Real-time experimental control of a system in its chaotic and nonchaotic regimes

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Current model-independent control techniques are limited, from a practical standpoint, by their dependence on a precontrol learning stage. Here we use a real-time, adaptive, model-independent (RTAMI) feedback control technique to control an experimental system — a driven magnetoelastic ribbon — in its nonchaotic and chaotic regimes. We show that the RTAMI technique is capable of tracking and stabilizing higher-order unstable periodic orbits. These results demonstrate that the RTAMI technique is practical for on-the-fly (i.e., no learning stage) control of real-world dynamical systems.

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Model-independent chaos control techniques, the first of which was developed by Ott, Grebogi, and Yorke [1], have been applied to a wide range of physical and physiological systems [2–11]. Recently, similar techniques have been developed to stabilize underlying unstable periodic orbits (UPO’s) in nonchaotic dynamical systems [12–18]. In general, model-independent control techniques use feedback perturbations to stabilize a dynamical system about one of its UPO’s. In contrast to traditional control techniques (which require knowledge of a system’s governing equations), model-independent techniques are inherently well-suited for “black-box” systems because they extract all necessary control information from a premeasured time series. The flexibility of model independence in current dynamical control techniques, however, does not come without limitations. The precontrol time-series measurement and the corresponding system-dynamics estimation comprise a “learning” stage. For some real-world systems (e.g., cardiac arrhythmias), however, unwanted dynamics must be eliminated quickly, and thus the time required for a learning stage may be unavailable.

Recently, a real-time, adaptive, model-independent (RTAMI) control technique, was developed [19] to stabilize flip-saddle UPO’s in chaotic and nonchaotic dynamical systems that can be described effectively by a unimodal one-dimensional map. Because the RTAMI technique does not require a precontrol learning stage (i.e., it operates in real time) it is practical for on-the-fly control of dynamical systems. In Ref. [19], the RTAMI technique was successfully applied to a wide range of model systems in their nonchaotic and chaotic regimes. Here, we apply the RTAMI control technique to an experimental system — a driven magnetoelastic ribbon — in its nonchaotic and chaotic regimes.

The RTAMI technique is designed to stabilize the flip-saddle unstable periodic fixed point \( \xi^* = [x^*, x^*]^T \) (where superscript \( T \) denotes transpose and \([x^*, x^*]^T\) is a \(2 \times 1\) column vector) of a system that can be described effectively by a unimodal one-dimensional map \( x_{n+1} = f(x_n, p_n) \), where \( x_n \) is the current value (scalar) of one measurable system variable, \( x_{n+1} \) is the next value of the same variable, and \( p_n \) is the value (scalar) of an accessible system parameter \( p \) at index \( n \). The control technique perturbs \( p \) such that \( p_n = \bar{p} + \delta p_n \), where \( \bar{p} \) is the nominal parameter value, and \( \delta p_n \) is a perturbation [3,4,20–22] given by

\[
\delta p_n = \frac{x_n - x_n^*}{g_n},
\]

where \( x_n^* \) is the current estimate of \( x^* \), and \( g_n \) is the control sensitivity \( g \) at index \( n \). The ideal value of \( g \) is the sensitivity of \( x^* \) to perturbations: \( g_{\text{ideal}} = \delta x^*/\delta p \). As described in Ref. [23], control can be achieved for nonideal values of \( g \) in the range \(|g|_{\text{min}} \leq |g| \leq |g|_{\text{max}} \). (Prior to control, it is not possible to determine \( g_{\text{min}} \) or \( g_{\text{max}} \) without an analytical system model or a learning stage.)

