 1992).

4.3.5.1. *Performance.* Luh *et al.* (1983) report that the Coulomb friction in their Stanford robot arm is 1072 oz-in, and that the apparent friction with joint torque feedback is 33.5 oz-in, a reduction of 32:1. Pfeffer *et al.* (1989) report that the apparent friction in joint 3 of a PUMA robot arm was reduced to 3% of its uncompensated level; they demonstrate free swinging motions of the robot link ($T_c = 0$ in Fig. 49, but T_u quite active to compensate for actuator and transmission friction during the motions). Hashimoto *et al.* (1992) evaluate joint torque control coupled with feedforward Coulomb-friction compensation, in both fixed and adaptive forms. Step torque input and sinusoid position input responses of their system are presented in Figs 50 and 51. The four presented responses are: open-loop, joint torque control with no friction feedforward, joint torque control with fixed friction feedforward, and joint torque control with adaptive friction feedforward. Numeric values for the data are not presented, but it is evident that the influence of friction is very greatly reduced. Hashimoto *et al.* (1992) also present vibration suppression data which show a factor of 10 reduction of vibration at the mechanical resonant frequency during a medium velocity motion. The resonant vibration couples the inertia with the compliance of the transmission, and is reduced because the transmission compliance appears within the joint torque control loop.

Robotics Research Corporation implemented joint torque control to compensate for the compliance of their harmonic drives. Using only sensing at the actuator, the drive compliance would result in lightly-damped, low frequency modes. Figure 52 is adapted from Karlen *et al.* (1990), and shows the magnitude of response to a sinusoidal disturbance at the toolplate (distal end) of the robot arm. Without joint torque control, there are two lightly damped modes between 10 and 20 Hz; with joint torque control the response is nearly flat through this frequency range.

In all of the trials reported with joint torque control, the quality of the system behavior as measured at the output—in terms of apparent friction, oscillatory behavior or mechanism nonlinearities—is very greatly improved. The data reported are not directly comparable to the results of other compensation techniques, such as the 5:1 improvement in telescope pointing achieved by adaptive control (Walrath, 1984), but show that when friction is sensed, it can be compensated by feedback control.

4.3.6. *Dual mode control*
High precision applications, such as semiconductor manufacturing and diamond turning of optical

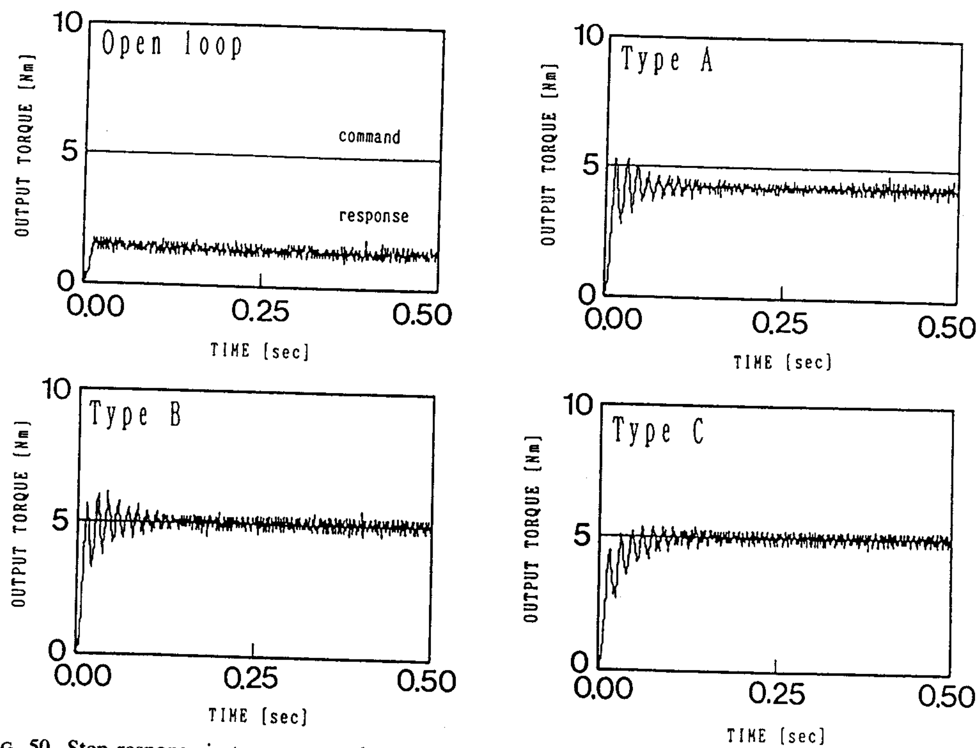

FIG. 50. Step response in torque control; open-loop, (a) joint torque control with no friction feedforward, (b) joint torque control with fixed friction feedforward, (c) joint torque control with adaptive friction feedforward [from Hashimoto et al. (1992), courtesy of the author].

elements, require nanometer positioning accuracy over millimeters of motion range. The standard technology for nanometer positioning involves two stage mechanisms. The coarse positioning stage might be a ball screw; and the fine motion stage might comprise a piezoelectric actuator [see Futami et al. (1990) and references for a brief discussion of two stage mechanisms]. The liabilities of the two stage mechanisms are weight, size and complexity: two actuators and two controllers are required per degree of freedom.

By capitalizing on presliding displacement, referred to as microdynamics in the nanotechnology literature, it is possible to achieve two modes of control in a

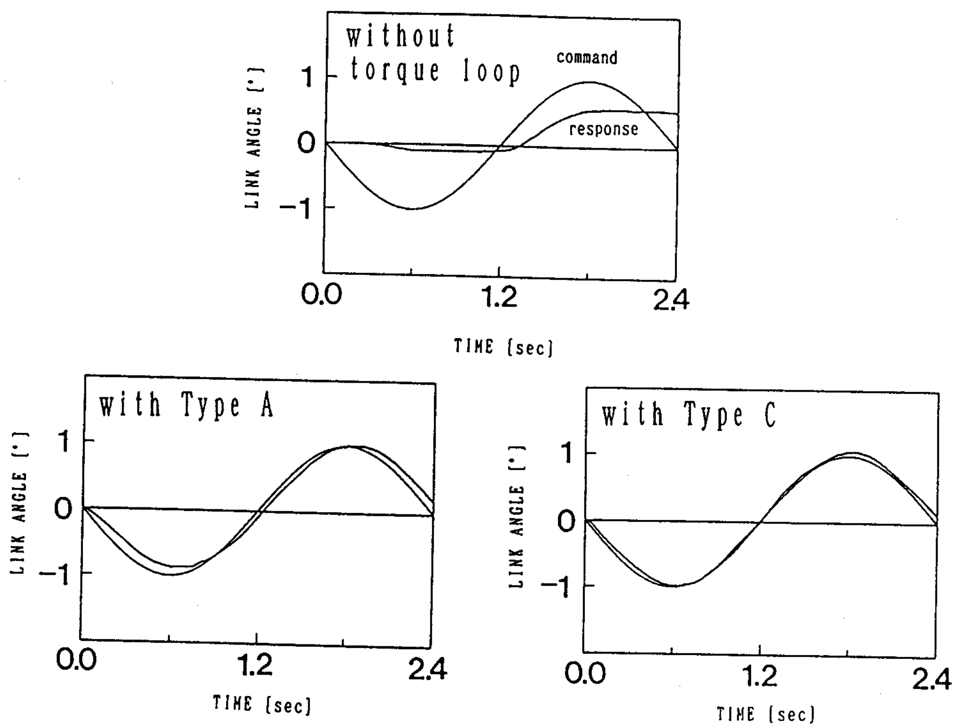

FIG. 51. Sinusoid tracking in torque control; (a) open-loop, (b) joint torque control with no friction feedforward, (c) joint torque control with adaptive friction feedforward [from Hashimoto et al. (1992), courtesy of the author].

