CAS MA 226 Differential Equations SPRING 2017

Instructor:

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Discussion Class Instructor:

Dr. Lars Ludwig, Institute of Scientific Computing, TU Dresden

Course objective:

In this course we study ordinary differential equations and how to solve them. In contrast to solutions of algebraic and other types of mathematical problems, solutions of differential equations are not simply numbers, but rather *functions or families of functions* that satisfy the differential equation(s) on a certain domain, that is, for all values of the independent variable (often called t for *time*) in the domain.

Solution functions (simply called *solutions* from here on) may be obtained or approximated using analytic, quantitative (in particular geometric), and numerical techniques. All three approaches have their advantages, and we will learn how to choose and use the appropriate technique. We begin by deriving a few classical examples with an emphasis on the phenomena that they model. We then discuss first-order equations using all of the techniques mentioned above. Next we study first-order systems of differential equations. Using techniques from linear algebra, we derive a systematic approach to the solution of linear systems (of differential equations).

Unfortunately, nonlinear systems are more difficult to investigate, but we will learn how to apply what we know about the linear case to the nonlinear case.

Often the solutions of differential equations *cannot* be written as expressions (formulas) involving arithmetic operations and elementary functions (e.g. trig, log, exp functions) only. Whenever we cannot find such *closed-form solutions* (regardless of whether they simply do not exist or whether we are not clever enough to find them), we might have to resort to numerical methods. For the so-called "integration" of differential equations we will study some simple numerical solution techniques.

Our overall goal is to be able to say as much as possible about the properties and the behavior of solutions of differential equations even if the solutions themselves cannot be expressed in closed-form (which is the norm in the real world).

IMPORTANT:

You are requested to

1. **activate the computer account you received with your student ID** (you will have to log in and change the password) at your earliest convenience,
2. **log into the OPAL system** at

   `https://bildungsportal.sachsen.de/opal/dmz/`

   and search for the course **TUDMATH SS2017 Differential Equations**, and

3. **register for this course** by signing up for your discussion group (which is on Wednesdays at 1:00 pm). You will then have access to all homework sets, other course material, news related to the course and a discussion forum — and we will be able to reach all of you easily by email.

Attendance at lectures and discussion sessions is mandatory!

Missing class or the discussions even with a serious written excuse more than three times will result in a failure in this course. Missing class or a discussion session without a serious written excuse more than once will also result in a failure.

Lectures:	Monday	1:00 pm – 2:30 pm	WIL C 133
	Thursday	11:10 am – 12:40 pm	WIL C 133
Discussion:	Wednesday	1:00 pm – 2:00 pm	WIL C 203

Textbook: Blanchard, Devaney, Hall: *Differential Equations.* **4th edition**, Brooks/Cole – CENGAGE Learning, 2012, ISBN 0-495-56198-3 .

Further suggested readings:

1. P. Deuflhard, F. Bornemann: *Scientific Computing with Ordinary Differential Equations.* Texts in Applied Mathematics, Springer Verlag, New York, 2010, ISBN 1-441-93011-6 .
2. R. Redheffer: *Introduction to Differential Equations.* Jones and Bartlett, 1992, ISBN 0-86720-289-0.
3. W. Walter: *Ordinary Differential Equations.* Graduate Texts in Mathematics, Springer Verlag, New York, 1998, ISBN 0-387-98459-3.

Examination/Grading:

We will have one **midterm** exam during normal lecture time. It will be held on **Thursday, May 18th, 2017**.

The **final** exam is scheduled for **Monday, July 10th, 2017**. It is Boston University's policy that you must take the final at the scheduled time.

In addition to the midterm and the final, you will be required to submit written work during the semester. This work will include solutions to **weekly homework sets** and **one project report**. You may also be asked to present your solution of a homework problem on the blackboard in your discussion session.

Normally, homework problems will be assigned during lecture on Monday. You must hand in your own solutions of these before or at the beginning of the discussion session the following week, i.e. no later than 1:00 pm on Wednesday.

Your final grade will be determined by applying the following weighting scheme:

Homework **30%**
Project report **10%**
Midterm exam **20%**
Final exam **40%**

Course schedule and contents based on the textbook

Week	Time	Lecture	Section	Discussion
1	03/04/17	What is an ODE, Examples	1.1	05/04/17
	06/04/17	Separation of Variables	1.2	05/04/17
2	10/04/17	Slope Fields, Nonanalytic Techniques	1.3 + 1.4	12/04/17
	13/04/17	Existence & Uniqueness of Solutions	1.5	12/04/17
3	17/04/17	**Holiday** (no lecture)		19/04/17
	20/04/17	Equilibria and Bifurcations	1.6 + 1.7	19/04/17
4	24/04/17	Linear Problems, Undetermined Coefficients	1.8	26/04/17
	27/04/17	Integrating Factors	1.9	26/04/17
5	01/05/17	**Holiday** (no lecture)		03/05/17
	04/05/17	First-Order Systems and their Geometry	2.1 + 2.2	03/05/17
6	08/05/17	Damped Harmonic Oscillator, Decoupling	2.3 + 2.4	10/05/17
	11/05/17	Existence, Uniqueness and Chaos	2.6 + 2.8	10/05/17
7	15/05/17	Explicit Numerical Methods	2.5 + 7.x	16/05/17
	18/05/17	**MIDTERM EXAM**		16/05/17
8	22/05/17	Linear Systems	3.1 + 3.2	24/05/17
	25/05/17	**Holiday** (no lecture)		24/05/17
9	29/05/17	Phaseplane for various Eigenvalues	3.3	31/05/17
	01/06/17	Complex Eigenvalues	3.4	31/05/17
10	05/06/17	**Holidays** (no lecture and no discussion)		07/06/17
	08/06/17	**Holidays** (no lecture and no discussion)		07/06/17
11	12/06/17	Repeated and Zero Eigenvalues	3.5	14/06/17
	15/06/17	Second-Order Linear Equations	3.6 + 3.7	14/06/17
12	19/06/17	Forced Harmonic Oscillators	4.1 + 4.2	21/06/17
	22/06/17	Resonance	4.3 + 4.4	21/06/17
13	26/06/17	Suspension Bridges & Chaotic Systems	4.5 + 2.8	28/06/17
	29/06/17	Equilibrium Point Analysis for Nonlinear Systems	5.1 + 5.2	28/06/17
14	03/07/17	Hamiltonian and Dissipative Systems	5.3 + 5.4	05/07/17
	06/07/17	Periodic Forcing and Chaos	5.6	05/07/17
15	10/07/17	**FINAL EXAM**		
	13/07/17	... *other finals* ...		