As shown in Fig. 1, the current state point \( \xi_n \) would move,

FIG. 1. First-return map showing that \( \delta p_n \) [Eq. (1), with \( g = g_{\text{ideal}} \)] shifts the map from \( f(x_n, p_n) \) to \( f(x_n, p_n + \delta p_n) \) such that the next system state point is forced to \( \xi_{n+1} = \xi^* \), rather than to its expected position \( \xi_{n+1} \). These data, shown for illustrative purposes, are from simulations of the Belousov-Zhabotinsky chemical reaction.
in the absence of a perturbation (i.e., $\delta p_n = 0$), to $\xi_{n+1}$ (via the dotted arrow). However, the control perturbation of Eq. (1) (corresponding to $g = g_{\text{ideal}}$) shifts $f(x_n, p_n) \rightarrow f(x_n, p_n + \delta p_n)$ such that $x_n$ maps to $x_{n+1}^g = x^*$, instead of $\hat{x}_{n+1}$. On the first-return map, this shift appears as the movement of $\xi_n$ to $\xi_n^g$ (via the solid vertical arrow for Fig. 1). When the map is returned to $f(x_n, p_n)$ for the next iteration, the next state point will be $\xi_{n+1}^g = \xi^*$, as desired for control. In a physical system, due to noise, measurement errors, and the instability of $\xi^*$, perturbations are required at each iteration to hold $\xi_n$ within the neighborhood of $\xi^*$.

Learning-stage dependent techniques use static values for $x^*$ and/or $g$, as estimated from a precontrol time-series measurement. In contrast, the RTAMI technique repeatedly estimates $x^*$ and $g$. In addition to eliminating the need for a learning stage, this adaptability allows for the control of non-stationary systems. When control is initiated, $g$ can be set to an arbitrary value (with the restriction that the sign of $g$ must match that of $g_{\text{ideal}}$; if the signs do not match, control will fail). After each measurement of $x_n$, $x^*$ is estimated using

$$x_n^* = \sum_{i=0}^{N-1} \frac{x_{n-i}}{N},$$  

where $N$ is the number of past data points included in the average [24]. Equation (2) converges to $x^*$ because consecutive $x_n$ alternate on either side of $x^*$ due to the flip-saddle nature of $\xi^*$.

At each iteration, after $x^*$ is re-estimated via Eq. (2), the RTAMI technique evaluates whether the estimate of $g$ should be adapted. The value of $g$ is not adapted if the desired control precision $\epsilon$ has been achieved. Control precision has not been achieved if

$$|x_n - x_n^*| > \epsilon$$  

is satisfied by at least $L$ data points out of the $N$ previous data points, where $x_n^*\equiv x_n$ is the estimate of $x^*$ that was targeted for a given $x_n$. The $L/N$ factor is used [instead of a single evaluation of Eq. (3)] to reduce the influence of noise and spurious data points.

If the desired control precision has not been achieved [i.e., Eq. (3) has been satisfied by at least $L$ data points out of the $N$ previous data points], then the magnitude of $g$ is adapted in accordance with the expected perturbation dynamics [19]. If $g = g_{\text{ideal}}$, then the perturbation moves the state point from its current position $\xi_n$ to $\xi_n^g$ (as in Fig. 1). If $|g|$ is too large (i.e., $\delta p$ is too small), then the state point moves from its current position $\xi_n$ to a position closer to $\xi^*$ than would be expected without a perturbation. If $|g|$ is too small (i.e., $\delta p$ is too large), then the state point moves from its current position $\xi_n$ to a position on the same side of the line of identity. (This is in contrast to the expected alternation, due to the flip-saddle nature of $\xi^*$, of consecutive state points on either side of the line of identity.) The criterion

$$\text{sgn}(x_n - x_{n-1}) = \text{sgn}(x_{n-1}^* - x_{n-2})$$  

is satisfied when two consecutive state points ($[x_{n-1}, x_{n-2}]$ and $[x_{n-1}, x_{n-2}]$) lie on the same side of the line of identity. The RTAMI technique increases the magnitude of $g$ (i.e., $g\rightarrow g_{\text{new}}$) by a factor of $\rho$ if Eq. (4) is satisfied for at least $L$ data points out of the $N$ previous data points. As with the evaluation of control precision [Eq. (3)], the $L/N$ factor is used [instead of a single evaluation of Eq. (4)] to reduce the influence of noise and spurious data points. If the magnitude of $g$ is not increased [as dictated by Eq. (4)], then the magnitude of $g$ is decreased if $\xi_n$ is not converging rapidly (at a rate governed by $r$) to $\xi^*$. Specifically, the magnitude of $g$ is decreased (i.e., $g_{n+1} = g_n / \rho$) if