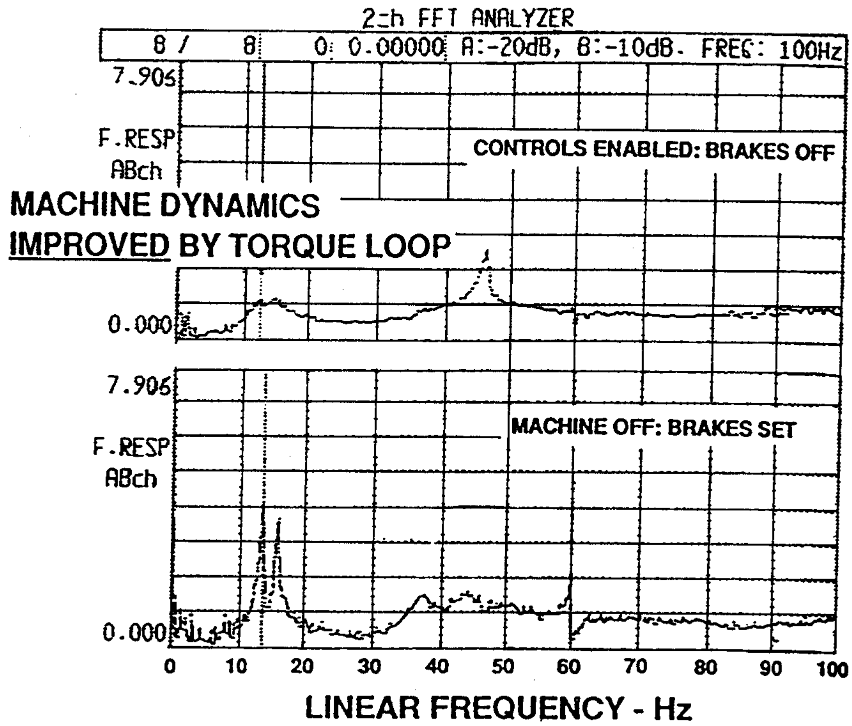

FIG. 52. Dynamic response of a robot with (above) and without (below) joint torque control [from Karlen *et al.* (1990), courtesy of the publisher].

single mechanism: gross motion in the standard way, and fine motion in the presliding displacement. Referring to Fig. 7 above, presliding displacement is motion that occurs by the deformation of asperities in the sliding interface. In this motion, position is a function of applied force; the junction appears to be a stiff spring rather than a sliding (or rolling) bearing. The result is two markedly different mechanism dynamics: 'macrodynamics,' the ordinary dynamics of the mechanism, and 'microdynamics,' which governs motions that depend upon elastic deformation in the frictional contact. Because the dynamics are drastically different, two different controller structures are required, thus dual mode control.

Futami *et al.* (1990) demonstrate accurate tracking of 1 nm step inputs with a slider on an air table and a linear AC motor and ball screw. The stroke of the mechanism is 250 mm, giving a dynamic range of $2.5 \times 10^8:1$. The authors identify three friction regimes, as seen in Fig. 53. Regime I exhibits the behavior of a linear spring, regime II the behavior of a nonlinear spring with damping, and regime III the normal sliding friction. The authors present controllers for regimes I and II. The coarse motion (regime III) controller is capable of positioning within 100 nm, which is the range of operation of the fine controller (regime I). After a coarse motion, 50 ms are allowed for vibrations to settle out, and the fine motion controller takes over. The authors have observed that the force–displacement characteristic of regime I is consistent throughout the travel of the mechanism.

Ro and Hubbel (1993) demonstrate dual mode control for a similar ball screw mechanism. They argue that the frictional characteristics are not constant throughout the workspace, and present a model reference adaptive control scheme. Demonstrating step input tracking, they find that the microdynamic controller is satisfactory for motions smaller than 400 nm, and the macrodynamic controller satisfactory for motions larger than 1000 nm. Intermediate size steps they find difficult to accurately control. Armstrong (1988) and Armstrong-Hélouvry (1991) has observed presliding displacement in an industrial manipulator and proposed dual mode control for that mechanism, suggesting that the method may be suitable for standard as well as high precision equipment. It appears that two and perhaps three orders of magnitude of improvement in positioning accuracy is possible with sufficiently accurate sensing and dual mode control.

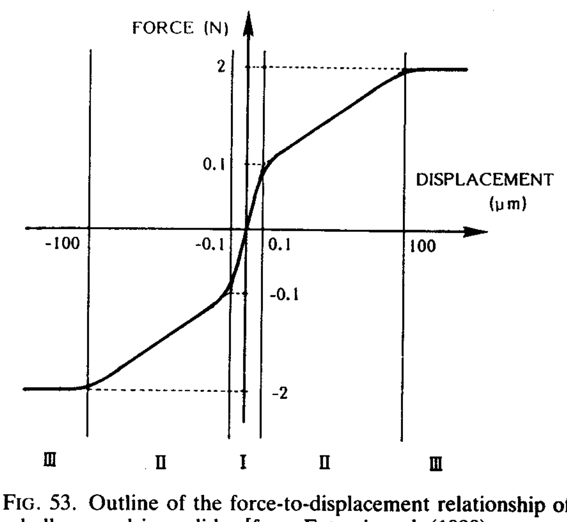

FIG. 53. Outline of the force-to-displacement relationship of a ball screw driven slider [from Futami *et al.* (1990) courtesy of the author].

4.4. Model-based Compensation for Friction

4.4.1. *Fixed compensation*

When a model of friction is available, it is possible to compensate for friction by applying a force/torque command equal and opposite to the instantaneous friction force. This presumes that force or torque actuation of adequate bandwidth is available and is stiffly coupled to the friction element. In many cases the dominant friction sources are the motor and transmission, and so adequate stiffness is assumed. Here compensation is addressed; questions of tuning the friction model parameters either off-line or adaptively, will be taken up below.

Model-based compensation has been reported in a number of studies (Gilbart and Winston, 1974; Walrath, 1984; Henrichfreise, 1985, 1992; Ackermann and Müller, 1986; Kubo *et al.*, 1986; Craig, 1987; Canudas de Wit *et al.*, 1987, 1989, 1991; Tomizuka *et al.*, 1988; Rattan *et al.*, 1989; Brandenburg *et al.*, 1988b, 1989, 1991; Armstrong, 1988; Armstrong-Hélouvry, 1991; Johnson and Lorenz, 1991; Suzuki and Tomizuka, 1991; Dupont, 1993; Maqueira and Masten, 1993); and is found in industrial control applications (see Section 4.5). All of these studies report experimental results, and many of them report significant improvement in performance when feedforward compensation is applied. The basic construction of model-based friction compensation is shown in Fig. 54. The model-based schemes can be classified according to what estimate of velocity is used to evaluate the friction model and what portions of the friction model are applied. In all of the studies above, as well as the reported industrial control applications, compensation for Coulomb friction is included; many of the studies include additional friction terms as well (Walrath, 1984; Armstrong, 1988; Armstrong-Hélouvry, 1991; Brandenburg and Schäfer, 1991; Canudas de Wit *et al.*, 1989, 1991; Johnson and Lorenz, 1991). Typical of the approaches where only Coulomb friction is compensated, Gilbart and Winston (1974) apply their scheme to optical tracking devices and report a factor of six reduction in RMS tracking error. The linear compensator is not affected by the feedforward compensation. Canudas de Wit *et al.* (1987) show a significant reduction in stand still during zero crossings of velocity with Coulomb friction compensation; as does Walrath (1984).

Because the Coulomb friction model is discontinuous, the choice of estimate of velocity with which to evaluate the model is significant. Gilbart and Winston (1974), Walrath (1984), Kubo *et al.* (1986) and Canudas de Wit *et al.* (1987) employ sensed velocity. Though these authors do not report undue difficulty with stability, there are no doubt sensor noise and stability issues which must be considered when an infinite gain operator is introduced in the feedback loop. Canudas de Wit *et al.* (1989) point out that model-based Coulomb friction may reduce the need for high servo gains, and thus reduce the impact of sensor noise. None-the-less, to reduce the impact of sensor uncertainties on the friction compensation a state estimator may be used. Brandenburg and Schäfer (1988b, 1989) and Schäfer and Brandenburg (1990), who term the feedforward Coulomb friction compensation a "disturbance observer", have shown remarkable performance with a system employing a state estimator for velocity. Here the principal concern is with the presence of limit cycles rather than tracking accuracy; the authors study a two mass system with backlash and report the presence of high and low frequency limit cycles. By use of the Coulomb friction compensation, the high frequency limit cycle is eliminated completely and the range of velocities over which a low frequency limit cycle is observed is halved (Brandenburg and Schäfer, 1988b).

