

Journal Review

Automatica

(Survey Paper) The early days of geometric nonlinear control

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- [Roger Brockett](#)
- School of Engineering and Applied Sciences, Harvard University, United States

Abstract

Around 1970 the study of nonlinear control systems took a sharp turn. In part, this was driven by the hope for a more inclusive theory which would be applicable to various newly emerging aerospace problems lying outside the scope of linear theory, and also by the gradual realization that tools from differential geometry, and Lie theory in particular, could be seen as providing a remarkably nice fit with what seemed to be needed for the wholesale extension of linear control theory into a nonlinear setting. This paper discusses an initial phase of the development of geometric nonlinear control, including material on the broader context from which it emerged. We limit our account to developments occurring up to the early 1980s, not because the field stopped developing at that point but rather to limit the scope of the project to something manageable. Even so, because of the volume and diversity of the literature we have had to be selective, even within the given time frame.

Consensus-based distributed cooperative learning control for a group of discrete-time nonlinear multi-agent systems using neural networks

Vol. 50, No.9, Page 2254-2268

- [Weisheng Chen^{a, 1}](#),
- [Shaoyong Hua^a](#),
- [Shuzhi Sam Ge^b](#)
- ^a School of Mathematics and Statistics, Xidian University Xi'an, 710071, PR China
- ^b Department of Electrical and Computer Engineering, National University of Singapore, 117576, Singapore

Abstract

This paper focuses on the cooperative learning capability of radial basis function neural networks in adaptive neural controllers for a group of uncertain discrete-time nonlinear systems where system structures are identical but reference signals are different. By constructing an interconnection topology among learning laws of NN weights in order to share their learned knowledge on-line, a novel adaptive NN control scheme, called distributed cooperative learning control scheme, is proposed. It is guaranteed that if the interconnection topology is undirected and connected, all closed-loop signals are uniform ultimate bounded and tracking errors of all systems can converge to a small neighborhood around the origin. Moreover, it is proved that all estimated NN weights converge to a small neighborhood of their common optimal value along the union of all state trajectories, which means that the estimated NN weights reach consensus with a

small consensus error. Thus, all learned NN models have the better generalization capability than ones obtained by the deterministic learning method. The learned knowledge is also adopted to control a class of uncertain systems with the same structure but different reference signals. Finally, a simulation example is provided to verify the effectiveness and advantages of the distributed cooperative learning control scheme developed in this paper.

Reachability for partially observable discrete time stochastic hybrid systems

Vol. 50, No.8, Page 1989-1998

- [Kendra Lesser[†]](#),
- [Meeko Oishi](#)
- Department of Electrical and Computer Engineering, University of New Mexico, Albuquerque, NM 87131, USA

Abstract

When designing optimal controllers for any system, it is often the case that the true state of the system is unknown to the controller. Imperfect state information must be taken into account in the controller's design in order to preserve its optimality. The same is true when performing reachability calculations. To estimate the probability that the state of a stochastic system reaches, or stays within, some set of interest in a given time horizon, it is necessary to find a controller that drives the system to that set with maximum probability, given the controller's knowledge of the true state of the system. To date, little work has been done on stochastic reachability calculations with partially observable states. The work that has been done relies on converting the reachability optimization problem to one with an additive cost function, for which theoretical results are well known. Our approach is to preserve the multiplicative cost structure when deriving a sufficient statistic that reduces the problem to one of perfect state information. Our transformation includes a change of measure that simplifies the distribution of the sufficient statistic conditioned on its previous value. We develop a dynamic programming recursion for the solution of the equivalent perfect information problem, proving that the recursion is valid, an optimal solution exists, and results in the same solution as to the original problem. We also show that our results are equivalent to those for the reformulated additive cost problem, and so such a reformulation is not required.

Probabilistic model validation for uncertain nonlinear systems

Vol. 50, No.8, Page 2038-2050

- [Abhishek Halder[†]](#),
- [Raktim Bhattacharya](#)
- Department of Aerospace Engineering, Texas A&M University, College Station, TX 77843-3141, United States

Abstract

This paper presents a probabilistic model validation methodology for nonlinear systems in time-domain. The proposed formulation is simple, intuitive, and accounts both deterministic and stochastic nonlinear systems with parametric and nonparametric uncertainties. Instead of hard invalidation methods available in the literature, a relaxed notion of validation in probability is introduced. To guarantee provably correct inference, algorithm for constructing probabilistically

robust validation certificate is given along with computational complexities. Several examples are worked out to illustrate its use.