$$\frac{1}{N} \sum_{i=0}^{N-1} \frac{|x_{n-i} - x_{n-i}^*|}{|x_{n-i-1} - x_{n-i-2}|} < r\%.$$  

Equation (5) is satisfied if, on average, the distance $|x_{n-i} - x_{n-i-1}^*|$ between a given data point $x_{n-i}$ and its corresponding fixed-point estimate $x_{n-i}^*$ is not at least $r\%$ smaller than the distance $|x_{n-i-1} - x_{n-i-2}^*|$ between the previous data point $x_{n-i-1}$ and the previous fixed-point estimate $x_{n-i-2}^*$. If neither Eq. (4) nor Eq. (5) is satisfied, then $g$ is not adapted because $x$ is properly approaching the estimate of $x^*$.

The experimental system we considered [25] consists of a gravitationally buckled magnetoelastic ribbon driven parametrically by a sinusoidally varying magnetic field. The ribbon is clamped at its lower end and its position $x$ is measured once per drive period at a point a short distance above the clamp. The ribbon’s Young’s modulus can be varied by applying an external magnetic field. The applied magnetic field

![Diagram](image)

**FIG. 2.** (a) $x_n$, (b) $H_{\text{dc}}$, and (c) $g_n$ versus drive cycle $n$ for a RTAMI control trial of the chaotic magnetoelastic ribbon. The respective control stages are annotated in (a), (b), and (c).
is $H_{\text{app}} = H_{\text{dc}} + H_{\text{ac}} \sin(2\pi ft)$, where $H_{\text{dc}}$ is the dc-field amplitude, $H_{\text{ac}}$ is the ac-field amplitude, and $f$ is the ac-field frequency. To apply the RTAMI control technique to the magnetoelastic ribbon, $H_{\text{dc}}$ was used as the control parameter [i.e., $p_n = H_{\text{dc}}$ such that $H_{\text{dc}} = \bar{H}_{\text{dc}} + \delta H_{\text{dc}}$].

Figure 2 shows a typical RTAMI control trial (with $\bar{H}_{\text{ac}} = 0.302$ Oe, $H_{\text{ac}} = 1.037$ Oe, $f = 0.9$ Hz, $N = 10$, $\epsilon = 0.01$, $L = 3$, $r = 5\%$, and $\rho = 1.025$). At $n = 250$, following a period of chaotic ribbon motion (corresponding to a two-piece attractor), control of the unstable period-1 fixed point was activated. The initial control perturbations [Fig. 2(b)] were too small (because $|g|$ was too large) to move the state point into the neighborhood of the fixed point (and hold it within that neighborhood) [Fig. 2(a)]. Thus, $|g|$ was decreased [as dictated by Eq. (5)] until the magnitude of the perturbations increased and the state point converged to the unstable period-1 fixed point. Note that although Eq. (1) is only valid in the linear region of $\xi^b$, the value of $g$ required to pull $\xi_n$ into the neighborhood of $\xi^b$ was also suitable for the stabilization of $\xi^b$ (i.e., $|g|_{\text{max}} = |g|_{\text{mean}}$). Also note that it is possible that the large parameter perturbations required to move $\xi_n$ into the neighborhood of $\xi^b$ could alter $p$ to a regime where $\xi^b$ is stable. However, because of the flip-saddle nature of $\xi^b$, consecutive perturbations (excluding those influenced by noise or when $|g|$ is too small) are opposite in polarity, thereby ensuring that a parameter-regime change into the stable regime of $\xi^b$ is followed by a parameter-regime change away from the stable regime of $\xi^b$. Thus, the large perturbations should not be mistaken for a parameter-regime shift that is used to capture $\xi^b$ when it is stable, in order to drag it back into the unstable regime.