Johnson and Lorenz (1991) and Armstrong-Hélouvry (1991) have considered the use of commanded velocity to generate the friction compensation. Johnson and Lorenz (1991) present experimental results for both feedforward and feedback systems; the two perform comparably, except near zero velocity. They report a factor of four reduction in position errors during step input trials for both systems. Armstrong-Hélouvry (1991) reports two different types of results: open-loop motions intended to demonstrate the potential for accurate friction modeling, and high precision force control with an

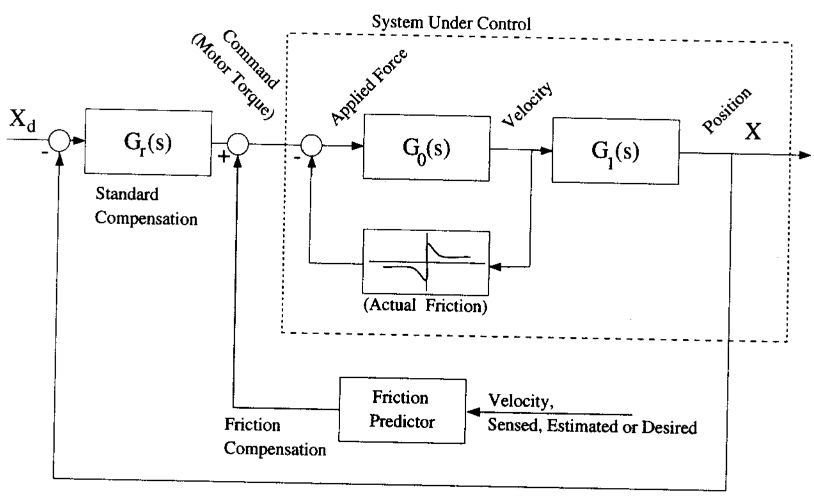

FIG. 54. Basic construction of model-based friction compensation.

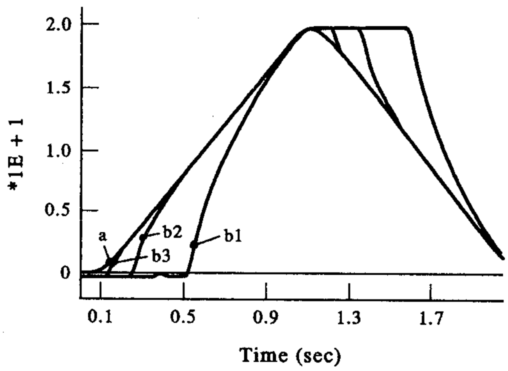

FIG. 55. Position as a function of time from experimental trials: (a) reference model; (b1) without nonlinear compensation; (b2) with Coulomb friction compensation; (b3) Coulomb plus Karnopp-like static friction compensation with an additional dynamic term [from Brandenburg and Schäfer (1991), courtesy of the authors].

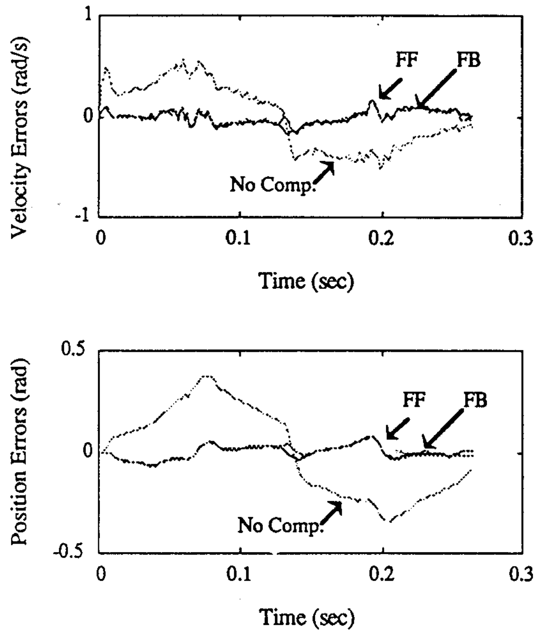

FIG. 56. State errors for three controllers: feedback friction compensation, feedforward friction compensation and uncompensated [from Johnson and Lorenz (1991), courtesy of the publisher].

impulsive controller. In these cases it was necessary to use command velocity. Tracking errors of 10% or less are demonstrated in the open-loop motions, corresponding to a very high fraction of the friction compensated by the model term.

While good performance has been shown with straight Coulomb friction compensation, improvements have been shown with the use of richer friction models. Brandenburg and Schäfer (1991) and Johnson and Lorenz (1991) present systems with static friction modeling and compensation. Both employ a Karnopp-like model (see Section 3.4, Fig. 39), with a higher level of friction compensation provided near zero velocity. In Fig. 55, experimental results from Brandenburg and Schäfer (1991) are presented. Their system is a complicated one, involving two masses, compliance, backlash and a multi-loop controller that simulates an industrial servo drive controller. Model reference adaptive compensation is used (a block diagram of this system is provided with the discussion of adaptive control, see Fig. 59). The figure shows that standstill at zero-crossings of velocity are substantially reduced during a sawtooth motion. Curve (a) is the track of the reference model, curve (b1) shows the case where no friction compensation is used. The curve (b2) indicates Coulomb friction modeling and feedforward compensation; and the curve (b3) Coulomb + static friction modeling and feedforward compensation, using a discrete time, single zero differentiating filter which compensates for frictional memory. In Fig. 56, taken from Johnson and Lorenz (1991), their results are presented for both feedforward and feedback compensation and position and velocity errors.

One of the major difficulties in performing friction compensation is the difficulty in modeling friction at very low velocities. Several practical problems can appear as a consequence of doing friction compensation on the basis of a discontinuous model. Conceptually, state variable models are better adapted to describe and hence to compensate for friction for very small velocities. They better reflect the fact that friction (or any force transmitted by a compliant member) is a continuous function of time. Examples of such models are the Dahl model (Dahl, 1977), the second-order Dahl model (Bliman and Sorine, 1991) and the modified first-order Dahl model (Canudas de Wit *et al.*, 1993). When these models are used as a basis for friction compensation, one major difficulty arises because the internal states of these models are not measurable. However, internal state observers can be designed and stability of the observer-based control schemes can be shown (Bliman and Sorine 1991; Canudas de Wit *et al.*, 1993). In connection with control design, these models enjoy the input–output property of passivity or, more precisely, dissipativity. These properties can be explicitly exploited during the control design leading to an explicit determination of the class of compensators that render the feedback loop stable.

Walrath (1984) reports a particularly sensitive and detailed study of friction phenomena during velocity reversal of a tracking telescope. The technical challenge is considerable because the telescope must be accurately pointed from a moving platform. Response to platform motions involves frequent velocity reversals. Unlike the machine tool or robotics systems studied above, most of the friction in Walrath's mechanism arises in the bearings. Walrath (1984) extends his model with an acceleration dependent decay of the friction from one Coulomb friction level to the other during zero crossing; thus, there is no instantaneous friction transition in his friction model or applied compensation. Walrath develops his model from the Dahl model and does not attempt to explain the underlying physics; the model, however, is consistent with the presence of frictional memory. Walrath reports a factor of five improvement in RMS pointing error.

Tung et al. (1993) report an interesting variant on the theme, they apply a corrective signal to the input of the compensator, $G_r(s)$ in Fig. 54. In this way compensation may be applied to standard industrial controls.

4.4.2. *Friction identification and adaptive control*

"Therefore always when you wish to know the quantity of the force that is required in order to drag the same weight over beds of different slope, you have to make the experiment and ascertain what amount of force is required to move the weight along a level road, that is to ascertain the nature of its friction."

Leonardo da Vinci (1452–1519),
The Notebooks, F II 106 r.

Although progress has been and continues to be made toward the ability to anticipate friction forces based on features of the mechanical design, e.g. (Kragelskii, 1988), this challenge remains a formidable one and for the foreseeable future it will continue to be necessary, as Leonardo has suggested, to ascertain by experiment the friction parameters of a particular machine.