The Minkowski–Lyapunov equation for linear dynamics: Theoretical foundations

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- [Saša V. Raković^{a, b, 1,}](#)
- [Mircea Lazar^c](#)
- ^a St. Edmund Hall, Oxford University, Oxford, UK
- ^b Supélec, Gif Sur Yvette, France
- ^c Eindhoven University of Technology, Eindhoven, The Netherlands

Abstract

We consider the Lyapunov equation for the linear dynamics, which arises naturally when one seeks for a Lyapunov function with a uniform, exact decrease. In this setting, a solution to the Lyapunov equation has been characterized only for quadratic Lyapunov functions. We demonstrate that the Lyapunov equation is a well-posed equation for strictly stable dynamics and a much more general class of Lyapunov functions specified via Minkowski functions of proper C -sets, which include Euclidean and weighted Euclidean vector norms, polytopic and weighted polytopic $(1, \infty)$ -vector norms as well as vector semi-norms induced by the Minkowski functions of proper C -sets. Furthermore, we establish that the Lyapunov equation admits a basic solution, i.e., the unique solution within the class of Minkowski functions associated with proper C -sets. Finally, we provide a characterization of the lower and upper approximations of the basic solution that converge pointwise and compactly to it, while, in addition, the upper approximations satisfy the classical Lyapunov inequality.

Optimal tracking control of nonlinear partially-unknown constrained-input systems using integral reinforcement learning

Vol. 50, No.7, Page 1780-1792

- [Hamidreza Modares^{1,}](#)
- [Frank L. Lewis](#)
- University of Texas at Arlington Research Institute, 7300 Jack Newell Blvd. S., Ft. Worth, TX 76118, USA

Abstract

In this paper, a new formulation for the optimal tracking control problem (OTCP) of continuous-time nonlinear systems is presented. This formulation extends the integral reinforcement learning (IRL) technique, a method for solving optimal regulation problems, to learn the solution to the OTCP. Unlike existing solutions to the OTCP, the proposed method does not need to have or to identify knowledge of the system drift dynamics, and it also takes into account the input constraints a priori. An augmented system composed of the error system dynamics and the command generator dynamics is used to introduce a new nonquadratic discounted performance function for the OTCP. This encodes the input constraints into the optimization problem. A tracking

Hamilton–Jacobi–Bellman (HJB) equation associated with this nonquadratic performance function is derived which gives the optimal control solution. An online IRL algorithm is presented to learn the solution to the tracking HJB equation without knowing the system drift dynamics. Convergence to a near-optimal control solution and stability of the whole system are shown under a persistence of excitation condition. Simulation examples are provided to show the effectiveness of the proposed method.

3D environmental extremum seeking navigation of a nonholonomic mobile robot

Vol. 50, No.7, Page 1802-1815

- [Alexey S. Matveev](#)^{a,1},
- [Michael C. Hoy](#)^b,
- [Andrey V. Savkin](#)^b.
- ^a Department of Mathematics and Mechanics, Saint Petersburg University, St. Petersburg, Russia
- ^b School of Electrical Engineering and Telecommunications, University of New South Wales, Sydney, Australia

Abstract

A nonholonomic under-actuated robot with bounded control travels in a 3D region. A single sensor provides the value of an unknown scalar field at the current location of the robot. We present a new kinematic control paradigm to drive the robot to the maximizer of the field, which is different from conventionally trying to align the velocity vector with the field gradient. The proposed strategy does not employ gradient estimation and is non-demanding with respect to both computation and motion. Its mathematically rigorous analysis and justification are provided. Simulation results confirm the applicability and performance of the proposed guidance approach.