Stabilization was maintained until $n = 1250$, when control was deactivated. At $n = 1500$, stabilization of the system’s unstable period-2 fixed point was activated [26]. Period-2 stabilization was quickly achieved by updating the estimates for $x_n^a$ and $g$ and applying control interventions at every other iterate rather than at every iterate.

Figure 3 shows a RTAMI control trial (with $\bar{H}_{\text{ac}} = 0.258$ Oe, $H_{\text{ac}} = 1.037$ Oe, $f = 0.9$ Hz, $N = 10$, $\epsilon = 0.00$ [27], $L = 3$, $r = 5\%$, and $\rho = 1.025$) that demonstrates: (i) on-the-fly control of a system that is switched rapidly between different parameter regimes and (ii) stabilization of UPO’s which underlie stable higher-period orbits in a nonchaotic system. At $n = 250$, following a period of stable period-4 ribbon oscillation, control of the system’s underlying unstable period-2 fixed point was activated. After $|g|$ was decreased, as dictated by Eq. (5), period-2 stabilization was achieved and maintained until $n = 500$, when the control target was switched from the underlying unstable period-2 fixed point to the underlying unstable period-1 fixed point. Period-1 stabilization was maintained until $n = 750$, when control was deactivated. At $n = 1000$, period-1 stabilization was reactivated directly from the stable period-4 oscillation. Period-1 stabilization was maintained until $n = 1250$, when control was deactivated and $\bar{H}_{\text{ac}}$ was changed to $\bar{H}_{\text{dc}} = 0.210$ Oe, corresponding to a stable period-2 oscillation. At $n = 1500$, period-1 stabilization was activated directly from the stable period-2 oscillation. Note that the magnitude of $g$ increased and decreased [Fig. 3(c)], as dictated by Eqs. (4) and (5), for the different unstable periodic fixed points and parameter regimes.

In addition to controlling a dynamical system in its non-
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Corresponding regime control failure resulted from the fact that the value of \( r \) required initially to move \( \xi_n \) into the neighborhood of \( \xi^* \) was suitable for control (i.e., \(|g|_{\min} \leq |g| \leq |g|_{\max}\)). When \( H_{dc} > 0.311 \) Oe, \(|g|_{\min} \leq |g| = 0.144 \) Oe was required to pull \( \xi_n \) into the neighborhood of \( \xi^* \). Thus, once \( \xi_n \) entered the neighborhood of \( \xi^* \), oversized perturbations [28] were delivered that promptly repelled \( \xi_n \) from \( \xi^* \) before the magnitude of \( g \) could be increased.

In this paper, we have shown that the RTAMI technique can be used to control an experimental system. Specifically, we have controlled the motion of a driven magnetoelastic ribbon in its period-2 regime, period-4 regime, and chaotic regime. We have demonstrated that the RTAMI control technique is capable of (i) on-the-fly control as a system is switched between parameter regimes, (ii) stabilizing higher-order UPO’s, and (iii) tracking a UPO through multiple bifurcations. These results demonstrate that the RTAMI technique is versatile and practical for real-time control of real-world systems.

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[26] Period-2 control can fail for systems which have two unstable period-2 fixed points that are characterized by g’s with opposite signs. In such systems, failure will occur if the initial value of g for the targeted fixed point has the sign corresponding to g for the other fixed point.
[27] Setting \( \varepsilon = 0.00 \) is equivalent to eliminating Eq. (3) from the RTAMI algorithm. This simplifies the real-world applicability of the technique by eliminating a parameter (i.e., \( \varepsilon \)).
[28] The perturbations were oversized because \(|g|\) was too small for the neighborhood of the fixed point. This resulted in consecutive state points that were forced onto the same side of the line of identity.