The friction parameters may be determined either off-line, following a data gathering experiment, or continuously, on-line as part of operation of the machine. If the parameters are then used in a model-based friction compensation, on-line identification becomes adaptive control. The off-line and on-line identification schemes are most distinguished by the type of experimental motion: the designer of the off-line identification is often free to specify the motions through which data will be gathered, whereas the on-line identification must normally use data from motions dictated by the operation of the machine. The distinction is important in that there are issues of excitation: with common friction models, the terms of the model, or basis functions, are not automatically independent or orthogonal but may be to a large degree orthogonalized by proper choice of experimental trajectories. Stated another way: when not using special trajectories, the friction model terms may be strongly coupled and difficult to accurately identify. But by proper choice of machine motion it is possible to more nearly orthogonalize the model basis functions, i.e. to increase the excitation. Several of the authors presenting schemes for off-line identification have capitalized on this possibility. The adaptive friction compensation has been proposed and demonstrated in many forms. The principal advantage of an adaptive scheme relative to an off-line scheme lies in its ability to track changes in friction.

4.4.2.1. *Off-line identification.* The tribology literature is rich in experimental work which might be described as off-line identification of friction models and model parameters, e.g. Rabinowicz (1965) and Bowden and Tabor (1973). A large part of this work has been carried out with simplified friction contacts (e.g. sphere or cylinder on flat), with controlled normal forces, and direct and precise sensing of contact displacements and forces, e.g. Polycarpou and Soom (1992). The controls engineer, on the other hand, must contend with a machine designed for other purposes and sensing better suited to the task of the machine than to friction identification. For these reasons, careful attention must be given to the design of the off-line friction identification experiment.

Friction identification, both off- and on-line, is made more challenging by the need for acceleration sensing to directly observe friction in mechanism motion. Acceleration sensing is not uncommon in the tribology literature, and has been employed by Armstrong-Hélouvry (1991) to identify friction in complex motions. Johnson and Lorenz (1991) take the alternative tack of using the desired acceleration as an estimate of the true acceleration. Their technique consists in a stepwise procedure for individually identifying the parameters of an inertia/friction model. The torque command signal from the feedback compensation is used. Following the notation of Johnson and Lorenz (1991), their compensation is given by:

[Total Command]

$$T^* = T_{fb} + T_{ff} + (T_{nldsfb} \quad \text{or} \quad T_{nldsff}) \tag{32}$$

[Standard Feedback]

$$T_{fb} = k_p(\theta^* - \theta) + k_v(\omega^* - \omega) \tag{33}$$

[Linear Feedforward]

$$T_{ff} = \hat{J}\dot{\omega}^* + \hat{b}\omega^* \tag{34}$$

[Friction Compensation/Feedback]

$$T_{nldsfb} = \begin{cases} \hat{T}_c \operatorname{sgn}(\omega) + b\omega; & |\omega| > \Delta\omega \\ \mathrm{MIN}\,(\hat{T}_{em}, \hat{T}_s \operatorname{sgn}(\omega)); & |\omega| < \Delta\omega \end{cases} \tag{35}$$

[Friction Compensation/Feedforward]

$$T_{nldsff} = \begin{cases} \hat{T}_c \operatorname{sgn}(\omega^*) + b\omega^*; & |\omega^*| > \Delta\omega \\ \mathrm{MIN}\,(\hat{T}_{em}, \hat{T}_s \operatorname{sgn}(\omega^*)); & |\omega^*| < \Delta\omega \end{cases} \tag{36}$$

where T^* is the total command torque; T_{fb} is the feedback torque from the error correction process; k_p, k_v, θ and ω are position and velocity gains and position and velocity, respectively; T_{ff} is a linear feedforward term that compensates for inertia and known damping through the parameters $\hat{J}$ and $\hat{b}$; $\dot{\omega}^*$ is the desired acceleration; T_{nldsfb} or T_{nldsff} are torque, nonlinear decoupling state feedback, and torque, nonlinear decoupling state feedforward, respectively. Only one of the terms T_{nldsfb} or T_{nldsff} would be used at a time. The authors implement the Karnopp friction model, where $\Delta\omega$ marks the small range of velocity over which static friction is taken to apply; $\hat{T}_{em}$ is the applied motor torque, estimated from commanded motor current T^*; $\hat{T}_c$ is the estimate of Coulomb friction; and $\hat{T}_s$ is the estimate of static friction. T_{nldsfb} or T_{nldsff} differ as to whether measured velocity, ω, or desired velocity, ω^*, is used.

In the procedure of Johnson and Lorenz, the inertial parameter, $\hat{J}$, is identified first, then the Coulomb and viscous friction parameters, $\hat{T}_c$, b, and finally the static friction parameter, $\hat{T}_s$. As each parameter is identified, that term is added to a feedforward or a feedback compensation. Ideally, as the entire model is identified the error correcting term of feedback control, T_{fb}, would go to zero. If the model is adequate, T_{fb} can indeed become quite small (Armstrong-Hélouvry, 1991). Once the inertial parameter is identified, its presence in the feedforward path brings the desired acceleration into the estimation of the subsequent parameters. The parameters are identified by minimizing the error correcting term of the feedback control, T_{fb}. The procedure is distinctly off-line in that the parameters are identified sequentially. Care is taken to select experimental trajectories which will maximize the sensitivity of the identification to each parameter as it is sought (Johnson and Lorenz, 1991). The stepwise identification can achieve higher excitation [coupling of the sought parameter(s) to the observed signal(s)] than can a simultaneous identification. While the lower excitation of a simultaneous identification might be partially compensated by allowing a longer run than the individual parameter identification experiments of Johnson and Lorenz; the biasing influences of sensor noise—as opposed to process noise—and systematic disturbances are not reduced by longer experimental runs (Armstrong, 1989a).

Johnson and Lorenz apply their technique to friction in the joint of a motorized robotic gripper. Their apparatus allows them high bandwidth torque (current) control and sensing of actuator position. The coupling of the actuator to the friction contacts, the motor and a preloaded transmission element, is presumed to be quite rigid. This combination of features is adequate for accurate friction identification (Armstrong-Hélouvry, 1991); and is common to many of the reported friction identification experiments, e.g. (Kubo *et al.*, 1986; Townsend and Salisbury, 1987). Johnson and Lorenz (1991) employ the Karnopp friction model, with viscous friction added, and different friction parameters in the two movement directions. They also observe rising static friction. Their identification technique, coupled with model-based compensation, is quite successful at reducing the impact of friction on the closed-loop performance of their machine, as shown by Fig. 56.

Armstrong-Hélouvry (1991) presents an experiment for identifying Coulomb + viscous friction that does not require acceleration, either measured or estimated. His experiment consists of long, open-loop glides at constant torque and measuring average velocity. The technique was applied to a PUMA manipulator; initial transients were reduced with a manually tuned acceleration torque. The experiment is repeated at several torque levels, corresponding to several velocities; and a Coulomb + viscous friction model is fit to the (friction-velocity) data points obtained, as shown in Fig. 57. The technique depends upon viscous friction in the machine to create a stable gliding velocity. Using a breakaway experiment, Armstrong-Hélouvry had identified position dependency in the friction and constructed a table lookup compensation. This compensation was applied in feedforward during the open-loop glides to better achieve a constant velocity.

Armstrong-Hélouvry (1991) also demonstrated a technique involving closed-loop constant velocity glides and measuring average torque. This technique resulted in a higher noise content than did the open-loop glides, but could be applied to a machine without viscous friction. The closed-loop technique is like that of Johnson and Lorenz with the (average) acceleration taken to be zero during the constant velocity glides.

Using a friction model comprising Coulomb + viscous + position-dependent friction, Armstrong (1988) and Armstrong-Hélouvry (1991) demonstrated three axis open-loop motions of the robot using joint-wise spline trajectories. Model-based, open-loop control was not proposed as a viable architecture, rather the experiments were made to demonstrate the possibility of accurate friction identification. Spatial motions with an accumulated error of less than 10% were demonstrated.