IEEE T-Ro

Cooperative Visibility Maintenance for Leader–Follower Formations in Obstacle Environments

Vol. 30, No.4, Page 831-844

[Panagou, D.](#) ; Coordinated Sci. Lab., Univ. of Illinois at Urbana-Champaign, Urbana, IL, USA ; [Kumar, V.](#)

Abstract

Vision-based formation control of multiple agents, such as mobile robots or fully autonomous cars, has recently received great interest due to its application in robotic networks and automated highways. This paper addresses the cooperative motion coordination of leader-follower formations of nonholonomic mobile robots, under visibility and communication constraints in known polygonal obstacle environments. We initially consider the case of $N = 2$ agents moving in L-F fashion and propose a feedback control strategy under which L ensures obstacle avoidance for both robots, while F ensures visibility maintenance with L and intervehicle collision avoidance.

The derived algorithms are based on set-theoretic methods to guarantee visibility maintenance, dipolar vector fields to maintain the formation shape, and the consideration of the formation as a tractor-trailer system to ensure obstacle avoidance. We furthermore show how the coordination and control design extends to the case of $N > 2$ agents, and provide simulation results, which demonstrate the efficacy of the control solutions. The proposed algorithms do not require information exchange among robots, but are instead based on information locally available to each agent. In this way, the desired tasks are executed and achieved in a decentralized manner, with each robot taking care of converging to a desired configuration, while maintaining visibility with its target.

An Information Potential Approach to Integrated Sensor Path Planning and Control

Vol. 30, No 4, Page 919-934

[Wenjie Lu](#) ; Dept. of Mech. Eng. & Mater. Sci., Duke Univ., Durham, NC, USA ; [Guoxian Zhang](#) ; [Ferrari, S.](#)

Abstract

This paper presents an information potential method for integrated path planning and control. The method is applicable to unicycle robotic sensors deployed to classify multiple targets in an obstacle-populated environment. A new navigation function, referred to as information potential, is generated from the target conditional mutual information, and used to design a closed-loop stable switched control law. The information potential is shown to obey the properties of potential navigation functions and to enable measurements that maximize the information value over time. The information potential is also used to construct a local roadmap for escaping local minima. The properties and computational complexity of the local roadmap algorithm are analyzed. Numerical simulation results show that the method outperforms other strategies, such as rapidly exploring random trees and classical potential field methods.

Balancing in Dynamic, Unstable Environments Without Direct Feedback of Environment Information

Vol. 30, No 5, Page 919-934

[Nagarajan, U.](#) ; [Yamane, K.](#)

Disney Research, Pittsburgh PA 15213 USA.

Abstract

This paper studies the balancing of simple planar bipedal robot models in dynamic, unstable environments such as seesaw, bongoboard, and board on a curved floor. This paper derives output feedback controllers that successfully stabilize seesaw, bongoboard, and curved floor models using only global robot information and with no direct feedback of the dynamic environment and, hence, demonstrates that direct feedback of environment information is not essential for successfully stabilizing the models considered in this paper. This paper presents an optimization to derive stabilizing output feedback controllers that are robust to disturbances on the board. It

analyzes the robustness of the derived output feedback controllers to disturbances and parameter uncertainties and compares their performance with similarly derived robust linear quadratic regulator controllers. This paper also presents nonlinear simulation results of the output feedback controllers' successful stabilization of bongoboard, seesaw, and curved floor models.

Scalable Multicore Motion Planning Using Lock-Free Concurrency

Vol. 30, No 5, Page 1123-1136

[Ichnowski, J.](#) ; Department of Computer Science, Chapel Hill, NC, USA ; [Alterovitz, R.](#)

Abstract

We present Parallel Rapidly Exploring Random Tree (PRRT) and Parallel RRT* (PRRT*), which are sampling-based methods for feasible and optimal motion planning designed for modern multicore CPUs. We parallelize RRT and RRT* such that all threads concurrently build a single-motion planning tree. Parallelization in this manner requires data structures, such as the nearest neighbor search tree and the motion planning tree, to be safely shared across multiple threads. Rather than relying on the traditional locks which can result in slowdowns due to lock contention, we introduce algorithms that are based on lock-free concurrency using atomic operations. We further improve scalability by using partition-based sampling (which shrinks each core's working dataset to improve cache efficiency) and parallel work-saving (in reducing the number of rewiring steps performed in PRRT*). Because PRRT and PRRT* are CPU-based, they can be directly integrated with existing libraries. In scenarios such as the Alpha Puzzle and Cubicles scenario and the Aldebaran Nao performing a two-handed task, we demonstrate that PRRT and PRRT* scale well as core counts increase, and in some cases they exhibit superlinear speedup.