4.4.2.2. *Full model identification.* In a Coulomb + viscous + static friction model, all of the parameters enter the model in a linear fashion and may thus be identified by standard techniques. This is, for example, true of $\hat{T}_c$, $\hat{b}$, and $\hat{T}_s$ in equations (32)–(36). When Stribeck or rising static friction are incorporated, the model parameters, specifically $\dot{x}_s$ and γ in equation (10), bear a nonlinear relationship to the friction torque. To identify these parameters, nonlinear techniques are appropriate. Cheok *et al.* (1988) employ the simplex method to determine the parameters of a Karnopp friction model, including the width parameter D_V of equation (25). Like $\dot{x}_s$ in equation (10), D_V of the Karnopp model appears in a nonlinear relationship to the friction torque. Cheok *et al.* (1988) point out that multiple minima are possible and pose a problem for gradient techniques. Armstrong-Hélouvry (1991) identified the parameters of a Coulomb + viscous + Stribeck + frictional memory + rising static friction + position-dependent friction model from friction data collected with the PUMA 560 robot. The frictional memory, rising static friction and position-dependent friction were identified separately, in a step-wise fashion comparable to that of Johnson and Lorenz (1991). The remaining friction

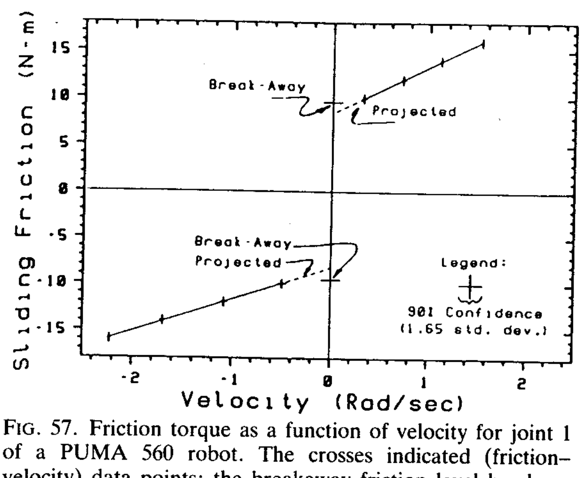

FIG. 57. Friction torque as a function of velocity for joint 1 of a PUMA 560 robot. The crosses indicated (friction–velocity) data points; the breakaway friction level has been recorded in a separate experiment [from Armstrong-Hélouvry (1991)].

versus velocity model contained both nonlinear and linear parameters. To identify the parameters which enter in a nonlinear fashion, an exhaustive search technique was used. At each point in the space of (two or four) nonlinear parameters, the four parameters entering linearly in the model were fit by regression analysis (the normal equation) and the residual error determined. The parameter set giving the smallest residual was selected. A search technique was certainly necessary; multiple local minima were observed and would have arrested a gradient technique. The presence of noise in the data is thought to have introduced local minima where intuitively there ought to have been none.

In addition to nonlinearity in certain parameters, some of the parameters of a full friction model may be very difficult to measure. Armstrong-Hélouvry (1991) reports a measurement of Stribeck and frictional memory parameters. Using sensitive acceleration and force instruments and statistical techniques, he was able to observe the Stribeck friction, which is dominant at velocities near 5 milliradians per second, a velocity range in which stable motion is not possible for the PUMA. Even with acceleration and force sensors, he was not able to identify the frictional memory parameter directly. Rather, he investigated the correlation between the best fit friction model and the actual friction signal (obtained with measurement of acceleration). A 10% improvement in correlation was observed with the introduction of a 51 ms frictional lag. Frictional memory has been measured directly in sensitive tribology experiments (Hess and Soom, 1990; Polycarpou and Soom, 1992), but with a quality of instrumentation and control of conditions that are probably unachievable in practical machines. It has been shown that all of the parameters of the seven parameter model, equation (10), influence the presence of stick slip (Derjaguin *et al.*, 1957; Armstrong-Hélouvry, 1991); it may be possible to indirectly identify the full friction model parameters by mapping the presence or absence of stick slip across a range of system stiffness, damping and velocity conditions (Armstrong, 1988). If possible, such an identification technique would function with nothing more than force actuation and position sensing.

4.4.2.3. *Adaptive control.* The challenges to adaptive control of a machine with friction are not unlike the general challenges of adaptive control: problems of stability, the need for persistent (indeed, sufficient) excitation, difficulties that arise when the true model is not in the model set, the want of methods for setting rate and other parameters in the adaptive algorithm, etc. With friction, the motivation for adaptive control is also not unlike the general motivation: friction will, in many cases, be a variable quantity which the controller must track. A very substantial number of papers concerning adaptive control in robotics and elsewhere have touched on friction; here we focus on several papers where friction has been a major concern in the design of an adaptive control algorithm. All of the papers surveyed employ a model-based friction compensation and adaptively update the model parameters. The adaptive algorithms, however, span a great range; including the recursive-least-square (RLS) and least-mean-square (LMS) algorithms of Walrath (1984) Craig, (1986, 1987), Canudas de Wit (1988) and Canudas de Wit *et al.* (1987, 1991); the model reference adaptive control (MRAC) algorithms of Gilbart and Winston (1974), Brandenburg and Schäfer (1988b, 1989), Schäfer and Brandenburg (1990), Yang and Tomizuka (1988) and Maqueira and Masten (1993); or the Lyapunov function based algorithm of Friedland and Park (1992); which has been extended to multi-mass systems by Friedland and Mentzelopoulou (1993). Adaptive control has been richly described in the literature, e.g. (Widrow and Sterns, 1985; Åström and Wittenmark, 1989). Only brief descriptions of algorithms will be given here. An interesting experimental comparison of several adaptive algorithms and PID control in a low-velocity position-tracking task with velocity reversal is presented in Leonard and Krishnaprasad (1992).

The Recursive Least Squares (RLS) and Least Mean Squares (LMS) algorithms

Intuitively the simplest algorithms, the recursive least squares (RLS) and least mean squares (LMS) algorithms function by determining the correlation between an error signal and the state dependent basis functions that make up the model. The model is constructed:

$$y(t) = \varphi_1(t)\theta_1 + \varphi_2(t)\theta_2 + \varphi_3(t)\theta_3 + \cdots, \quad (37)$$

where y is an output signal, the $\varphi_i(t)$ are model basis functions of the system state; as in the case of Coulomb friction, the basis functions need not be linear; and θ_i are the (time invariant or slowly varying) model parameters. Note that the θ_i make a linear contribution to y, whether or not the $\varphi_i(t)$ are linear. Following an example from Canudas de Wit *et al.* (1991) (with gravity and Stribeck friction terms omitted for simplicity) one might have for a single mass system: where $\Phi(t)$ is the regressor vector, the vector of φ_i.

$$m\ddot{x} = \tau - F_v\dot{x} - F_c\,\mathrm{sgn}(\dot{x}) \quad \text{[System Dynamics]}$$

$$\tau(t) = \theta^T\Phi(t) \quad \text{[Model in } \theta/\Phi \text{ Form]}$$

$$\begin{aligned}\Phi(t) &= [\ddot{x}(t), \dot{x}(t), \mathrm{sgn}(\dot{x}(t))]^T \\ \theta^T &= [m, F_v, F_c]\end{aligned} \quad (40)$$

and where $\tau(t)$ is the applied torque; $x(t)$ is the position variable; and m, F_v and F_c are mass, viscous and Coulomb friction parameters, respectively. An error signal is constructed:

$$e(t) = \tau(t) - \hat{\theta}^T\Phi(t), \quad (41)$$

where $\hat{\theta}$ is the vector of estimated parameters. An update equation is constructed:

$$\begin{aligned}\dot{\hat{\theta}}^T &= \lambda(t)P(t)\Phi(t)e(t) \\ &= \lambda(t)P(t)\Phi(t)(\tau(t) - \hat{\theta}^T\Phi(t)),\end{aligned} \quad (42)$$

where $\lambda(t)$ is a rate gain; and $P(t)$ is the inverse of the input correlation matrix for the RLS algorithm or the identity matrix for the LMS algorithm.

Good convergence properties can be shown for the RLS and LMS algorithms when conditions are ideal, i.e. $|P(t)|_2$ is bounded (persistent excitation) and not too big (sufficient excitation); and the true system is in the model set; that is to say, there exists a choice of parameters, θ^*, such that $\tau(t) = \theta^{*T}\Phi(t)$.

Canudas de Wit *et al.* (1987) study a basic case and show convergence and performance improvement for a DC motor/transmission mechanism. Canudas de Wit *et al.* (1991) extend this result to the important case of Stribeck friction, and show the possibility of destabilizing over-compensation when adaptation occurs at low velocities. The problem arises when a Coulomb friction term adapts to compensate for high friction at very low velocities and then over-compensates at higher velocities. Because models for Stribeck friction contain parameters which do not appear linearly in the output, such as $\dot{x}_s$ in equation (10), Canudas de Wit *et al.* (1991) propose the use of a square-root velocity term:

$$\tau_f(\dot{x}) = \hat{\theta}^T\Phi; \quad \Phi = [\operatorname{sgn}(\dot{x}), |\dot{x}|^{1/2}\operatorname{sgn}(\dot{x}), \dot{x}]^T \\ \hat{\theta}^T = [F_c, F_s, F_v], \tag{43}$$

where $\tau_f(t)$ is the friction term.

They show that the square-root-velocity term can be used to closely match (friction–velocity) curves proposed by Tustin (1947) and others; as seen in Fig. 58. It appears, however, that this will only be the case if the break in the Stribeck friction curve lies in a velocity range comparable to that of the break (the greatest second derivative) in the square-root-velocity basis function; and that the basis functions might have to be replaced with

$$\Phi = \left[\operatorname{sgn}(\dot{x}), \left|\frac{\dot{x}}{\dot{x}_s}\right|^{1/2}\operatorname{sgn}(\dot{x}), \dot{x}\right]^T, \tag{44}$$

where $\dot{x}_s$ is a scaling parameter in velocity that can be used to locate the velocity range of the break in the curve. Even though $\dot{x}_s$ could not be adjusted by a linear scheme, if it could be set *a priori* to an appropriate range, the linear identification scheme might perform satisfactorily.

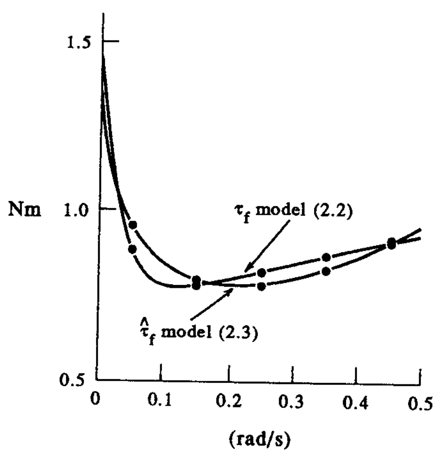

FIG. 58. Comparison of the exponential friction model [of Tustin (1947)] and the estimated linear model, equation (43) [from Canudas de Wit *et al.* (1991)].

Craig (1986, 1987) presents an LMS algorithm comparable to the RLS algorithm of Canudas de Wit *et al.* (1987). Craig's (1986, 1987) compensation for friction is part of a larger identification of the inertial parameters of the first two degrees of freedom of an Adept robot. Experimental results are presented, including identified friction parameters; but from the results it is difficult to determine whether the adapted parameters either reflect true friction values or significantly improve performance.

In practical implementations, the use of acceleration in $\Phi(t)$ poses a considerable challenge. Versions of the algorithm with filtered signals have been presented which do not employ acceleration signals, but require a high pass filtering step on the velocity, (Hsu *et al.*, 1987). As did Johnson and Lorenz (1991) with the off-line identification, Slotine and Li (1987, 1989) have employed command acceleration rather than actual acceleration in the regressor vector of a sliding mode adaptive algorithm. The authors present a proof of the stability of their algorithm using Lyapunov's direct method; and they present very successful adaptive control of a robot manipulator, of which friction compensation is a part.

Model Reference Adaptive Control (MRAC)

Gilbart and Winston (1974) were the first to implement adaptive friction compensation, and do so with a model reference adaptive controller. Åström and Wittenmark (1989) present a good discussion of the MRAC. Brandenburg and Schäfer (1988b, 1989), Schäfer and Brandenburg (1990), Held and Maron (1988) and Maron (1989a, b) also present friction compensation in an MRAC framework.

Gilbart and Winston (1974) implement an MRAC with a first-order reference model and a Lyapunov function chosen to eliminate acceleration terms. The result is Coulomb friction compensation given by:

$$u = (\text{PD Control}) + K_3\operatorname{sgn}(\dot{x}) \\ K_3 = B_3\int_0^t \dot{e}\operatorname{sgn}(\dot{x})\,dt + C_3\,\dot{e}\operatorname{sgn}(\dot{x}) \tag{45} \\ \dot{e} = \dot{x}_m - \dot{x},$$

where $\dot{x}$ is velocity; $\dot{x}_m$ is model velocity; K_3 is a Coulomb friction compensation parameter; and B_3 and C_3 are positive constants which are determined by the parameters of the Lyapunov function. As opposed to equation (40), (45) does not involve acceleration. Gilbart and Winston present a remarkable analog computer implementation that achieves a 6X reduction in RMS pointing error of a tracking telescope.

Brandenburg and Schäfer (1988b, 1989) and Schäfer and Brandenburg (1989, 1990, 1993) employ an MRAC structure to adapt the parameters of a Coulomb friction compensating disturbance observer for a two mass flexible system with backlash. A block diagram is shown in Fig. 59. Without friction compensation, the system exhibits two stick-slip limit cycles. Though the details are not specified, it is

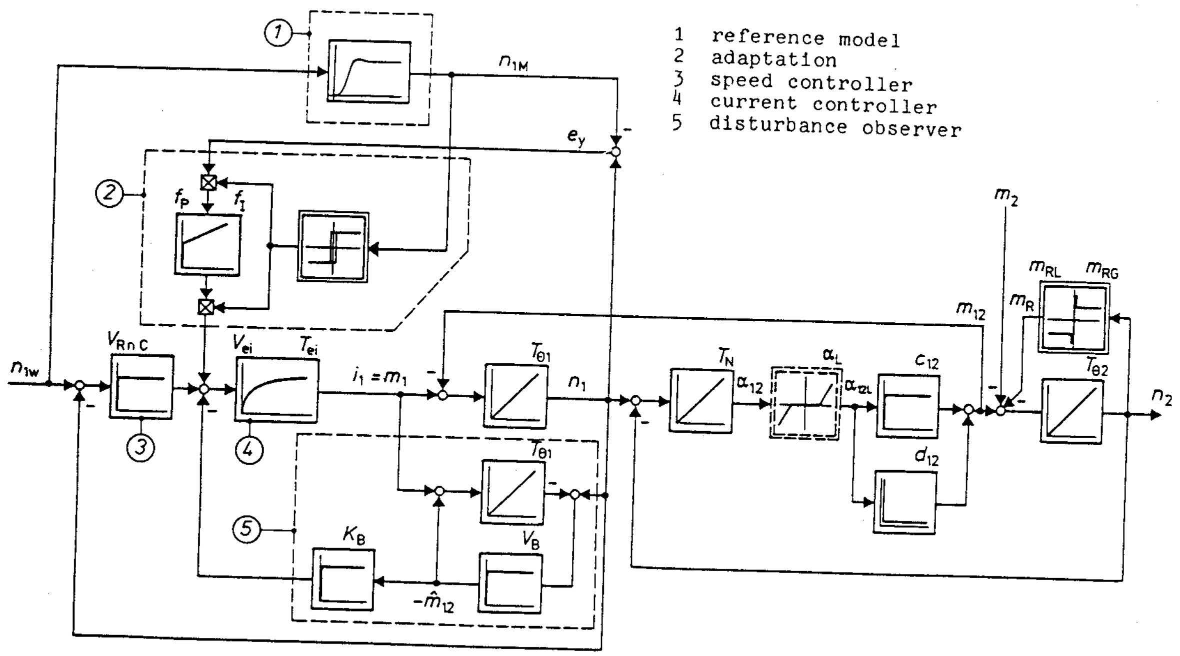

FIG. 59. Speed-controlled elastic two-mass system with reference model [from Brandenburg and Schäfer (1988b), courtesy of the authors].

indicated that the MRAC design is based upon a Lyapunov function; the result is a friction compensation made by applying a lag (PI) filter to $\dot{e}$, the difference between model and true velocity; just as in equation (45). Combined with integrator deadband, their algorithm eliminates stick slip and reduces standstill intervals at speed reversals by a factor of five.

Lyapunov and Lyapunov-like methods

Friedland and Park (1992) present an adaptive friction compensation scheme which is based upon a Lyapunov-like argument involving the position error. The result is an update law that does not depend upon acceleration measurement or estimation. The friction compensation is given by [modifying the notation of Friedland and Park (1992)]:

$$m\ddot{x} = u - f(\dot{x}, F_c^*)$$

[System Dynamics, single mass]

$$f(\dot{x}, \hat{F}_c) = \hat{F}_c \operatorname{sgn}(\dot{x})$$

[Friction Model]

$$u(t) = (\text{Standard Control}) + \hat{F}_c \operatorname{sgn}(\dot{x})$$

[Control Law]

$$\hat{F}_c = z - k\,|\dot{x}|^{\mu}$$

[Friction Estimator]

$$\dot{z} = k\mu\,|\dot{x}|^{\mu-1}\frac{1}{m}(u - f(\dot{x}, \hat{F}_c))\operatorname{sgn}(\dot{x})$$

[Friction Estimator Update Law],

where m is system mass; u is the control input; z is a defined function; $\hat{F}_c$ is the estimated Coulomb friction; and μ and k are tunable gains. Defining the model misadjustment:

$$e = F_c^* - \hat{F}_c \tag{51}$$

one finds that

$$\dot{e} = -k\mu\,|\dot{x}|^{\mu-1}e \tag{52}$$

making $e = 0$ the stable fixed point of the process. Friedland and Park (1992) do not present experimental results, but report simulations of a single degree of freedom system with Coulomb friction and the adaptive algorithm run with several values of μ and k. The algorithm significantly improves dynamic response.

The RLS/LMS, MRAC and Lyapunov-based adaptive laws have in common a relationship between the integral of acceleration error and the estimated friction constants. In the RLS and LMS algorithms, acceleration appears explicitly and, scaled by mass, is subtracted from the applied torque to give torque or acceleration error, which determines the rate of change of the estimated parameter. The estimated parameter is thus in proportion with the integral of acceleration error. In terms of physical dimensions, this situation holds whether the actual acceleration, desired acceleration or filtered velocity is used.

In the MRAC algorithms, a PI filter is applied to the velocity error signal. The 'P' term gives a contribution proportional to the integral of acceleration error. In the algorithm of Friedland and Park (1992), the term $\dot{z}$ is proportional to the time derivative of $k\,|\dot{x}|^{\mu}$, where $(1/m)(u - f(\dot{x}, \hat{F}_c))$ has the physical dimension of $\ddot{x}$. Since $\dot{z}$ is in proportion with the derivative of a function of velocity, $z - k\,|\dot{x}|^{\mu}$ is in proportion with the integral of acceleration error.

The RLS algorithm is a recursive version of the normal equation from statistics, which gives the squared-error optimal estimate of the parameters, but requires that the basis functions, i.e. acceleration, be known. The other algorithms may be seen as implementing filtered versions of the normal equations; versions which are perhaps not squared-error optimal, but which are implementable without measurement of acceleration. A thorough analysis of the statistical implications of the several estimators has not been presented.

Learning Control

Learning control—sometimes called repetitive control—is a process of developing feedforward corrections for a specific trajectory. It has been proposed for robotics and other servos (Craig, 1986; Kuc *et al.*, 1991) and is available on some industrial controllers (see Section 4.5). Generally, the learning control compensation takes the form of a table of corrections to be added to the control signal during the execution of a motion. The table must be learned during executions of the precise motions to be corrected, something which will not be a limitation when the application involves repetitive motions. A correction table developed in this way will include frictional forces.

4.5. Input from Engineers in Industry who have Controlled Machines with Friction

An informal survey has been conducted of engineers in industry who have controlled machines with friction. Engineers were contacted through the professional societies, references in the literature and referrals. While not all contacted were equally concerned about privacy, it was sometimes an issue; and so the results of this survey will be reported without reference to the specific contributor.

The contributors include engineers with 23 companies in Europe, Japan and the United States, and several government laboratories. The machine tool industry was the most represented among the respondents, followed by precision pointing applications (often for government systems), robotics and all others. A great breadth of techniques were found to be in service:

- system hardware modification;
- high servo gains (stiff position and velocity control);
- modifications to integral control;
- linear adaptive control;
- model-based friction compensation:
    - —fixed;
    - —adaptive;
- dither;
- table lookup compensation;
- learning control;
- joint torque control; and
- variable structure control.

System hardware modification is by far the most common and successful approach to overcoming frictional disturbances. As opposed to the controls literature, controls engineers often mentioned issues of machine design and particularly lubrication as the first and perhaps only necessary step to achieving the needed performance. Machine modification did not always consist of reducing the overall friction. In the machine tool industry, for example, "friction materials" such as Rulon®, are used to provide a high Coulomb friction with a reduced excess of static over Coulomb friction. The friction materials consume substantial energy, but serve to reduce mass-spring oscillations as well as frictional limit cycles, and thereby increase the robustness of the controller. A number of cases were encountered in which a new lubricant choice solved a stick-slip problem, reflecting the fact that friction modification is not always considered in the initial lubricant selection.

Dither is most successful in cases where the vibration can be applied without great attenuation to the friction contact. Hydraulic systems are suitable, and several standard hydraulic servo controllers include provisions for dither in the spool valve command signal. The earliest applications of dither were not in the control signals, but were made by attaching a vibrating element—typically an eccentric wheel—to the machine (Bennet, 1979). No applications of this type were reported.

In each field, cases can be found where machine design and lubrication have been pushed to their natural limits and improvement in performance requires enhancements to the servo control. In stiff systems, high position and velocity servo gains are used to overcome frictional disturbances. The technique is applied with the greatest success in pointing devices, which do not carry a payload and can be designed specifically to meet mechanical rigidity requirements. When high gains have been applied successfully in robotics, it has been observed that many features of the system must function harmoniously, including sensor characteristics and properly tuned lead–lag compensation. In some applications, variable structure systems have been engineered which apply higher servo gains near zero velocity, in order to meet the demands of nonlinear friction at low velocities and the desired performance characteristics at higher velocities.

Joint torque control has been discussed in Section 4.3.5. Engineers working with joint torque control in commercial robotics report that the residual friction is imperceptible, and that the closed-loop controllers function well in position, force and transitional tasks.

Integral control, or lag compensation, is certainly very common in practice; much more so than is suggested by its standing in the controls literature. The integral control term aggravates stick slip and introduces hunting; and a great many mechanisms for modifying integral control are available in off-the-shelf servo controllers. A deadband in the position-error input to the integrator eliminates hunting, but introduces a threshold in the precision with which a servo can be positioned. The use of lag compensation with high but finite DC gain will accomplish something of the same end without introducing a nonlinearity. Integral control terms are also modified by resetting the integrator when motion is detected (appropriate when short motions without overshoot are sought), and saturating the integral term as a function of position or velocity error terms, a deadband-like construction which allows full integral control action during motion.

One very substantial challenge introduced by integral control arises because during motion in one direction the integrator winds up to compensate for Coulomb friction in that direction, and when there is a reversal, the integral control suddenly compounds rather than compensates for Coulomb friction. This behavior enlarges (but does not create) quadrature glitches in multi-axis machining stations—probably the frictional disturbance of greatest economic impact. One application was found in which the integral control term is multiplied by the sign of velocity. This technique will likely be successful so long as the integral control is not compensating for anything other than Coulomb friction. A more common technique is to reset the integrator at velocity reversal.

Adaptive control is present in a growing number of industrial applications. In its linear form, the adaptive algorithms are certainly responding to friction. The possible dangers of over-compensation are not insubstantial, because the true model is not in the model set for these systems. Model-based friction compensation is also employed and offsets some of the limitations of both integral and linear adaptive control. The Coulomb friction term explicitly multiplies the sign of velocity in these systems, allowing the integral control term, if present, to accommodate other disturbances. Applications were found in which the Coulomb friction compensation was based on desired velocity or upon a model velocity, in a model reference adaptive controller. No cases were found in which Coulomb friction compensation was based on sensed or estimated velocity, suggesting that problems arise when the abrupt transition is based on possibly noisy measurements.

In practice, it seems, the Coulomb friction compensation is always coupled to an adaptive, or to an auto-tuning controller, which adjusts the Coulomb friction parameter. Indications of the range over which friction would vary spanned from tens of percent for machine tools to hundreds of percent in aerospace applications that function over a very wide temperature range. The question of range of friction variations to expect, however, remains a remarkably elusive one.

Although problems with over-compensation were reported, the range of applications to which model-based Coulomb friction compensation has been applied suggests that it is an effective tool. Model-based feedforward schemes using more complete friction models were not reported, but interest was found in such a possibility.

Table lookup compensation has been used in machine tools for some time, to compensate for variable backlash. The tables are filled in lengthy auto-tuning procedures; and certainly, in conjunction with the servo stiffness and tuning procedure, include a factor that is dependent on the friction. Compensation tables tuned specifically to friction were not reported.

Learning control schemes are coming on-line in commercial controllers. These systems identify and compensate for a disturbance signal which certainly includes friction. Because so many applications in industry are repeated, the learning controllers will certainly become wide spread if the performance enhancement is near what the manufacturer's claim.

Although considerable research attention has been given to the analytic treatment of stick slip, engineers in industry working with precision machining, optical tracking, disk head positioning or robotic force control rarely report stick slip as the problematic frictional behavior. Stick slip, where it arises, is most often dispached with appropriate choice of lubricant, or modifications to the mechanism or integral control.

Frictional forces, none-the-less, introduce disturbances such as lost motion at velocity reversal or overshoot due to integrator windup. These disturbances are most important when accurate tracking is required. Thus, attention needs to be given to accurate friction modeling in practical machines, and to aggressive, and in many cases model-based, disturbance rejection.

Many of the engineers surveyed reported reliance upon, and dissatisfaction with, by-guess-and-by-gosh methods used for friction compensation design. By far the most systematic techniques lie in the realm of problem avoidance, e.g. machine design for favorable friction characteristics. When it comes to control, there seemed to be no tools useful even for choosing between the variety of deadbands and saturation functions available for integral control—not to mention tuning these systems. No one contributing to this survey reported an analysis technique consistently giving results useful for application. Of the analysis tools, interest was greatest in extensive simulation. But selection of controller design and parameter tuning remain lengthy, hand procedures.

In contrast to the availability tools for analysis, however, was the frequency with which successful application of the various compensations were reported. In robotics, in precision pointing and positioning devices and in sophisticated machine tools, control functions included specifically for friction compensation are not uncommon.

5. CONCLUSION

Like the elephant encountered by six blind men, friction in machines is a multifaceted phenomenon, incorporating Coulomb and viscous friction, nonlinear friction at low velocities, temporal phenomena and the elasticity of the interface. In any given circumstance, some features may dominate over others, and some features may not be detectable with the available sensing. But all of these phenomena are present all of the time in fluid lubricated metal contacts and, in many cases, present in dry contacts as well. The use of a more complete friction model will extend the range of applicability of analytic results and resolve discrepancies that arise when different investigations are based on different phenomena, each of which dominate under some circumstances.

Analyses based on a complete friction model will be able to consistently account for the observed behavior of systems.

Successful servo-control techniques have been demonstrated which compensate for friction in machines. Stiff control, model-based feedforward control, adaptive strategies and variable structure control have all received attention in both theory and application oriented efforts. Indeed, the compensation techniques seem further advanced than the analysis techniques for controller design. Friction compensators are too often tuned by hand. Tools, as yet, do not exist which can correctly and over a broad range of conditions predict the presence of stick slip, let alone performance in subtler measures, such as tracking error or optimality.

5.1. A Paradigm for Future Work

The opportunity exists for very substantial progress in the control of machines with friction—progress which may ultimately identify and reach the limits of performance achievable with a given machine. But to realize this progress, a new paradigm must be applied.

5.1.1. *Friction models must have a theoretical and experimental foundation*

The mechanics of sliding contacts is certainly less well established than the mechanics of inertial bodies. None-the-less, friction models and frictional behavior must be selected with justification. In some cases in the past, elements of a friction model have been selected because they are able to account for behavior under immediate investigation. While this might be adequate if the slate were blank the tribology, mechanics and lubrication engineering literatures provide a broad theoretical and experimental foundation upon which details of a friction model may be based. In many cases, the experiments presented have been conducted with a level of sensing beyond what is normally available to the controls engineer.

This survey of the tribology literature is certainly not exhaustive; and the integrated friction model presented will certainly be superceded as new ideas and results emerge. Thus the model presented is neither the only model supported by the literature, nor the final word on the question. But alternatives that are proposed or employed as a basis for analysis must be grounded in the science of friction.

5.1.2. *In experimental work, lubrication must be addressed*

The details of lubrication absolutely dominate frictional instability and play a leading role in all other frictional behavior. Yet papers in the controls literature rarely mention the lubricant employed. An experiment reported without reference to the lubricant is not a reproducible experiment. Even if the reference is nothing more than a manufacturer's part number, the possibility will remain open for other investigators to reproduce the results or investigate the role played by the lubricant.

The issue of lubrication is important on more than one level. Lubrication is not normally a concern of the controls engineer; but then stick slip is not normally a concern of the lubrication engineer, who is more often concerned with limiting machine wear and issues of the lubricant environment, service life and delivery. Someone must give attention to the stick-slip properties of the lubricant. One lubrication engineer, interviewed in the course of this project, went so far as to say that stick slip could be eliminated in any application by appropriate choice of lubricant. This perhaps overstates the possibilities, but the point is made that the impact of lubricant cannot be neglected.

5.1.3. *Analyses must be verified*

An analysis tool cannot reach its potential utility until it is established that the tool correctly predicts behavior across the intended range of application. Theory, simulation and experiment are mutually supportive in this regard; and analysis tools should, at a minimum, be verified against many simulations carried out across a range of parameters. While the analysis tool/simulation synergy cannot verify the correctness of a friction model, it can verify that the approximations incorporated within the analysis itself are valid. This is an issue with describing function analysis, for example, where very general results may be possible, but the utility of the results quite limited.

Verification by experiment is more involved than simulation, particularly across a broad range of operating conditions. There are opportunities, however, in the literature to find experiments reported in sufficient detail that new analysis tools may be applied and verified using the results of reported experiments. This is perhaps an appropriate standard for future articles reporting experimental investigations: that the experiments be adequately described so that other investigators can use the results to apply and verify new and developing analyses and models. The work of Brandenburg *et al.* demonstrates this possibility.

5.1.4. *Data showing the repeatability of friction are needed*

One of the assumptions underlying model-based control, and perhaps any control analysis, regards the repeatability of the process in question. Questions of the feasibility of model-based compensation techniques, the importance of adaptive control, and the robustness required of controllers all hinge on the repeatability of friction. And yet the literature is nearly silent on this question. Every report of experimental work with friction and control should include mention of the repeatability of the observed frictional behavior. It is clear that a number of factors influence friction, principal among them constancy of lubrication, temperature of operation and the state of wear. And that these factors will give rise to variations over time or operating conditions, and perhaps to variations which appear random. As reports of repeatability under specific experimental conditions become available, it will be possible to synthesize an informed view of the anticipated repeatability of friction in a mechanism. Such a result will mark very considerable progress in the area of friction and control.

With a tribophysically justified model, rigorously verified tools for analysis, consideration of all of the system attributes that affect friction, and available data regarding such issues as repeatability; there is great possibility of progress in the control of machines with friction. By combining the strengths of existing analysis tools, such as the integrated plant/friction describing function, with a more comprehensive friction model, more general tools may be developed. Other avenues, such as generalizing the projective phase plane techniques of Radcliffe and Southward (1990), may also lead to this goal. Better analysis tools will permit better utilization of demonstrated compensation techniques, such as adaptive, model-based Coulomb friction compensation; and compensation techniques on the horizon, such as impulsive control, will open new possibilities. Better models and analysis tools, coupled with compensation strategies, will lead to precision motion achieved by lower cost machines—the original and continuing objective of servo control.

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